Capital and Goods Market Integration and the Inequality of Nations

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Editorial

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Abstract

A 2-country model with two groups of agents, workers and capitalists is presented in which economic integration results in an initial phase of catch-up, where the less industrialised country experiences the rise in both capital and labour income. Then, after a certain level of integration has been reached, the less industrialised country is completely de-industrialised. This has detrimental effects on the income of this country’s workers, but the capital owners of this country gain from specialisation, as do the workers in the industrialised country. Both the capital and the goods markets are subject to imperfections. The structure of the equilibrium sets during integration is characterised completely.

Keywords
Globalisation, trade, market imperfections, integration

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F12, F15
Comments
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1 Introduction

This paper extends the model described in Krugman and Venables (1995) and, in a slightly different way, in Fujita, Krugman and Venables (1998). It includes capital as an additional factor of production and it identifies capital owners as a distinct group of agents. In addition to transportation costs, the paper introduces imperfect capital mobility as a further wedge between countries.

Krugman and Venables (1995) present a 2-country model that predicts a U-shaped pattern of real incomes during successive phases of economic integration of core and periphery regions. This pattern is meant to rationalise changing positions in the discussion of the effects of (global) economic integration. In the 1960s and 1970s many people argued that integration would harm the developing countries, whereas in recent years many people have come to believe that the difficult economic situation in the OECD countries has been affected by market integration that has favoured newly industrialised countries, as for example the countries of South East Asia.

The modelling device that generates this outcome is a model with intermediate goods and transportation costs, which is based on Krugman (1980) and Ethier (1982).\(^1\) The usage of intermediate goods in the manufacturing sector creates linkages. First, so-called forward linkages: a greater variety of intermediate products available reduces the production costs. The costs are lowered the more, the more goods are produced in the same country, because they are not subject to transportation costs. Second, there are backward linkages in which an increase in the share of manufacturing in one country, i.e. more firms, increases the demand for all the varieties produced in this country through higher demand for intermediate usage by all the other firms in the same country.

To this, we add imperfectly mobile capital owned by another group of agents named capitalists. This simplified set-up perfectly splits investment from consumption decisions. The capitalists are only interested in maximising their return on investment and do not take into account the effects of their decisions on the workers' income.\(^2\) The restrictions to capital mobility in the model shall grossly modo reflect all the restrictions that are prevalent in the real world, like quantitative restrictions to capital import and export,\(^3\) costs of monitoring foreign investment,\(^4\) and maybe risk considerations

\(^1\) A variant of this model has been used to analyse issues of industrial clustering, see Krugman and Venables (1996).

\(^2\) It may be regarded as a nice interpretation to consider the capitalists as a financial sector that is investing the savings of the consumers and does not take into account the repercussions of their decisions, which mostly arise from the effects of investment on labour income, on the owners of the capital. This interpretation comes rather close to the popular fear that a globalised financial sector is controlling the real sides of the economies without any control. If it appears more convenient, one can think of two classes of individuals, proletarians and capitalists.

\(^3\) Up to some years ago many countries did have tight quantity constraints on the import and export of capital. And if one is thinking over longer horizons of course all the former communist countries have more or less been isolated from the western financial markets.

\(^4\) Like having a lawyer or an investment bank that is taking charge of the foreign investment.
associated with foreign investment.\footnote{Our model has no risk. One can, however, still regard risk as a factor that reduces the effective, or average, rate of return on investment.}

Paralleling the approach of Krugman and Venables, we will think of a sequence of equilibria parameterised by declining market restrictions, to assess the effects of continuing integration. The sequences may be best thought of as sequences of steady states, which again shows that the model is intended to analyse long-term developments.

It turns out that the model has a continuum of equilibria. All the equilibrium sets are characterised and analytical expressions for all equilibria are given for all the variables except the price indices (and therefore the real wages). The numerical computation of the price indices is also described.

Following the popular idea that the (uncontrollable) financial sector is controlling the worldwide evolution of economies, the sequence of equilibria that is analysed is the capitalists' income maximising sequence. This means that given the state of integration the capitalists (of country 1) have the power to choose the equilibrium that gives them the largest income.\footnote{Due to the fact that there is a continuum of equilibria an equilibrium has to be chosen at each point of the integration process. In our model the most intrinsically consistent choice is to choose the equilibrium that maximises the return on investment. The results do not change qualitatively if the worldwide income of capitalists is taken as the criterion for selecting equilibria. Besides the above, "natural" candidates would be equilibria with a specified allocation of revenue between capitalists and workers, or equilibria maximising a social welfare function at each point of the sequence. It may also be an interesting task to compare these equilibrium sequences.}

This leads to the following pattern of industrial agglomeration: In a situation with large restrictions one country is specialised in manufacturing whereas the other country is active on both sectors. During the first stage of integration, the manufacturing sector is shrinking in the industrialised country and growing in the other one. This corresponds to the situation that is seen as a threat to high living standards in industrialised countries today. This phase is characterised by a rising income of workers and capitalists in the country that experiences industrialisation. Later, after the restriction to capital mobility has dropped below a critical value, all the worldwide manufacturing is concentrated in one country. This has severe adverse effects on real income of the workers in the de-industrialised country, while it has positive effects on the real income of the workers in the country that has captured the whole manufacturing sector. During the course of integration, the income of the capital owners of the two countries is equalised. In this model integration thus favours the owners of the mobile factor capital in contrast to the immobile factor labour. Thus workers living in the country which has managed to attract the manufacturing sector therefore happen to profit from the process of economic integration.

*Section 2* describes the model. In *Section 3* the equilibrium structure is described, in
Section 4 the sequence of equilibria during ongoing integration is discussed and Section 5 presents some final remarks. There are two appendices. In Appendix A a special case of an equilibrium that can be treated analytically is shortly discussed and Appendix B reports all the comparative statics results.

2 Description of the Model

In my presentation I closely follow the lines of Fujita, Krugman and Venables (1998). The world is assumed to consist of two economies which will be labelled 1 and 2 throughout the paper. Subscripts are used to refer to the countries. The two countries are identical with respect to their endowments of labour, capital and the available technology. In each country there are two types of agents: consumers (workers) and capitalists. The countries are also identical with respect to the preferences of consumers and capitalists. The following description omits country indices if the described relationship holds for both countries. Where necessary, countries will be distinguished.

The workers in the economies all share the same preferences

\[ V = M^\mu A^{1-\mu} \]  

(1)

where \( A \) is the consumption of the agricultural good and \( M \) is a quantity index of manufactured goods consumption. Both countries are endowed with a total of \( L = 1 \) units of labour. Labour is perfectly mobile between the two sectors and completely immobile across the two countries. The agricultural good is taken to be the numeraire, i.e. its price is set to one. \( \mu \) is the constant expenditure share of manufactures. \( M \) is defined over a continuum of varieties of manufactured goods

\[ M = \left( \int_0^N m(i)^\rho di \right)^{1/\rho}, 0 < \rho < 1 \]  

(2)

where \( m(i) \) denotes the consumption of the variety \( i \) and \( N \) is the range of varieties produced. \( \rho \) is a parameter reflecting the preference for variety. The smaller \( \rho \) the larger the preference for variety. This formulation has been popularised by Dixit and Stiglitz (1977).

