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Deterministic Chaos versus Stochastic Processes: An Empirical Study on the Austrian Schilling - US Dollar Exchange Rate

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Abstract

The classical theory about foreign exchange rate explains its fluctuations as the resulting of a random walk motion. In this paper, such a theory is put into question by performing Brock, Dechert and Scheinkman's (1987) test on the Austrian Schilling-US Dollar exchange rate for the period 1971–1998, giving us strong evidence of nonlinearities in its behaviour. By further analysing, features such as the correlation dimension will be estimated in order to better understand the characteristics of the underlying process.

Keywords
Chaos, BDS test, correlation dimension, exchange rate

JEL-Classifications
C14, C49, F31

Comments

The author acknowledges the helpful comments of Robert Kunst during the confection of the paper.

Contents

"Thus the anima and life itself are meaningless as they offer
no interpretation. Yet they have a nature that can be interpreted,
for in all chaos there is a cosmos, in all disorder a secret order,
in all caprice a fixed law, for everything that works
is grounded on its opposite."
C.G.Jung ("Über die Archetypen des kollektiven Unbewussten")

1 Introduction

Chaotic motion is often characterized, despite its deterministic nature, by having properties which are impossible to differentiate from those of stochastic processes by using linear methods. In particular, first and second order moment properties of certain deterministic processes producing chaos are the same as those of white noise (these processes are also denominated "white chaos"). Thus, a test for distinguishing such two processes in observed data is of capital interest for analysis and prediction of economic time series.

It is a non-parametric test, the BDS test (Brock, Dechert, Scheinkman, 1987), that has been proved to be not only useful for such a purpose, but also to have good power against departures from i.i.d.ness and nice finite sample properties (Brock, Hsieh, LeBaron, 1991).

The paper is organised in two parts: Firstly, section 2 introduces the main definitions and instruments for the analysis of nonlinearity and chaos in economic time series, as well as the BDS test itself. The second part is concerned with the application of such tools to the exchange rate between the US Dollar and the Austrian Schilling between January 1971 and July 1998. Such a study is performed in section 3, together with a further analysis that will enable us to estimate the correlation dimension of the strange attractor behind the data generating process of the data. Finally, section 4 comments the conclusions of the empirical study performed in section 3.

2 Definitions and tools

In this section the main definitions concerning deterministic chaos will be briefly exposed, as well as some of the instruments that can be used for distinguishing deterministic from stochastic time-series.

2.1 Deterministic chaos, fractal dimension, and Lyapunov exponents

2.1.1 Definition of deterministic chaos

A time series $\{a_t\}$ has a C^2 *deterministic chaotic explanation* if there exists (h,F,x_o) such that $a_t=h(x_t)$, $x_{t+1}= F(x_t)$ and $x(0)=x_o$, with h: $\Re^n \rightarrow \Re^1$, F: $\Re^n \rightarrow \Re^n$, both C^2, and there exists an ergodic invariant measure μ for F, absolutely continuous with respect to Lebesgue measure.

The definition of chaos is not unique, and there are stochastic versions of this definition, where h and F are random functions. Such refinements of this definition are frequently related to Kalman filtering frameworks.

2.1.2 Measuring chaos: fractal dimension

When plotting the trajectory of a time series with a deterministic chaotic explanation, its distribution differs from a simple random distribution of points. The figures obtained by graphing chaotic systems into a phase space (the so called "fractals", which have given rise to an incredibly vast amount of publications since the eighties) only occupy a fraction of the totality of the phase space.

DeGrawe, Dewachter and Embrechts (1993) in a loose but intuitive way define a *strange attractor* as "an intriguing phase space trajectory plot with fractal properties", and the *fractal dimension* as the fraction of the phase space occupied by such an attractor.

The *correlation dimension* (see section 2.2.1) is often referred to as an estimate of the fractal dimension (in fact it corresponds to a lower bound of the fractal dimension) and a way of differentiating deterministic processes from stochastic ones, as the correlation dimension of the attractor in the case of white noise goes into infinity.

2.1.3 Definition of Lyapunov exponents

The *largest Lyapunov exponent*, L, of a function F, is defined by:

$$L = \lim_{t\rightarrow\infty} \left[\frac{\ln(\|DF^t(x) \cdot v\|)}{t}\right], \tag{2.1}$$

where D indicates the derivative of the function, $F^t(x)(= F(F^{t-1}(x))$ is the application of the function F t times to x, $\|\|$ is a norm and v is a direction vector.

L is a measure of how fast the initial measurement error multiplies into error in forecast, and the definition of deterministic chaos requires it to be greater than zero. The idea of sensitive dependence upon initial conditions (SDIC) hides behind such a requirement.

The idea of chaos will be more easily understood after going through the following example.

2.1.4 Example: The tent map

Let us consider the following function, $\varphi(x)$:

$$\varphi(x) = \begin{cases} \eta x, & if\ x \in [0, \frac{1}{\eta}] \\ \frac{\eta}{\eta-1}(1-x) & if\ x \in [\frac{1}{\eta}, 1] \end{cases}, \tag{2.2}$$

with $\eta > 1$. Such a simple mapping can perfectly illustrate many of the characteristics of chaotic motion. Considering the tent map for $\eta = 2$, that is:

$$\varphi(x) = \begin{cases} 2x, & if\ x \in [0, \frac{1}{2}] \\ 2-2x & if\ x \in [\frac{1}{2}, 1] \end{cases}, \tag{2.3}$$

many of the properties of chaos can be explained.