The representative consumer’s problem, given income \( Y \) and all the prices \( p(i) \), can be solved in two steps, because the consumer preferences are separable between agriculture and manufacturing. First one has to minimise the expenditures necessary to obtain \( M \), for any given value of \( M \). This results in demands \( m(j) = \frac{p(j)^{1/\rho - 1}}{\int_0^N p(i)^{1/\rho - 1} di} M \) for all the different varieties \( m(j) \). Multiplying \( m(j) \) with its price \( p(j) \) and integrating over all the varieties yields

\[ \int_0^N p(j)m(j) dj = \left[ \int_0^N p(i)^{\rho/\rho - 1} di \right]^{\rho - 1} M \]  

(3)

The term multiplying \( M \) on the right hand side of equation (3) can be interpreted as a price index. This interpretation implies that the expenditure on manufacturing is equal
to the price index of the composite good times the quantity index of consumption of manufactured goods.

Defining \( \sigma = \frac{1}{1-\alpha} \), gives \( P = \left[ \int_0^N p(i) \left( \frac{\sigma}{\alpha} \right) \, di \right]^{\alpha-1} = \left[ \int_0^N p(i)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}} \). From the Cobb-Douglas preferences, we immediately know the allocation of income between \( M \) and \( A \): \( M = \mu Y/P \) and \( A = (1 - \mu)Y \). Substituting this expression shows that the consumer demand for each of the varieties of manufactures is given by \( m(j) = \mu Y p(j)^{-\sigma}, \forall j \in [0, N] \). The consumers’ sole source of income is labour, so \( Y = w \), where \( w \) is the nominal, i.e. measured in units of the agricultural good, wage. Real wage \( \omega \), i.e. nominal wage divided by the consumer price index, is given by

\[
\omega = wP^{-\mu}
\] (4)

The agricultural good is tradable without transportation costs. This implies that there is one worldwide market for the agricultural good. The manufactured goods are subject to transportation costs. For simplicity these costs are modelled as iceberg transport costs: only a fraction \( \frac{1}{T}, T \geq 1 \), of a unit shipped, arrives in the other country. With a simple specification like this, we avoid the necessity to model an additional transportation industry. So, if a specific variety \( k \) is produced in country 1, its prices in 1 and 2 are given by \( p_2(k) = p_1(k)T \).

In equilibrium all varieties produced in one country must sell at the same price, because they enter utility symmetrically and are produced with the same technology. Thus, the manufacturing price indices can be expressed as

\[
P_1 = (n_1 p_1^{-(\sigma-1)} + n_2 (p_2 T)^{-(\sigma-1)})^{-\frac{1}{\sigma-1}}
\]

\[
P_2 = (n_1 (p_1 T)^{-(\sigma-1)} + n_2 p_2^{-(\sigma-1)})^{-\frac{1}{\sigma-1}}
\]

where \( n_i \) denotes the "number" of varieties produced in country \( i \) and \( P_1 \) stands for the price indices in the two countries.

The capitalists in the two economies are endowed with \( W_1 = W_2 = \frac{\beta}{n_1 - \beta \gamma} \) units of capital. In this paper we will use \( K_i \) for the amount of capital employed in country \( i \).\(^8\)

The capitalists’ sole objective is maximising the return on capital.\(^9\) Capital is imperfectly mobile. In this paper we model the restrictions on capital mobility in a fashion similar to the transportation costs prevalent on the manufactured goods markets. In each country there is only one capital market, i.e. there are no separate markets for capital owned by either domestic or foreign investors. This implies that there is only one interest rate per country. Now for each unit of capital invested abroad an investor

\(^7\)The parameters \( \beta \) and \( \gamma \) will appear later in the production function for manufactures. This choice of endowment with capital turns out to be very convenient when solving for equilibrium.

\(^8\)This need not be equal to \( W_i \), the amount of capital owned by the capitalists resident in the country \( i \). Let \( W_{ij} \) denote the amount of capital owned by capitalists resident in \( i \) that is invested in the country \( j \). Then of course \( K_1 + K_2 = (W_{11} + W_{21}) + (W_{12} + W_{22}) = W_1 + W_2 \) has to hold (in equilibrium).

\(^9\)With this simple specification the capitalists are only interested in obtaining as much of the numeraire, i.e. of the agricultural good, as possible. Therefore there are no demand effects on the manufactured goods market from this side.
does not earn \( r \) units of the agricultural good, but only \( r/U \), with \( U \) being \( \geq 1 \), units. Solving the problem of the representative capitalist (resident in country 1) trivially leads to the following capital supply function:

\[
W_{11}^s = \begin{cases} 
W_1 & \text{if } r_1 > r_2/U \\
\in [0, W_1] & \text{if } r_1 = r_2/U \\
0 & \text{if } r_1 < r_2/U
\end{cases}
\]

The incomes of capitalists resident in country 1 and 2 are given by

\[
C_1 = r_1 W_{11} + \frac{r_2}{U} W_{12} \\
C_2 = \frac{r_1}{U} W_{21} + r_2 W_{22}
\]

Next we turn to the description of the behaviour of the producers in the economy. Agriculture uses only labour as an input and is assumed to be a perfectly competitive CRS sector. For simplicity, the agricultural good is produced with a linear technology, where normalisation is such that one unit of labour is producing one unit of the agricultural good. The economy-wide production of agriculture is then given by \( A = (1 - \lambda) \), where \( \lambda \) denotes the share of labour of the economy devoted to manufacturing.

Manufacturing is a monopolistically competitive IRS sector that uses labour, capital and intermediate goods as inputs to produce the varieties of manufactures. Technology is Cobb-Douglas and assumed to be identical for all the varieties and both countries. The scale effects thus occur at the level of production of the different varieties. For this reason, and because we assume free entry and exit and an unlimited potential of varieties, two competitive firms won’t choose to produce the same variety. The production function of an operating firm is given by

\[
F + cq = Dl^{1-\beta-\gamma} k^\beta \int_0^N g(i)^\rho di^{\gamma/\rho}
\]

where \( q \) is the quantity produced, \( l \) the amount of labour used, \( k \) the amount of capital used; the integral term represents the quantity index of the manufactured goods used as intermediates. \( g(i) \) denotes the quantity used of variety \( i \). With this specification of intermediate demand of the firm sector, we find that the intermediate composite good used as an input in production has the same composition as the aggregate of manufactured varieties demanded by the consumers (in the same country). This set-up is convenient when deriving total demand for manufactures. \( F \) stands for the fixed costs that an operating firm incurs, \( c \) is the marginal input requirement.\(^\text{10}\)

Firms are profit maximising, price takers on their input markets and price setters on their output market. Furthermore, when taking their decision, firms consider the price index \( P \) fixed. The profit function of a firm is given by

\[
\Pi(i) = p(i)q(i) - w^{1-\beta-\gamma} r^\beta P^\gamma (F + cq(i))
\]

\(^{10}\)The constant \( D \) is set to \( \left( \frac{1}{1-\beta-\gamma} \right)^{1-\beta-\gamma} \left( \frac{1}{\rho} \right)^{\rho/\gamma} \).
where \( p(i) \) denotes the price of the produced variety, \( q(i) \) the quantity produced and \( w \) and \( r \) are the wage and interest rates.

Taking into account the pricing behaviour of the monopolistic firm and noting that \( \sigma \) is the elasticity of demand perceived (keeping \( P \) fixed), \( p(1 - \frac{1}{\sigma}) = w^{1-\beta-\gamma}P^{-\beta}P^{-\gamma}c \) has to hold. Choosing units so that \( c = (1 - \frac{1}{\sigma}) \) we obtain

\[
p = w^{1-\beta-\gamma}P^{-\beta}P^{-\gamma}
\]  

(11)

Inserting the price in the profit function yields \( \Pi = w^{1-\beta-\gamma}P^{-\beta}P^{-\gamma}[\frac{q}{\sigma} - F] \). Free entry drives profits down to zero in equilibrium, so that the equilibrium quantity produced by all the firms is \( q^* = F\sigma \).

The technology described above implies that every firm uses all the varieties of manufactured products as intermediates. So the producers, as well as the consumers gain from having a larger worldwide variety of manufactured goods. This is a forward linkage. In addition, as already mentioned in the introduction, the larger the share of varieties produced in one country, the larger the reduction in the price index that the firms (and consumers) experience in this country, because less varieties are subject to transportation costs.

Now we collect all the equations and identities describing equilibrium. The income of the workers \( Y \) is the sum of workers’ earnings in agriculture and in manufacturing

\[
Y = (1 - \lambda) + w\lambda
\]  

(12)

\( Y \) can exceed one only if all labour is devoted to manufacturing and manufacturing pays a wage higher than one. Total expenditure \( E \) on manufactures in the two countries is composed of consumer demand - fraction \( \mu \) of consumer income - and the firms’ intermediate demand - fraction \( \gamma \) of total costs or, equivalent in equilibrium, of total revenue - and is given by

\[
E_i = \mu Y_i + \gamma n_i p_i q_i^4
\]

The above equation can be modified by using the following equilibrium conditions on factor payments that directly follow from the assumption of the Cobb-Douglas technology.

\[
(1 - \beta - \gamma)n_i p_i q_i^4 = w_i \lambda_i
\]

\[
\beta n_i p_i q_i^4 = r_i K_i
\]

\[
\gamma n_i p_i q_i^4 = P_i M_i
\]

where \( K_i \) is the capital employed in country \( i \) and \( M_i \) is the economy wide intermediate demand in \( i \). Now choose units so that \( q_i^* = \frac{1}{1-\beta-\gamma} \), i.e. set \( F \) to \( \frac{1}{\sigma(1-\beta-\gamma)} \). Then \( P_i M_i = \frac{\gamma}{1-\beta-\gamma}w_i \lambda_i \) and \( n_i = \frac{w_i \lambda_i}{p_i} \). The (gross) interest earned on capital in country \( i \) is then \( r_i K_i = \frac{\beta}{1-\beta-\gamma}w_i \lambda_i \). Because of imperfect capital mobility, this only equals the amount that the owners of the capital get paid, if there is no cross border investment or
the restrictions are degenerated, i.e. $U = 1$. Using the above expressions, the expenditure equation can be expressed as

$$E_i = \mu Y_i + \frac{\gamma}{1 - \beta - \gamma} w_i \lambda_i$$

(13)

The above normalisations have been made because the focus is on the allocation of labour between the sectors and on the allocation of capital between the countries, not on the number of firms or the prices.\(^1\)

Inserting the above relations, we can express the price index for country 1 (analogous for country 2) as

$$P_1^{1-\sigma} = (n_1 p_1^{1-\sigma} + n_2 (p_2 T)^{1-\sigma})$$
$$= (w_1 \lambda_1 p_1^{1-\sigma} + w_2 \lambda_2 p_2^{1-\sigma} T^{1-\sigma})$$
$$= (w_1 \lambda_1 (w_1^{1-\beta-\gamma} r_1^{\beta} P_1^{1-\gamma})^{1-\sigma} + w_2 \lambda_2 (w_2^{1-\beta-\gamma} r_2^{\beta} P_2^{1-\gamma})^{1-\sigma} T^{1-\sigma})$$

(14)

Use $r_i K_i = \frac{\beta}{1-\beta-\gamma} w_i \lambda_i$, define $\tilde{K}_i = K_i \frac{1-\beta-\gamma}{\beta}$ to further modify the price index equations to

$$P_1^{1-\sigma} = (w_1^{1-\sigma(1-\gamma)} \lambda_1^{1-\beta-\gamma} K_1^{\beta} P_1^{-\gamma} + w_2^{1-\sigma(1-\gamma)} \lambda_2^{1-\beta-\gamma} K_2^{\beta} P_2^{-\gamma} T^{1-\sigma})$$
$$P_2^{1-\sigma} = (w_1^{1-\sigma(1-\gamma)} \lambda_1^{1-\beta-\gamma} K_1^{\beta} P_1^{-\gamma} T^{1-\sigma} + w_2^{1-\sigma(1-\gamma)} \lambda_2^{1-\beta-\gamma} K_2^{\beta} P_2^{-\gamma} T^{1-\sigma})$$

(15)

(16)

Noting that $\tilde{K}_1 + \tilde{K}_2 = \frac{1-\beta-\gamma}{\beta} (K_1 + K_2) = \frac{1-\beta-\gamma}{\beta} (W_1 + W_2) = 1$, we see that the quantities $\tilde{K}_i$ are the shares of worldwide capital allocated to country $i$. The demand for a single variety $v$ produced in country one is given by

$$q(v)^D = E_1 P_1^{-\sigma} P_1^{\sigma-1} + E_2 (p_1 T)^{-\sigma} P_2^{\sigma-1} T$$

(17)

The first term is the demand for consumption and intermediate usage from country 1, the second term is the demand from the other country. Now we use the fact that firms sell $q = \frac{1}{1-\beta-\gamma}$ units in equilibrium and the price equation (11) to obtain

$$(w_1^{1-\beta-\gamma} r_1^{\beta} P_1^{1-\gamma})^{\sigma} = (1 - \beta - \gamma) (E_1 P_1^{\sigma-1} + E_2 P_2^{\sigma-1} T^{1-\sigma})$$
$$(w_1^{1-\beta-\gamma} r_1^{\beta} P_2^{1-\gamma})^{\sigma} = (1 - \beta - \gamma) (E_1 P_1^{\sigma-1} T^{1-\sigma} + E_2 P_2^{\sigma-1})$$

(18)

(19)

We will refer to the above equations as the wage-interest equations, since they give combinations of wages and interest rates consistent with zero profits.\(^2\) For our purposes, it is convenient to rewrite these equations as follows

$$(w_1^{1-\beta-\gamma} \lambda_1^{1-\beta-\gamma} K_1^{\beta} P_1^{1-\gamma})^{\sigma} = (1 - \beta - \gamma) (E_1 P_1^{\sigma-1} + E_2 P_2^{\sigma-1} T^{1-\sigma})$$
$$(w_1^{1-\beta-\gamma} \lambda_1^{1-\beta-\gamma} K_2^{\beta} P_2^{1-\gamma})^{\sigma} = (1 - \beta - \gamma) (E_1 P_1^{\sigma-1} T^{1-\sigma} + E_2 P_2^{\sigma-1})$$

(20)

(21)

\(^1\)In this respect we follow the lines of Fujita, Krugman and Venables (1998). We do not refer to the approach in Krugman and Venables (1995).