Let $x = x_t$ and $\varphi(x_t) = x_{t+1}$. Then,

i) $\frac{\#\{x_t, 1\leq t\leq T \mid x_t \in [a,b]\}}{T} \to (b-a)$, that is, the fraction of x_ts in the interval [a,b] converges to (b-a). Therefore, the hypothesis of existence of a non-degenerate invariant measure[1] μ is fulfilled just by defining $\mu([a,b]) = b-a$.

ii) The tent map materializes, in its simplicity, the idea of SDIC. Suppose that the initial conditions can only be measured with an error $\varepsilon > 0$ $[\tilde{x}_o = x_o \pm \varepsilon]$. The error in forecasting x_t , $t \geq 1$ grows exponentially with t. Such a loss of precision is measured by the Lyapunov exponent, which is L=ln(2)

[1] Recall the following theorem (Lasota-Yorke, 1973): Let h: I→I be piecewise C^2 and expansive, i. e., such that $\inf_{x\in I} |h'(x)| > 1$. Then h has a positive Lebesgue measure. Moreover, if h is unimodal, then the measure is ergodic, so that for almost all initial conditions this measure describes the long-run frequency with which different neighbourhoods are visited.

in this case. In the long run, the only feature we could report of our forecast is that it lies in the [0,1] interval.

iii) The series $\{x_t\}$ has similar characteristics to white noise concerning empirical spectrum and correlogram. This feature will be discussed in the following section.

2.2 Testing for deterministic chaos

Plotting x_t against t in the aforementioned "tent-map" example leads to a graph which looks like the realization of a stochastic process. Furthermore, the estimated autocorrelations are similar to the ones of white noise [for the results of a simulation with $\eta \cong 2$, see Liu, Granger and Heller (1993)], and the empirical spectrum appears flat. Therefore, some instrument in order to distinguish chaos from stochastic systems is required.

2.2.1 The correlation integral and the correlation dimension

The *correlation integral*, $C_{m,T}(\varepsilon)$, is defined (Grassberger and Procaccia, 1983) as follows:

$$C_{m,T}(\varepsilon) = \sum_{t<s} I_\varepsilon(x_t^m, x_s^m) \frac{2}{[T-(m-1)][T-m]}, \tag{2.4}$$

being $x_t^m = (x_t, x_{t+1}, ..., x_{t+m-1})$, and I_ε an indicator function such that:

$$I_\varepsilon(x_t^m, x_s^m) = \begin{cases} 1 & \text{if } \|x_t^m - x_s^m\| < \varepsilon \\ 0 & \text{otherwise} \end{cases}. \tag{2.5}$$

From the correlation integral, the correlation dimension is estimated by plotting $log(C_{m,T}(\varepsilon))$ against $log(\varepsilon)$, and looking for constant slope zones, where the correlation dimension for embedding dimension m will be defined as:

$$d_m = \lim_{\varepsilon \to 0} \left\{ \lim_{T \to \infty} \frac{\partial \log[C_{m,T}(\varepsilon)]}{\partial \log \varepsilon} \right\}, \tag{2.6}$$

and the correlation dimension itself is:

$$d = \lim_{m \to \infty} d_m \tag{2.7}$$

2.2.2 The BDS statistic

The BDS statistic is defined as:[2]

$$W_{m,T}(\varepsilon) = \frac{T^{\frac{1}{2}}\ [C_{m,T}(\varepsilon) - C_{1,T}(\varepsilon)^m]}{\sigma_{m,T}(\varepsilon)}, \tag{2.8}$$

where $\sigma_{m,T}(\varepsilon)$ is the consistent estimator of the standard deviation of the asymptotic (normal) distribution of $C_{m,T}(\varepsilon) - C(\varepsilon)^m$, with $C(\varepsilon)$ having been replaced by its consistent estimator, $C_{1,T}(\varepsilon)$, and $\phi(\varepsilon) = \int[F(z+\varepsilon) - F(z-\varepsilon)]^2 dF(z)$ by $\phi_T(\varepsilon) = \sum_{t<s<r} h_\varepsilon(x_t^m, x_s^m, x_r^m)6/[(T-m+1)(T-m)(T-m-1)]$, where $h_\varepsilon(x_t^m, x_s^m, x_r^m) = [I_\varepsilon(x_t^m, x_s^m)I_\varepsilon(x_s^m, x_r^m) + I_\varepsilon(x_t^m, x_r^m)I_\varepsilon(x_r^m, x_s^m) + I_\varepsilon(x_s^m, x_t^m)I_\varepsilon(x_t^m, x_r^m)]/3$. $W_{m,T}(\varepsilon)$ converges to a standard normal distribution under the hypothesis of i.i.d.ness.

3 On the evidence of nonlinearities in the Austrian Schilling-US Dollar exchange rate

3.1 Theory and evidence

[2]Brock, Dechert and Scheinkman (1987) prove that, under the null hypothesis that $\{x_t\}$ is i.i.d. with a distribution function F,

$$C_{m,T}(\varepsilon) - C(\varepsilon)^m \xrightarrow{d} N[0, \sigma_m^2(\varepsilon)]$$

where:

$$\begin{aligned} C(\varepsilon) &= \int [F(z+\varepsilon) - F(z-\varepsilon)]dF(z), \\ \sigma_m^2(\varepsilon) &= 4\{\int [F(z+\varepsilon) - F(z-\varepsilon)]^2 dF(z)\}^m \\ &\quad +2\sum_{j=1}^{m-1}\{\int [F(z+\varepsilon) - F(z-\varepsilon)]^2 dF(z)\}^{m-j}C(\varepsilon)^{2j} \\ &\quad +(m-1)^2 C(\varepsilon)^{2m} - m^2\{\int [F \\ &\quad (z+\varepsilon) - F(z-\varepsilon)]^2 dF(z)\}C(\varepsilon)^{2m-2} \end{aligned}$$

For a proof using the theory of U-statistics, see Brock, Hsieh and LeBaron (1991).