\(^2\)Fujita, Krugman and Venables derive similar equations without an interest rate term and label them wage equations.
For completeness’ sake, it remains to characterise equilibrium on the agricultural (numeraire) market. The worldwide supply of agricultural goods is given by \((1 - \lambda_1) + (1 - \lambda_2)\). Demand is the sum of two components: Consumption demand, which is given by \((1 - \mu)(Y_1 + Y_2)\), and the (gross) interest payments \(r_1K_1 + r_2K_2\). Equating supply and demand gives

\[
(1 - \lambda_1) + (1 - \lambda_2) = (1 - \mu)(Y_1 + Y_2) + (r_1K_1 + r_2K_2)
\]

Finally, in equilibrium capital demand has to equal capital supply in both countries.\(^{13}\)

### 3 Equilibrium Structure: Wages, Interest Rates and Income

When discussing asymmetric equilibria we will focus on the case where country 1 is the more industrialised country.\(^{14}\) Differences in equilibria may depend on whether both countries have both sectors active and on whether there are cross border capital flows. Of course, in a situation in which all the manufacturing is concentrated in one country, all the capital must have been invested in this country.

Both workers and capitalists gain from a clustering of manufacturing in one country. The reduction of the price index that is implied by this agglomeration increases the real income of workers \(\omega\) and the profitability of firms via reduced expenditures on intermediates and an increased demand for intermediate usage from other firms located in the same country. If the agglomeration effects are strong, this potentially allows firms in the country with the larger manufacturing share to pay higher interest rates than the firms in the other country, which in turn attracts foreign capital. Of course, it is advantageous for capitalists to be resident in the country that is relatively more specialised in manufacturing and has a higher interest rate. Because only in-flowing capital is subject to restrictions on the capital market, the capitalists based in the industrialised country have higher (net) returns than foreign investors.

For all the values of \(T\) and \(U\) the symmetric situation, in which the two countries are identical with respect to the values of all variables, is an equilibrium. The values of all the variables can be easily determined for this equilibrium. First note that \(w_1 = w_2 = 1\) and that therefore \(Y_1 = Y_2 = 1\) have to hold because agriculture is active in both countries. Next, use (22) to determine the symmetric \(\lambda = \frac{\omega(1-\beta-\gamma)}{1-\gamma}\). Thus we derive

\(^{13}\)Using Walras’ law we could skip one market in the description of equilibrium.

\(^{14}\)It is a fairly general feature of the so called New Trade Theory models that there is a substantial amount of indeterminacy due to the usually completely symmetric set-up of the (2-country) models. The models therefore have the ability to generate asymmetries but they do not endogenously explain the reason why one symmetric country attains the favourable position rather than the other. According to our view this is a nice feature of these models because it leaves room for other aspects like culture or history that cannot be modelled directly to serve as explanatory factors of asymmetric evolution.
Now insert the known quantities in (15), (16) or (18), (19) and use $K_1 = K_2 = 1/2$ to obtain

$$P_1^{1-\sigma(1-\gamma)} = P_2^{1-\sigma(1-\gamma)} = \left(\frac{1}{2}\right)^{\beta\gamma} \lambda^{1-\beta\gamma} (1 + T^{1-\sigma})$$  \hspace{1cm} (23)$$

The interest rate is given by $r = 2\lambda = \frac{2\beta(1-\beta-\gamma)}{1-\gamma}$. The capitalists’ income $C$ is equal to $\frac{\beta\gamma}{1-\gamma}$, real wages are given by $\omega = P^{-\mu}$.

We can also prove the existence of a specialised equilibrium. The meaning of specialized depends on the level of manufacturing necessary to satisfy worldwide demand with manufacturing being either solely concentrated in one country or, if there has to be a larger manufacturing sector, in both countries with one country having only manufacturing. As indicated, we will take country 1 to be specialised. To be able to proceed analytically, we will restrict ourselves (at the moment) to an analysis of the case in which all manufacturing goods can be produced by country 1. Thus, we can still use $w_1 = w_2 = 1$ when solving for equilibrium. To solve for the values of the variables in a specialised situation, we proceed as follows. From $w_1 = w_2 = 1$ we immediately derive $Y_1 = Y_2 = 1$. We also know by assumption that $\lambda_2 = 0$ has to hold. Use (22) to determine $\lambda_1 = \frac{2\beta(1-\beta-\gamma)}{1-\gamma}$. For this being smaller (or equal) than one, $\mu < \frac{1-\gamma}{2(1-\beta-\gamma)}$ has to hold, i.e. we have to impose an upper bound on the share of manufactures $\mu$ in consumption to guarantee that worldwide demand for manufacturing can be satisfied by one country. Next substitute the expression for $\lambda_1$ in (13) to obtain $E_1 = \mu \frac{1+\gamma}{1-\gamma}$ and $E_2 = \mu$. Now solve for the price indices by inserting in (15) and (16) to get $P_1 = \lambda_1^{1-\beta\gamma(1-\gamma)}$ and $P_2 = P_1 T$. The equation $P_2 = P_1 T$ is derived from the fact that in the situation described all the varieties of manufactures have to be shipped from 1 to 2. Real wages are given by $\omega_1 = P_1^{-\mu}$ and $\omega_2 = \omega_1 T^{-\mu}$. The capitalists’ income is given by $C_1 = r_1 W_1 = \frac{\beta\gamma}{1-\gamma}$ and $C_2 = C_1 / U = \frac{\beta\gamma}{1-\gamma}$. In this situation workers and capitalists resident in country 1 are better off because they have higher real wages and a higher rate of return.

Now it remains to clarify when this situation is an equilibrium. It can only be an equilibrium when it is not profitable to invest in country 2 and start manufacturing production there. This profitability question can be decided by looking at the wage-

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15The case where all manufacturing is concentrated in one country can be divided in two sub-cases, $\lambda_1 < 1$ and $\lambda_1 = 1$. The second case will be referred to as the perfectly asymmetric case, where there is only manufacturing in country 1 and only agriculture in country 2.

16The case with a large manufacturing sector is discussed later on.

17In the perfectly asymmetric case we still can find the value of $w_1 \geq 1$ and can also proceed analytically.

18Strictly speaking one has to choose the parameters $\beta, \gamma$ and $\mu$ in a way that this inequality is fulfilled but the interpretation given in the text is very natural. In the context of the paper of Krugman and Venables, where $\beta$ would be 0, this condition would be reduced to $\mu < \frac{1}{2}$.

19The results for the perfectly asymmetric case are given in Appendix A.
interest equation of country 2 in the context of the specialised situation

\[
(w_2^{1-\beta-\gamma}r_2^{1-\gamma}P_2)^{\sigma} = (1 - \beta - \gamma)(E_1 P_1^{\sigma-1} T^{1-\sigma} + E_2 P_2) = (1 - \beta - \gamma)(\mu \frac{1-\gamma}{(1-\gamma-\mu)} P_1^{\sigma-1} T^{1-\sigma} + \mu P_1^{\sigma-1} T^{\gamma-1})
\]  

(24)

To attract labour, a manufacturing firm planning to start production in 2 has to pay a wage of \(w_2 = 1\). Using this, \(P_1 = \lambda_1 \frac{1-\beta}{1-\gamma-\mu}\) and \(r_1 = \lambda_1\) (24) further simplifies to

\[
r_2 = r_1 \left( \frac{1+\gamma}{2} T^{1-\sigma} + \frac{1-\gamma}{2} T^{\gamma-1} \right)^{\frac{1}{\sigma}} T^{-\frac{\beta}{\sigma}}
\]

(25)

Here \(r_2\) stands for the maximum interest rate a firm planning to start production in 2 would be able to pay its capital owners while achieving zero profits. This implies that the specialised situation is an equilibrium if

\[
U^{-\beta\sigma} > (\frac{1+\gamma}{2} T^{1-\sigma} + \frac{1-\gamma}{2} T^{\gamma-1}) T^{-\sigma\gamma}
\]

(26)

(26) describes the feasible region of \(T\) and \(U\), given \(\beta, \gamma, \mu\) and \(\sigma\), that allow for a specialised equilibrium with \(w_1 = 1\). See Figure 1 for an illustration.