Until the end of the eighties, the behaviour of exchange rates was generally explained by the hypothesis that they followed a *random walk*. Mussa (1979) and Meese and Rogoff (1983) are outstanding examples of such a point of view. This explanation implies some unattractive properties for the data generating process of exchange rates, namely the existence of unboundedness in the unconditional variance of the levels of the variable.

On the other hand, in the end of the eighties many empirical studies showed some evidence of nonlinear dependence on exchange rate changes [Hsieh (1988), Hsieh and LeBaron, (1989)].

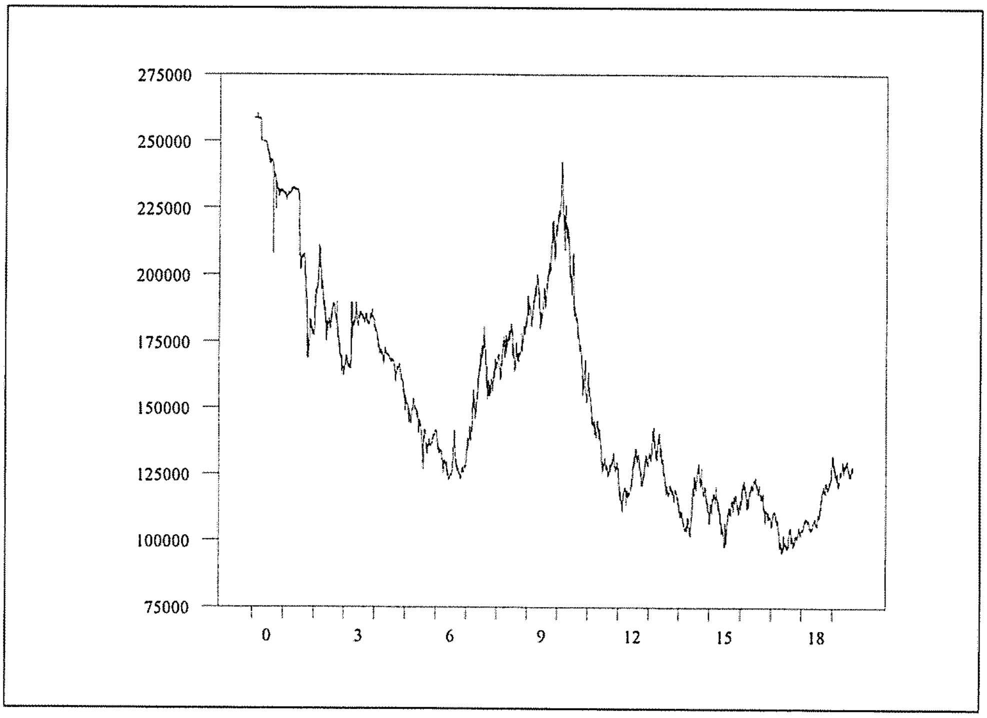

Figure 1: Exchange rate ATS-US$ (4/I/1971-2/VII/1998)

3.1.1 The data

In this study, daily exchange rates ATS-US$ from January 4, 1971, until July 2, 1998, (6888 data points) have been used (the US Federal Reserve is the source of information). Figure 1 shows the graph of this daily spot exchange rate looks. Daily returns were obtained by using the following formula:

$$R_t = A[\log(S_t) - \log(S_{t-1})], \tag{3.1}$$

where:

R_t: daily return at time t .

A: scaling factor, which has actually no influence on our conclusions, and we will fix equal to one.

S_t: exchange rate at time t .

The plot of the daily returns is shown in Figure 2.

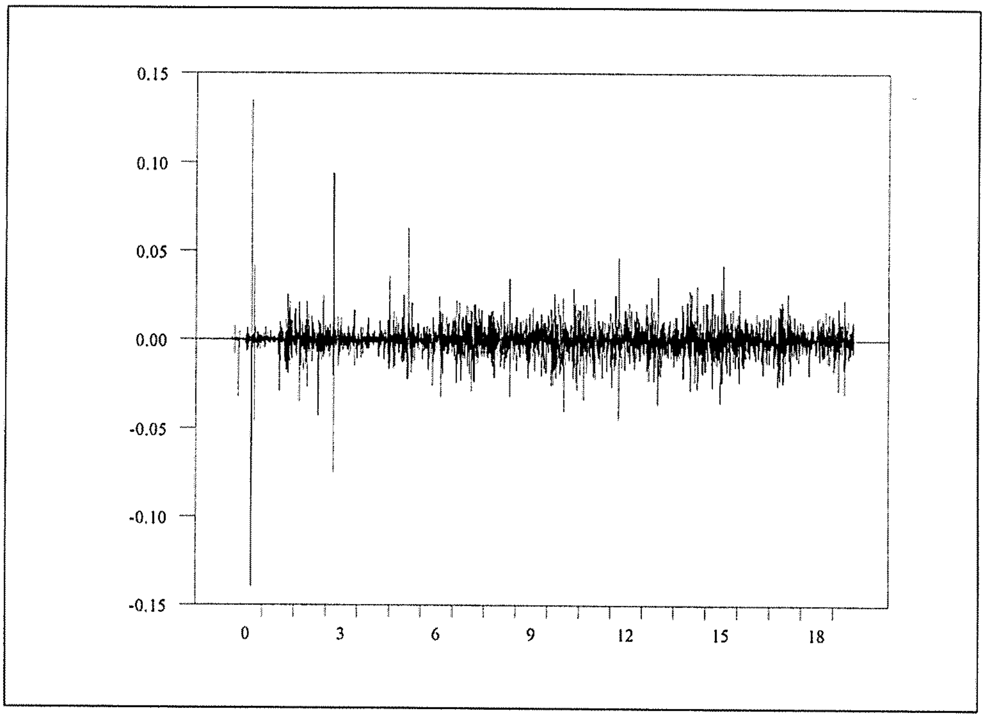

Figure 2: Daily returns AS-US$ (4/I/1971 - 2/VII/1998)