Within the specialised region (the workers’ real income, the capitalists’ income and the price level in country 2) vary according to the values of \(T\) and \(U\). The equilibrium quantities in country 1 do not vary throughout the whole region, because in the situation described the integration parameters \(T\) and \(U\) influence the situation in this country neither on the goods nor on the capital market.

We next explain equilibrium which involve manufacturing in both countries and capital flows between the countries.\(^{20}\) Two cases have to be distinguished: The case where both sectors, manufacturing and agriculture, are active in both countries, and the case where only manufacturing is active in one country and agriculture and manufacturing are active in the other one. Again we will take country 1 to be the more industrialised country. The existence of equilibria of these types depends on the values of certain parameters. To have capital flows from 2 to 1, the interest rates have to be related by \(r_1 = r_2 U\).

Let us first look at the case where country 1 is specialised in manufacturing and country 2 has agriculture and manufacturing.\(^{21}\) We solve for this equilibrium by using the following indirect approach. Regarding the case under investigation, we know that \(w_1\) either has to equal 1 or has to be larger than one. Thus, we set \(w_1 = \bar{\omega}\) with \(\bar{\omega} \geq 1\). Using (22), we get \(\lambda_2 = \mu (1-\beta-\gamma) (1+\bar{\omega}) - \bar{\omega}\). For \(0 \leq \lambda_2 \leq 1\) \(\bar{\omega} \leq \bar{\omega}_{\max} = \frac{\mu (1-\beta-\gamma)}{(1-\gamma-\mu (1-\beta-\gamma)}\) has to hold. On the other hand for \(\bar{\omega}_{\max} \geq 1\), we have to impose \(\mu \geq \frac{1-\gamma}{2(1-\beta-\gamma)}\) which is the

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\(^{20}\)At the end of this section we will also look at equilibria that have an uneven allocation of manufacturing but no capital flows.

\(^{21}\)So this is the specialised case for a large manufacturing sector. If \(\mu \geq \frac{1-\gamma}{2(1-\beta-\gamma)}\) this kind of equilibrium is possible for all values of \(T\) and \(U\), not only for a bounded set like the one described by equation (26).
Figure 1: The feasible region of $T$ and $U$ allowing for a small specialised equilibrium is bounded by the two solid lines. Parameter values: $\alpha = 0.25$, $\beta = 0.35$, $\mu = 0.5$, $\sigma = 5$.

The parabolic dotted lines are two integration paths, i.e. paths of $T$ and $U$ converging to (1, 1). The convex curve is given by $T = U^2 - 2U + 2$, the concave curve is given by $T = 0.3U^2 + 1$.

already familiar restriction for a sufficiently large manufacturing sector. Use $r_i \tilde{K}_i = w_i \lambda_i$ to get $\tilde{K}_2 = \frac{U \lambda_2}{w_1 + \lambda_2}$. From $r_1 = r_2 U$ we know that $\tilde{K}_2$ has to be smaller than $\frac{1}{2}$ or equal to $\frac{1}{2}$. This implies that $U \leq U_{max}(\bar{w}) = \frac{\bar{w}^2(1-\gamma)}{\mu(1-\beta-\gamma)(1+\bar{w})-\bar{w}(1-\gamma)}$. For $U_{max}(\bar{w}) \geq 1$ the condition $\bar{w} \geq \frac{\mu(1-\beta-\gamma)}{2(1-\gamma)(1+\bar{w})-\bar{w}(1-\gamma)}$ has to be imposed. Combining all the formulas used above, we have equilibria with only manufacturing in 1 and with agriculture and manufacturing in 2 for $\mu \geq \frac{1-\gamma}{2(1-\gamma)(1+\bar{w})-\bar{w}(1-\gamma)}$, $w_1 \in [1, \frac{\mu(1-\beta-\gamma)}{1-\gamma(1-\bar{w})}]$ and $U \in [1, U_{max}(w_1)]$.

Straightforward substitution then yields $\tilde{K}_1 = \frac{\bar{w}^2}{w_1 + \lambda_2}$, $r_1 = \bar{w} + U \lambda_2$ and $r_2 = \frac{\bar{w} + U \lambda_2}{U}$.

The price indices in the two countries can be found by solving the price index equations which can be transformed to

$$P_1^{1-\sigma} = r_2^{-\beta\sigma} (U^{-\beta\sigma} \bar{w}_1^{-1-\sigma(1-\beta-\gamma)} P_1^{1-\gamma\sigma} + \lambda_2 P_2^{-\gamma\sigma} T^{-\gamma(\sigma-1)})$$

$$P_2^{1-\sigma} = r_2^{-\beta\sigma} (U^{-\beta\sigma} \bar{w}_1^{-1-\sigma(1-\beta-\gamma)} P_1^{-\gamma\sigma} T^{-\gamma(\sigma-1)} + \lambda_2 P_2^{-\gamma\sigma})$$

(27) (28)
using \( n_3 p_i = w_i \lambda_i \) and (11).

There is a continuum of equilibria given the parameter values. We have parameterised this equilibrium set by choosing \( w_1 \). This is equivalent to a specific allocation of the revenue of the manufacturing sector to workers and capital owners because \( r_1 \) is also depending on \( w_1 \). Therefore the continuum of equilibria arises from different possibilities of distributing the manufacturing revenue.22

Now equilibria in which both sectors are active in both countries and which involve capital flows between the countries can be found by performing similar manipulations. Here \( \lambda_1 \) is set to an undetermined \( \tilde{\lambda}_1 \). The results are in brief: \( \frac{\mu(1-\beta-\gamma)}{1-\gamma} < \tilde{\lambda}_1 \leq 1^{23} \) and \( U \leq U_{\text{max}}(\tilde{\lambda}_1) = \frac{\lambda_1 (1-\gamma)}{2\beta(1-\beta-\gamma)-\lambda_1 (1-\gamma)} \). We also get \( r_1 = \tilde{\lambda}_1 + U \lambda_2 \) and \( \tilde{K}_1 = \frac{\lambda_1}{\lambda_1 + U \lambda_2} \). The price indices can now be found by solving

\[
\begin{align*}
P_1^{1-\sigma} &= r_2^{\beta \sigma} (U^{-\beta \sigma} \tilde{\lambda}_1 P_1^{-\gamma \sigma} + \lambda_2 P_2^{-\gamma \sigma} T^{-(\sigma-1)}) \\
P_2^{1-\sigma} &= r_2^{\beta \sigma} (U^{-\beta \sigma} \tilde{\lambda}_1 P_1^{-\gamma \sigma} T^{-(\sigma-1)} + \lambda_2 P_2^{-\gamma \sigma})
\end{align*}
\] (29)

for \( P_1 \) and \( P_2 \).