3.1.2 Testing for i.i.d.ness

The BDS test, as explained in section 2.2.2, was performed on the raw data for values of ε equal to 0.5, 1 and 1.5 times the standard error of the sample and for embedding dimensions m from 2 to 10. The results are shown in Table

1 and reject strongly the null hypothesis, namely that our observations are independent and identically distributed, for all ε and all m at all conceivable significance levels.

m	BDS statistic $\varepsilon = 0.5 * SE$	BDS statistic $\varepsilon = SE$	BDS statistic $\varepsilon = 1.5 * SE$
2	17.8	14.3	13.7
3	18.1	19.3	16.9
4	40.6	23.5	19.6
5	59.5	28.3	22.1
6	90.3	33.4	24.5
7	143.2	39.3	26.7
8	240.1	46.4	28.8
9	418.2	54.9	30.9
10	753.5	65.3	33.1

Table 1: BDS statistic for raw data

Furthermore, an autoregressive model such as (3.2) was fitted to the data. The number of lags was selected according to Akaike's AIC(1973) criterion, and the BDS test was applied to the residuals of such a regression:

$$\begin{aligned} R_t &= a + \sum_{i=1}^{p} b_i R_{t-i} + \sum_{i=1}^{5} c^i D_t^i + u_t \\ u_t &\sim N(0, \sigma^2), \end{aligned} \tag{3.2}$$

where D_t^i are dummy variables for the different days of the working week and p is chosen to be equal to 1 according to the AIC.

The results are in Table 2 and, once again, leave no doubt about the rejection of the null hypothesis, being the BDS statistics in the far extreme of the positive tail of the standard normal distribution.

m	BDS statistic $\varepsilon = 0.5 * SE$	BDS statistic $\varepsilon = SE$	BDS statistic $\varepsilon = 1.5 * SE$
2	19.4	13.9	10.1
3	24.3	17.03	12.06
4	32.9	19.9	13.5
5	49.9	23.8	15.04
6	85.7	28.8	16.4
7	164.8	35.6	18
8	350.2	45.04	20.24
9	828.9	59.7	22.7
10	2101.2	81.7	25.49

Table 2: BDS statistic for residuals of the AR(1) model

The rejection of i.i.d.ness, nevertheless, is not enough to conclude that our data are low-dimensional chaotic (see Hsieh, 1989). Other features could lead to the rejection of the null hypothesis in the BDS test: the existence of linear stochastic systems, such as an autoregressive or moving average model, or non-stationarity. The first case (AR, MA or ARMA systems) can be easily rejected by looking at the correlogram of the series (Figure 3).

In order to treat the case of non-stationarity, let us divide our set of observations into five subsamples and apply the test to each one of them. They approximately correspond to the following dates:

Subsample 1: January 1971 to May 1975
Subsample 2: May 1975 to September 1980
Subsample 3: September 1980 to February 1986
Subsample 4: February 1986 to June 1991
Subsample 5: June 1991 to July 1998

Table 5 shows the results for ε equal to the standard deviation of each subsample and embedding dimension going from 2 to 10. Notice that, apart from the period September 1980 to February 1986, where the one-sided test would accept the null hypothesis for a significance level smaller than 10%, and only with an embedding dimension of 2, the values of the BDS statistic are high enough to reject the hypothesis of i.i.d.-ness in each one of the subperiods we have divided our sample. More evidence is needed, in any case, in order to be able to affirm that our data could be the result of a low-dimensional chaotic system.

m	BDS ss1	BDS ss2	BDS ss3	BDS ss4	BDS ss5
2	11.07	8.3	1.32	4.8	4.7
3	12.6	12.8	2.96	4.8	5.2
4	13.7	16.5	4.35	5.3	5
5	14.97	20.2	5.6	5.4	5.65
6	15.98	24.5	6.7	5.7	6.5
7	16.88	29.4	7.82	5.86	7.5
8	17.96	35.4	8.95	6.07	8.5
9	19.04	42.8	10.1	6.4	9.5
10	20.2	52.1	11.1	6.7	10.57

Table 3: BDS test for subsamples

3.2 Reconstructing the strange attractor

The *time delay method* (Packard et al., 1980, Takens, 1981) was already used in economics by De Grawe, Dewachter and Embrechts (1993) in order to reconstruct the strange attractor for exchange rate data as well as for determining an estimate of the fractal dimension (the correlation dimension). Let us briefly explain the steps to be taken when using this procedure and the results for the data we are dealing with.

The idea behind the time delay method is to create n-dimensional vectors out of the data (with n not known a priori), being the components of such vectors the observations themselves with a proper time delay. These components form the directions of the phase space in which the attractor will be reconstructed. Takens proved the reconstruction of the strange attractor performed by using the time delay method to be topologically equivalent to the attractor in phase space.

Firstly, we will choose the time delay to be such that we eliminate any existing correlation among the observations. By observing the correlogram of the daily returns (Figure 3), we can assure that a time delay of two periods will do. Therefore, our n-tuples will look like:

$$
\begin{aligned}
\vec{x}_1 &= \left\{x_1, x_3, x_5 \ldots x_{1+2(n-1)}\right\} \\
\vec{x}_2 &= \left\{x_3, x_5, x_7 \ldots x_{1+2n}\right\} \\
&\ldots \\
\vec{x}_M &= \left\{x_{N-2(n-1)}, x_{N-2(n-2)}, \ldots x_N\right\},
\end{aligned}
$$

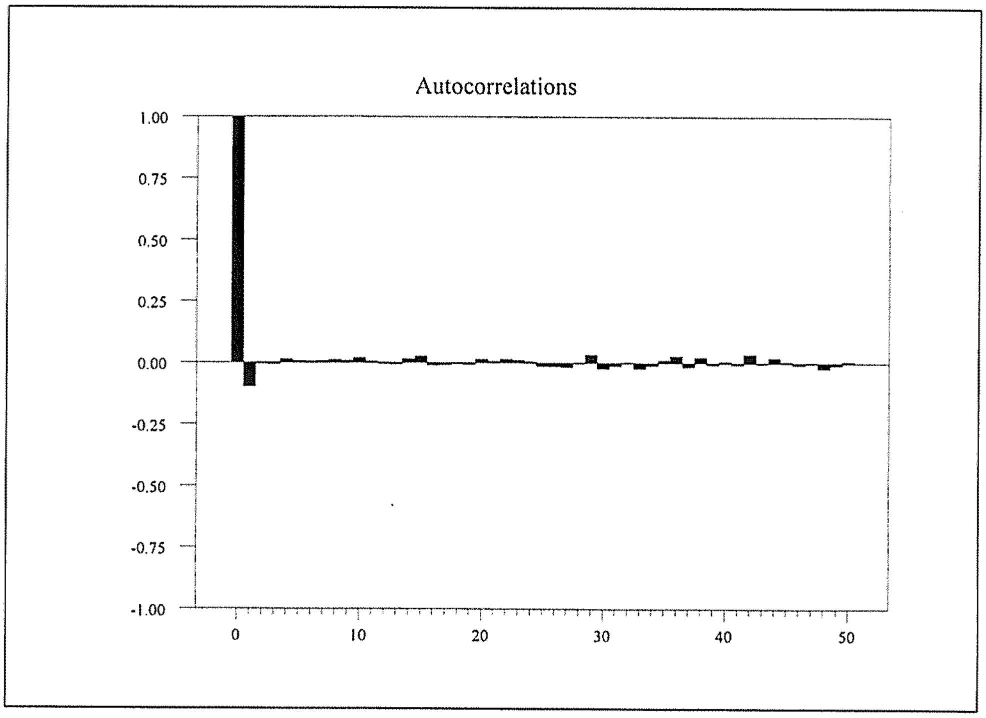

Figure 3: Correlogram of the daily return series

where N is the total number of observations.

So that we can compute the fractal dimension of the attractor, we need to plot the correlation integral versus the value of epsilon in a log-log axis for different dimensions (as the value of n is unknown). Empirically, the correlation dimension in embedding dimension m can be estimated from the slope of such a plot as (recall section 2.2.1):

$$d_m = \lim_{\varepsilon \to 0} \left\{ \lim_{T \to \infty} \frac{\partial \log[C_{m,T}(\varepsilon)]}{\partial \log \varepsilon} \right\}, \tag{3.3}$$

and the correlation dimension, *d*:

$$d = \lim_{m \to \infty} d_m \tag{3.4}$$

$C_{m,T}(\varepsilon)$ was computed for our observations, with increasing values of ε, and the result is plotted ($\log(C_{m,T}(\varepsilon))$ versus $\log \varepsilon$) in Figure 4 for m ranging

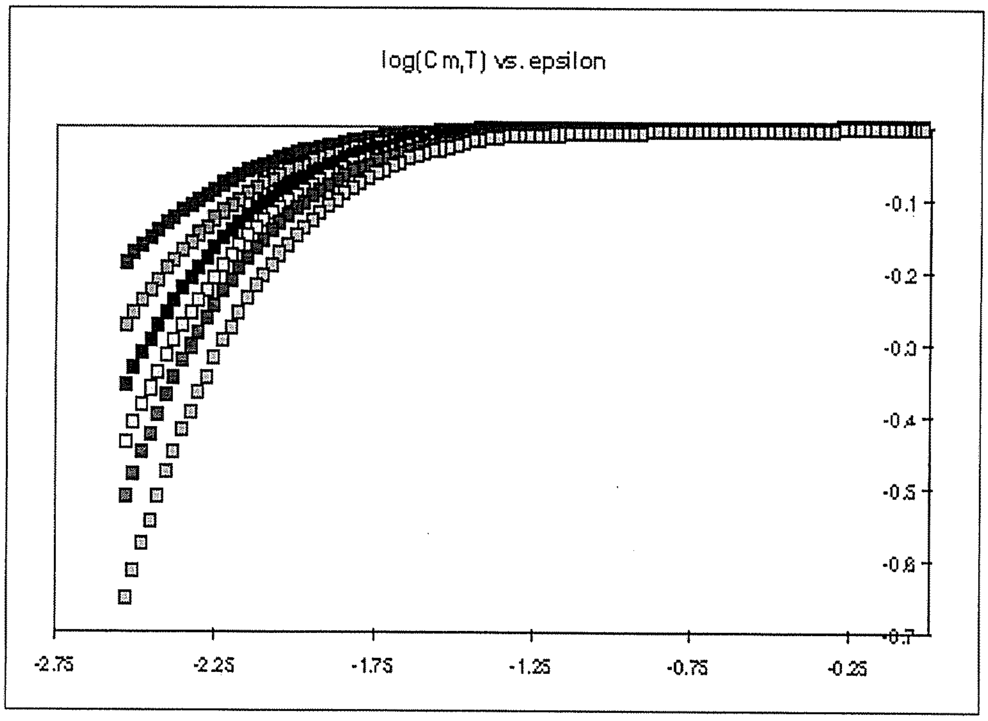

Figure 4: log of corr. integral vs. log epsilon

from 2 to 7, being m=2 the upper curve and m=7 the lowest one. In order not to make the graph too confusing no higher embedding dimensions were added. DeGrawe, Dewachter and Embrechts (1993) among others compute the same plots for different exchange rates, and the following features are reported:[3]

i) For certain subsamples of some exchange rate time series the slope of the $\log(C_{m,T}(\varepsilon))$ versus $\log\varepsilon$ plot tends to saturate at some levels for increasing embedding dimensions. Such regimes are labeled as "chaotic", and the level of saturation would be an estimate of the fractal dimension.

ii) Some other exchange rate series show a tendency to form horizontal areas when plotting the slope of $\log(C_{m,T}(\varepsilon))$ against $\log\varepsilon$, and no signal of saturation appears. Such regimes appear very similar to the results of the $\log(C_{m,T}(\varepsilon))$ -$\log\varepsilon$ plot for the Brownian motion, and are therefore labeled

[3]Notice that the samples in the referred studies are, in general, considerably smaller than the one used in this paper.