It finally remains to characterise equilibrium without capital flows but an uneven allocation of manufacturing to the two countries. First we note that there are no capital flows for \( r_1/U < r_2 < r_1 U \). We can therefore write \( r_2 = r_1 S \) for \( S \in (\frac{1}{U}, U) \). This implies \( \lambda_1 = \frac{2 \mu(1-\beta-\gamma)}{(1-\gamma)(1+\gamma)} \) and \( \lambda_2 = \frac{2 \mu(1-\beta-\gamma) S}{(1-\gamma)(1+\gamma)} \). Also \( 0 < \lambda_i \leq 1 \) has to hold which is guaranteed by \( \frac{2 \mu(1-\beta-\gamma)}{(1-\gamma)(1+\gamma)} - 1 < S < \frac{1}{2 \mu(1-\beta-\gamma) - (1-\gamma)} \), i.e. we have bounds on the value of \( S \) that support equilibria of this type. The other variables are found by inserting \( \lambda_i \) and \( r_1 \) in the corresponding equations, the price indices are now found by solving

\[
\begin{align*}
P_1^{1-\sigma} &= 2^{-\beta \sigma} (\lambda_1^{-\beta \sigma} P_1^{-\gamma \sigma} + \lambda_2^{-\beta \sigma} P_2^{-\gamma \sigma} T^{-(\sigma-1)}) \\
P_2^{1-\sigma} &= 2^{-\beta \sigma} (\lambda_1^{-\beta \sigma} P_1^{-\gamma \sigma} T^{-(\sigma-1)} + \lambda_2^{-\beta \sigma} P_2^{-\gamma \sigma})
\end{align*}
\] (31)

In all the above cases the modified price index equations can be differentiated with respect to \( P_1, P_2, T \) and \( U \) in order to get the comparative statics of the price indices (and therefore real income) with respect to all the parameters evaluated at these equilibria.26 Comparative statics for all the other variables is straightforward since analytical expressions are available for them.

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22Technically the non-uniqueness of equilibrium stems from non-(strict)-convexities like linear technology in agriculture, the demand of capitalists and fixed costs in manufacturing. As we have seen before, the linear specification of the capitalists leads to a capital supply function that is indeterminate for equal net interest rates between the two countries.

23We assume again \( \mu > \frac{1-\gamma}{2(1-\beta-\gamma)} \). For \( \mu < \frac{1-\gamma}{2(1-\beta-\gamma)} \) the same reasoning applies with \( \frac{\mu(1-\beta-\gamma)}{1-\gamma} < \lambda_1 \leq \frac{2 \mu(1-\beta-\gamma) S}{(1-\gamma)(1+\gamma)} \). This type of equilibrium is of course possible for manufacturing sectors of any size.

24For country 1 being the country with the higher interest rate, we have to restrict \( S \) to be in \((\frac{1}{U}, 1) \).

25As usual we assume \( \mu > \frac{1-\gamma}{2 \mu(1-\beta-\gamma) - (1-\gamma)} \). The values of \( S \) that are feasible are \( S \in (\frac{1}{U}, U) \cap (\frac{2 \mu(1-\beta-\gamma)}{1-\gamma}, 1) \). The case \( \mu < \frac{1-\gamma}{2 \mu(1-\beta-\gamma) - (1-\gamma)} \) can be handled in a completely similar way.

26Because the price indices are found numerically one has to insert the numerical values of \( P_i \) and the analytically given values of the other variables into two-dimensional systems of linear equations. See Appendix B.
4 Market Integration

In this section we report the consequences of economic integration on the two countries by drawing on the results from Section 3. We assume that the capitalists do have the power to choose the equilibrium that maximises the return on investment, i.e. there are no legal or institutional restrictions on the actions of the capitalists. Another interpretation is that this equilibrium selection is the result of a bargaining process where all the bargaining power is in the hands of the capitalists. From the preferences of the capitalists, it follows that only the imperfections on the capital market, given by $U$, exert influence on their choices. The paths have a structure as described in the following table where we see the dependence of the chosen equilibrium on $U$.

Table 1: The capitalists’ income maximising equilibria during economic integration.

<table>
<thead>
<tr>
<th>Equilibrium type</th>
<th>Wage in 1</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 specialised, 2 both sectors</td>
<td>$w_{\min}(U)$</td>
<td>$U \geq U^* = \frac{1-\gamma}{2\mu(1-\beta-\gamma)}$</td>
</tr>
<tr>
<td>Both sectors in 1 and 2</td>
<td>1</td>
<td>$U_s = \frac{1-\gamma}{3(1-\gamma)-4\mu(1-\beta-\gamma)} \leq U \leq U^*$</td>
</tr>
<tr>
<td>1 specialised, 2 only agri.</td>
<td>$w_{\max}$</td>
<td>$U \leq U_s$</td>
</tr>
</tbody>
</table>

It is interesting to note that for $U \leq U^*$, the optimal (nominal) wage level set by the capitalists is the maximum possible wage which leads to the limiting case $\lambda_2(w_1) = 0$ in this situation. So all the capital owned by capitalists resident in country 2 is invested in country 1. This means that for $U \leq U^*$, the interest rate that can be generated by concentrating all the manufacturing is large enough to make foreign investment profitable. The high wages are accompanied by high prices which are the basis for generating higher interest rates than in a situation where manufacturing would be allocated to both countries.

This set-up implies that according to the investment decisions taken by capital owners who decide about industrial agglomerations workers are directly influenced by integration on the goods markets and indirectly by integration on the capital market. In the

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27 One could e.g. think of unions that may influence wage negotiations or the like.
28 See the discussion after Figure 3 for what is happening when bargaining power is attached to both groups.
29 $w_{\min}(U) = \frac{\mu(1-\beta-\gamma)}{(1-\gamma)-\mu(1-\beta-\gamma)}$. The above table assumes $\mu \leq \frac{2(1-\gamma)}{1-(\gamma-\mu)1-\beta-\gamma}$, for $\mu$ larger than this value country 1 will be specialised throughout the whole sequence of integration, and country 2 will have both sectors for $U \geq \frac{1-\gamma}{(1-\gamma)\mu(1-\beta-\gamma)}$, and only agriculture for $U$ smaller than this value.
30 In this section the sequence of equilibria is computed along the concave path shown in Figure 1. Of course any path $(T, U) \rightarrow (1, 1)$ serves the same purpose, but this one is already drawn in Figure 1.
following graphs we will show and discuss the qualitative results of integration on the equilibrium sequence.\footnote{For maximising the worldwide income of capitalists, the qualitative behaviour is structurally analogous.} In Figure 2 we see the development of the capitalists' income in the two countries. During the process of integration the income of the capitalists of the less industrialised country approaches the income of the capitalists of country 1 which is constant for small $U$.\footnote{GAUSS and Mathematica programmes for all the computations performed in this paper are available from the author upon request.} Clearly, as soon as all the capital is invested in country 1, the capitalists resident in this country have no additional gains from further integration on the capital market. This is the reason for a perfect catch-up on the side of the capitalists. Concerning the fraction of labour employed in the two countries in

Figure 2: The capitalists' income $C_i$ during the process of integration. The solid line represents $C_1$, the horizontal dashed line is the constant income of capitalists in the symmetric equilibrium and the upper dashed line is $C_1 + C_2$. The parameter values are $\beta = 0.11$, $\gamma = 0.25$, $\mu = 0.75$ and $\sigma = 5$. \footnote{The income is constant for $U \leq \frac{1-\gamma}{(1-\gamma)\mu(1-\beta-\gamma)}$, the situation described in the third line of Table 1.}
avoid the complete de-industrialisation of one country. This is again a consequence of the fact that the continuum of equilibria arises precisely because of a continuum of possibilities regarding the distribution of manufacturing revenue to workers or capitalists.