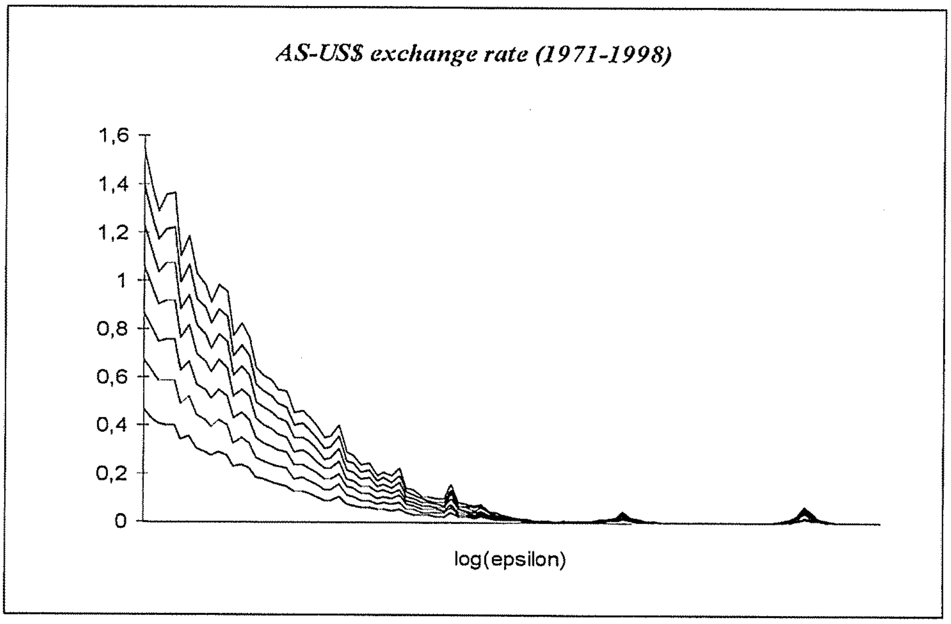

Figure 5: Instantaneous slope for log $[C_{m,T}(\varepsilon)] - \log \varepsilon$

"random walk".

iii) Finally, in some of the plots elements of i) and ii) appeared together, making it difficult for the scientist to give a statement about the nature of the process that generated the data.

Figure 5 shows the slope of the $\log(C_{m,T}(\varepsilon))$ for increasing ε for our ATS-US$ exchange rate data from 1971 to 1998 . The fluctuating slope for low ε is usually attributed to the noise that economic observations present (in comparison to time series in Physics, where this kind of analysis is very common). Notice that, except for a small interval where there seem to be some unclear signs of saturation at a very small level ($\approx 0,1$) and the lack of similarity with Brownian motion, the graph does not offer us definite signs of the existence of a strange attractor. The same graph was plotted for the five subsamples described in section 3.1.1, and they are represented in Figure 6 to 10.

In the first subsample (January 1971-May 1975), the results are much more concluding. On the one hand, a saw-tooth like structure repeats itself all through Figure 6. Repeating the observation done some lines before,

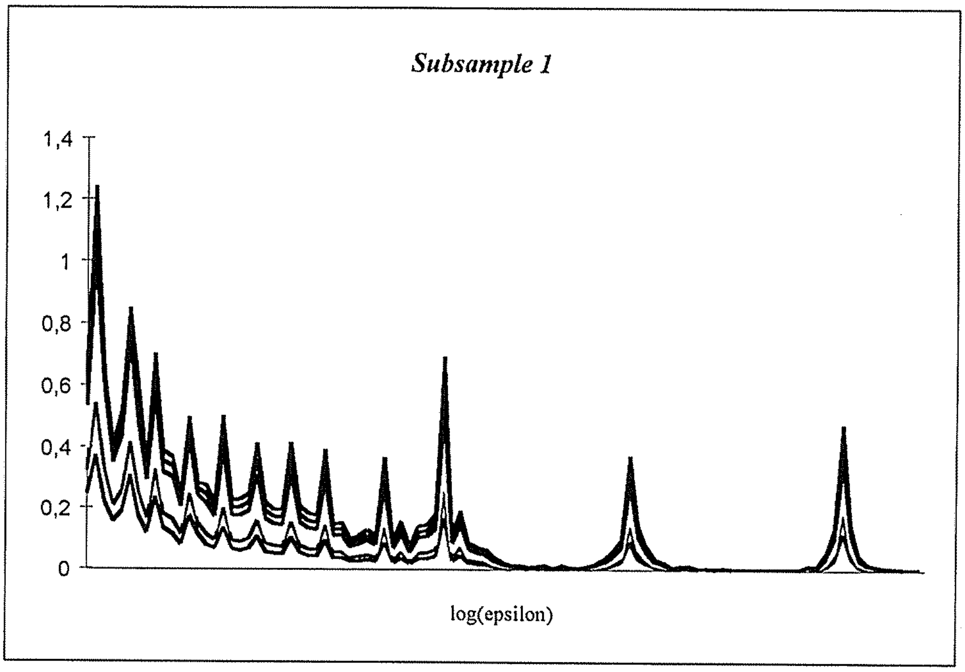

Figure 6: Instantaneous slope for log $(C_{m,T}(\varepsilon))$-log(ε)

this is a quite common feature in economic data, which have a more noisy nature than data used for this analysis in natural sciences. On the other hand, in this plot we do find stronger patterns of saturation in the slope of the $\log(C_{m,T}(\varepsilon))$ -logε plot for increasing embedding dimensions at a level of around 0.175. This would imply the existence of a strange attractor with a (low) non-integer fractal dimension for the considered subperiod.

The analysis for the period May 1975-September 1980 is represented in Figure 7. The result is more intriguing and difficult to interpret than represented in Figure 6: we can observe small repeated areas of pointwise convergence of slope for increasing embedding dimensions at a level of approximately 1.6, but this saturation is not continuously maintained in the neighbourhoods of $\log(\varepsilon)$ in which it appears. It would be highly speculative to assure that a strange attractor is hidden behind this feature, but at the same time the plot differs qualitatively from the classic Brownian-motion-like $\partial \log(C_{m,T}(\varepsilon))/\partial \log(\varepsilon)$-log$\varepsilon$ graph.

Figure 8, representing the same plot for the third subsample, namely the

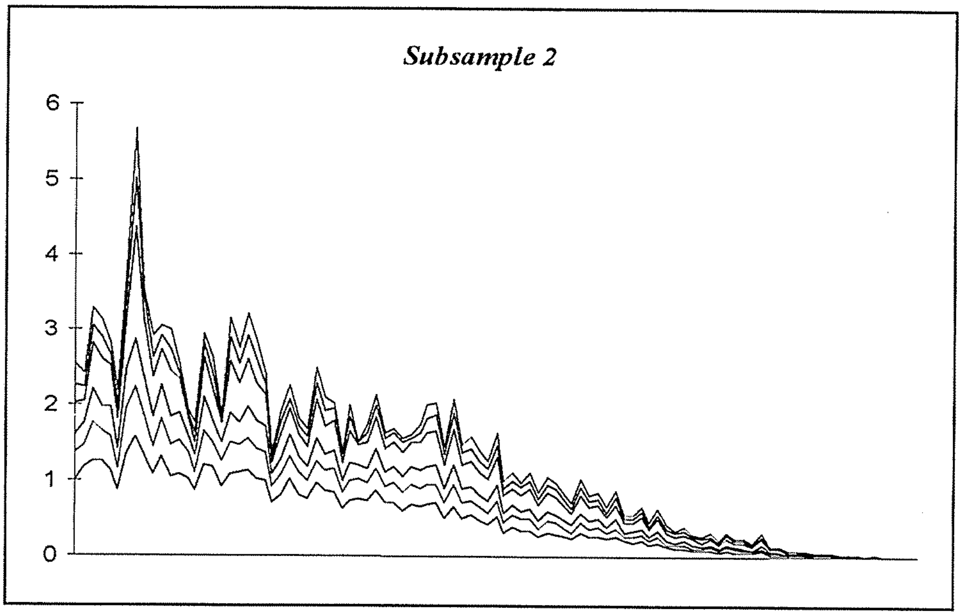

Figure 7: Instantaneous slope for log $(C_{m,T}(\varepsilon))$-log(ε)

one for the period from September 1980 to February 1986, only presents relative signs of pointwise saturation for a small value of log(ε). For higher values of ε, the slopes of log$(C_{m,T}(\varepsilon))$ for different embedding dimensions m keep a quasi-constant separation among each other, with no indication of further saturation whatsoever. There are no conclusive signs of low-dimensional attractors.

No signs of saturation at all appear, however, in Figure 9, in which the subsample running from February 1986 to June 1991 is represented. This regime is, thus, clearly non-chaotic.

Similar conclusions can be applied to Figure 10, which represents the analysis for the June 1991 to July 1998 subsample. In this case the non-chaotic behaviour is even clearer, as the slope curves tend to acquire almost horizontal shape in many intervals and saturation for increasing embedding dimensions is absent.

As an overall view, we can report the loss of chaotic nature in the skeleton of the data generating process for the period starting in the beginning of the eighties until nowadays, while the observations of the seventies show clear signs of the existence of a low-dimensional strange attractor.

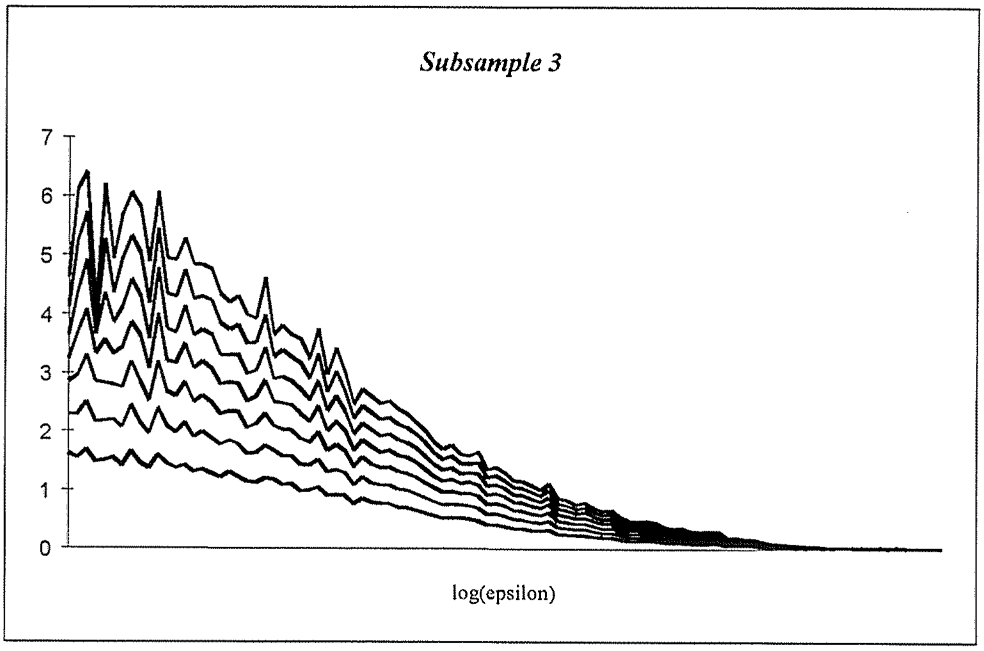

Figure 8: Instantaneous slope for log $(C_{m,T}(\varepsilon))$-log(ε)