Figure 3: Shares $\lambda_i$ devoted to manufacturing during the process of integration. The solid line represents $\lambda_1$. Parameter values as in Fig. 2.

It may be interesting to note why it is possible that country 1 is also specialised in manufacturing for large values of $U$, but without cross border capital flows. This is possible because the agricultural goods that are demanded in country 1 can be purchased without trade costs on the worldwide agricultural market.

We next turn to the real income of workers in the two countries. Here we have a more complicated picture than for the capitalists' income. The workers of the poorer country first experience a phase of catching-up. At the point of de-industrialisation of their country they are facing a large decline in real income. After that their income is rising again but is never getting equal to the income of the workers in the industrialized country. We also see that the gain in real income of the workers in country 1 is smaller than the loss that occurs in country 2. That gain in real income is solely caused by the decline in transportation costs.

The predictions of the model can therefore be summarised as follows. There is an initial phase of catch-up of the less industrialised country. This is a situation in which there is trade between the specialised country and the country that has both sectors active. At this stage of integration there is only trade in goods, manufactures and agriculture, but no trade in production factors, i.e. there are no capital flows. During that phase we observe rising income of the capitalists and rising real wages in country 2. In country
Figure 4: The workers’ real income $\omega_1$ during the process of integration. The solid line represents $\omega_1$ and the upper dashed line is $\omega_1 + \omega_2$. Parameter values as in Fig. 2.

1 the real wages are also increasing due to lowered transportation costs but the capitalists’ income is steadily decreasing there.

After that stage the next phase of integration is characterised by the fact that in both countries both sectors are active. Here we see a declining labour share in manufacturing in country 1 and an increasing share in country 2. With regard to this phase of integration, many observers are worried about an export of manufacturing jobs to developing countries. The pattern of income changes is as in stage 1.

Then, at a certain point of integration, when the restrictions to capital mobility are declining below a critical value, it is becoming profitable to locate all the manufacturing in one country. This is the point in the process of economic integration when not only trade in goods but also in production factors is observed. When that happens and all the capital is invested in country 1, the workers of country 2 experience a substantial decrease in real income while those in country 1 face a rise in real income. This development occurs because transportation costs only have to be paid for consumption purposes in country 2. This is also the reason why real wages do not change in country 1 from that point onwards.

The same holds true for the capitalists in country 1. Further integration on the capital markets only favours the capitalists resident in country 2 because they experience an equalisation of gross and net interest rates when the restrictions to capital mobility are vanishing.

The above description shows that in our model the picture presented by Krugman and Venables (1995) is modified in several important respects: The first difference to
be noted is that the starting point is no longer a symmetric situation but a situation where one country is specialised in manufacturing and the other country has some manufacturing, too. This is probably a more realistic starting point to discuss economic integration. After that one observes a smooth reallocation of manufacturing to the less industrialised country that undergoes a period of catch-up. Only in the final stage that leads to perfect integration in the end is manufacturing completely concentrated in one country.

The other important difference is that in our model integration only equalises the income of capital owners. The income of the workers in the different countries is not equalised but the concentration of manufacturing widens the income gap between the workers in the two countries.

This means that integration does favour the owners of the more mobile factor, i.e. capital, compared to the owners of the immobile factor labour. The workers in country 1 gain from integration because they happen to live in the right country. Once again, in contrast to Krugman and Venables (1995) not everybody gains from economic integration.

5 Final Remarks

Adding another group of agents to the Krugman and Venables model delivers a much more diversified picture than the starting point model. Our crude introduction of agent heterogeneity indicates that the effects of economic integration on income distribution deserve attention. In the context of the present model, it may be worth looking at social welfare maximising sequences of equilibria, in which of course, the choice of an appropriate social welfare function is an essential question. Another possibility would be to introduce explicitly some sort of bargaining to explain the distribution of national income.

Although the model is intended to be concerned with long-run developments, it is still restrictive since the static character of the model makes it impossible to include capital accumulation dynamics, which are of course important in the long run and would provide a link to growth theory.

Some of the suggestions for further research mentioned at the end of Krugman and Venables (1995) are still relevant. The main points are: More geography and some empirical work to confront the model with. Empirical work could be based on the work of Hummels and Levinsohn (1995).

Nevertheless, despite all these problems, we think it is useful to construct a simple model which makes global analysis possible analytically, and which gives rise to relatively complex patterns during the integration of the world economy.

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34 It is a well-known fact in economics that at the aggregate level steps towards free trade usually increase welfare but some groups may end worse off.

35 Maybe not the suggestion to include capital mobility.
A The perfectly asymmetric case

In this appendix we derive the equilibrium quantities for the perfectly asymmetric case. For this case we know $\lambda_1 = 1$, $\lambda_2 = 0$ and $w_2 = 1$. For labour income we know $Y_1 = w_1$ and $Y_2 = 1$. Next use (22) to obtain $w_1 = \frac{\mu (1-\beta-\gamma)}{(1-\mu)(1-\beta-\gamma)+\beta}$. For this to be greater than one or equal to one $\mu \geq \frac{1-\gamma}{2(1-\beta-\gamma)}$ has to hold. Insert the known quantities in the expenditure equations (13) to get $E_1 = \mu \frac{(1-\beta-\gamma)+\gamma}{(1-\mu)(1-\beta-\gamma)+\beta}$ and $E_2 = \mu$. From the expenditure equations (15) we obtain $P_1 = w_1$. This implies $\omega_1 = w_1^{-\mu}$, $\omega_2 = w_1^{-\mu} T^{-\mu}$, $C_1 = \frac{\mu(1-\beta-\gamma)+\gamma}{(1-\mu)(1-\beta-\gamma)+\beta}$ and $C_2 = C_1 / U$. Again by inserting in the wage-interest equation for country 2 at the described point we can decide when it is an equilibrium. The wage-interest equation can now be transformed to

$$r_2^{\beta \sigma} = r_1^{\sigma(1-\gamma)} [(\mu(1-\beta-\gamma)+\gamma)T_1^{1-\sigma} + ((1-\mu)(1-\beta-\gamma)+\beta)T^{\sigma-1}]T^{\sigma\gamma}$$

(33)

This yields

$$U^{-\beta \sigma} > r_1^{\sigma(1-\beta-\gamma)} [(\mu(1-\beta-\gamma)+\gamma)T_1^{1-\sigma} + ((1-\mu)(1-\beta-\gamma)+\beta)T^{\sigma-1}]T^{\sigma\gamma}$$

(34)

with $r_1 = w_1$ as the curve describing the feasible region of $T$ and $U$ that allow for perfectly asymmetric specialisation given the other parameters.

B Some Comparative Statics

In this appendix we summarize the results of comparative statics in tables. The variables that are investigated are $Y_i$, $E_i$, $\lambda_i$, $r_i$, $w_i$, $C_i$, $P_i$ and $\omega_i$, the parameters with respect to which differentiation is carried out are $\beta$, $\gamma$, $\mu$, $\sigma$, $T$ and $U$. The following tables have to be read as follows, the rows correspond to the variables that are written in the first entry and the columns correspond to the parameters with respect to which differentiation is taking place. Rows that would contain only 0s have been omitted, e.g. the row for $Y$ in Table 2.