4 Summary and conclusions

In the study performed, evidence of nonlinearities in the behaviour of the ATS-US$ foreign exchange rate has been discovered for both the totality of the daily returns series from 1971 to 1998 and for five subperiods of the sample. While the BDS test strongly rejects the hypothesis of non i.i.d. for all subsamples, the existence of a low-dimensional strange attractor in the data generating process does not seem equally acceptable for all of them.

The approach used in the paper, namely the search of the correlation dimension through the time delay method, does not give a conclusive answer to the question whether a strange attractor exists in the whole of the sample, but is able to recognize what seems to be a low-dimensional attractor at least for the first subsample of data, that is, the one covering the first half of the seventies. In the second half of the seventies the evidence is not that clear, but some indications of chaotic behaviour could be found. The rest of the subsamples shows signs of rejection of the hypothesis of low-dimensional chaotic motion.

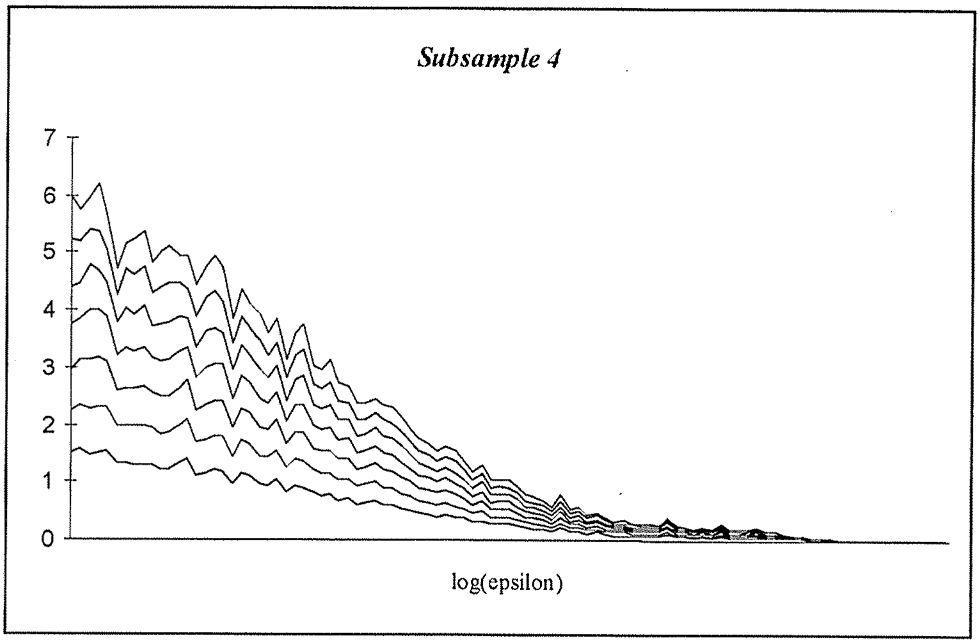

Figure 9: Instantaneous slope for log $(C_{m,T}(\varepsilon))$-log(ε)

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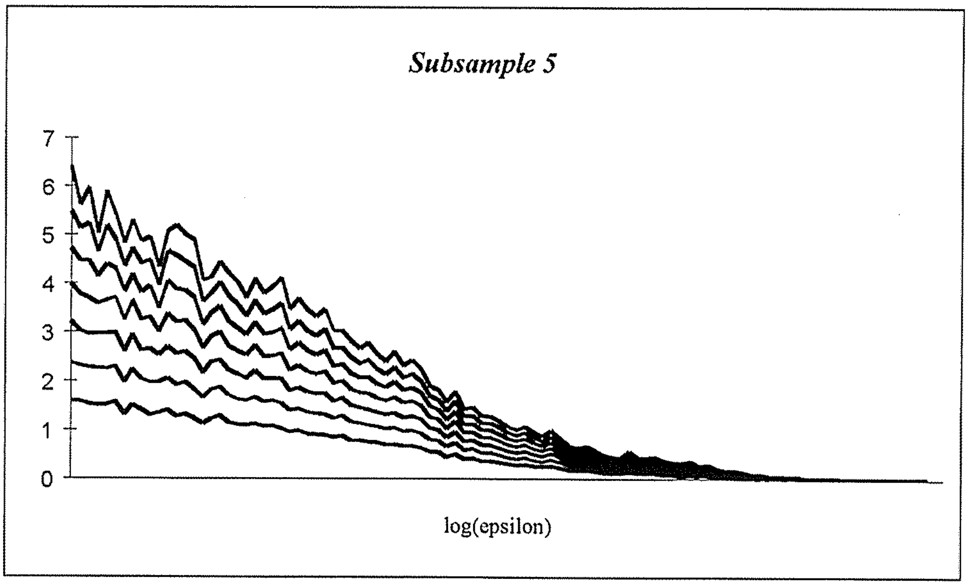

Figure 10: Instantaneous slope for log ($C_{m,T}(\varepsilon)$)-log(ε)

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