With two assumptions, or restrictions on the parameters, one arrives at relatively clear results for the required derivatives. The two restrictions that have to be made are $1 - \sigma (1 - \gamma) < 0$ and $1 - \beta \sigma > 0$. They have been imposed in the derivation of all the results of the comparative statics analysis.

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36 For $\mu = \frac{1-\gamma}{2(1-\beta-\gamma)}$ we have $w_1 = 1$.

37 Fujita, Krugman and Venables (1998) have an analogous restriction and call it the No Black Hole condition.
Table 2: Comparative Statics for the symmetric equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$T$</th>
<th>$U$</th>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>$&lt;0$</td>
<td>$&gt;0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C$</td>
<td>$&gt;0$</td>
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<td>$&gt;0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P$</td>
<td>$&gt;0$</td>
<td>$0^{(s)}$</td>
<td>$\gamma^{(ss)}$</td>
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<td>$&gt;0$</td>
<td>0</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$&gt;0$</td>
<td>$0^{(s)}$</td>
<td>$&lt;0$</td>
<td>?</td>
<td>$&lt;0$</td>
<td>0</td>
</tr>
</tbody>
</table>

($s$) in Table 2 means that the sign of $\frac{\partial P}{\partial \gamma}$ and $\frac{\partial \omega}{\partial \mu}$ is only determined for $T^{1-\sigma} < 2^{3\sigma} - 1$. Because of $\beta \sigma < 1$ and $\sigma > 1$ this inequality is fulfilled for $T$ larger than $T^{(s)}$ with $T^{(s)} = T^{(s)}(\beta, \gamma, \sigma)$ and $T^{(s)} > 1$.

($ss$) shall indicate that $\frac{\partial P}{\partial \mu}$ has a sign that can only be directly determined when $T^{1-\sigma} \geq 2^{3\sigma} \lambda^{\beta \sigma - 1} - 1$ holds. This inequality is true for $T < T^{(ss)}$ where $T^{(ss)} = T^{(ss)}(\beta, \gamma, \mu, \sigma)$. $T^{(ss)}$ is only larger than 1 when $\lambda > \frac{1}{2}$.

For fixed $\beta, \gamma$ and $\mu$ $T^{(ss)} > T^{(s)}$ holds, so at most one of the two described cases, ($s$) or ($ss$), can hold.

Table 3: Comparative Statics for the small $\mu$ specialised equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$T$</th>
<th>$U$</th>
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<td>$&gt;0$</td>
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<td>$\omega_2$</td>
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<td>$&lt;0$</td>
<td>?</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
<td>0</td>
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</tbody>
</table>

For completeness’ sake we also report the results for the perfectly asymmetric case.
Table 4: Comparative Statics for the perfectly asymmetric equilibrium

<table>
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<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$T$</th>
<th>$U$</th>
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</table>

\(^{***}\): For $w_1 \leq \exp \left( \frac{1-\mu}{\mu(1-\gamma)} \right) \frac{\partial \omega_1}{\partial \mu}$ is positive.

We now present the (analytical part of the) comparative statics for the price indices for the equilibria having manufacturing in both countries. We will exemplify the analysis for the case where there is only manufacturing in country 1 and both sectors are active in country 2.\(^{38}\) We restrict the analysis to differentiating with respect to 3 parameters or variables, $T$, $U$ and $\bar{w}_1$, which is in fact a parameter for the present case as well.\(^{39}\) If $x$ stands for one of the parameters, then the solution for $\frac{\partial P_1}{\partial x}$ can be found by solving

\[
A \left( \frac{\partial P_1}{\partial P_2} \right) = \begin{pmatrix} b^T_1 \\ b^U_1 \end{pmatrix}
\]

$A$ is given by

\[
\begin{pmatrix}
(1-\sigma)P_1^{-\sigma} + \gamma \sigma U^{-\beta \sigma \bar{w}_1 \sigma(1-\beta-\gamma)} P_1^{-1+\sigma(1+\gamma)} & \gamma \sigma \lambda_2 P_2^{-1+\gamma(1+\sigma)} T^{-(\sigma-1)} \\
\gamma \sigma U^{-\beta \sigma \bar{w}_1 \sigma(1-\beta-\gamma)} P_1^{-1+\sigma(1+\gamma)} & (1-\sigma)P_2^{-\sigma} + \gamma \sigma r_2^{-\beta \sigma \lambda_2 P_2^{-1+\gamma}}
\end{pmatrix}
\]

and the $b^x$ are given by

\[
b^T = \begin{pmatrix}
(1-\sigma)\lambda_2 P_2^{-\gamma(1+\sigma)} T^{-(\sigma-1)} \\
(1-\sigma)U^{-\beta \sigma \bar{w}_1 \sigma(1-\beta-\gamma)} P_1^{-\gamma(1+\sigma)} T^{-(\sigma-1)}
\end{pmatrix}
\]

\[
b^U = \beta \sigma r_2^{-\beta \sigma U^{-1+\beta \sigma \bar{w}_1 \sigma(1-\beta-\gamma)} P_1^{-\gamma(1+\sigma)}} \left( \begin{pmatrix} RHS_{S17} \\ RHS_{S18} \end{pmatrix} + \frac{1}{T^{-(\sigma-1)}} \right)
\]

\(^{38}\)This is the case where the price indices are found by solving equations (27) and (28).

\(^{39}\)For the two other cases we would have to differentiate w.r.t. $T$, $U$ and $\bar{w}_1$ or $S$. 
\[ \tilde{v}^p = -\beta \sigma r_2^{1-\beta \sigma} \frac{\partial \lambda_3}{\partial \tilde{b}} \left( \frac{RHS_{27}}{RHS_{28}} \right) + \\
\quad + (1 - \sigma (1 - \beta - \gamma)) r_2^{-\beta \sigma} U^{-\beta \sigma} \tilde{w}^{-\sigma(1-\beta-\gamma)} P_1^{-\sigma} \left( \frac{1}{T^{-(\sigma-1)}} \right) + \\
\quad + r_2^{-\beta \sigma} P_2^{-\sigma} \frac{\partial \lambda_3}{\partial \tilde{b}} \left( T^{-(\sigma-1)} \right) \]

where \( RHS_{27} \) and \( RHS_{28} \) are the expressions on the right hand side of equations (27) and (28). \( \frac{\partial \lambda_3}{\partial \tilde{b}} = \mu (1-\beta-\gamma) > 0 \) and \( \frac{\partial r_2}{\partial \tilde{b}} = \frac{1}{U} + \frac{\partial \lambda_3}{\partial \tilde{b}} \). \( \frac{\partial \lambda_3}{\partial \tilde{b}} \) is 0 for \( U = \frac{1}{(1-\gamma)-\mu(1-\beta-\gamma)} \), is larger than 0 for \( U \) smaller than this value and smaller than 0 for larger \( U \). Numerical experiments for this case yield definite results: \( \frac{\partial P_1}{\partial T} > 0 \), \( \frac{\partial P_1}{\partial U} < 0 \) and \( \frac{\partial P_1}{\partial \tilde{b}} > 0 \).

Doing completely similar manipulations, one can derive the required partial derivatives for the other parameters and the other cases.\(^{40}\)

\(^{40}\)For a given case for all the parameters \( x \) the matrix \( A \) is fixed, only the vectors \( \tilde{b}^p \) change.
References


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