

THE PIGOUVIAN APPROACH TO THE
TREATMENT OF EXTERNALITIES

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1. Introduction

When Jean Jacques Rousseau conceived of his Social Contract, he confronted an important social problem -- a problem that has continued to haunt mankind to the present day.

Individuals in a society are inevitably interdependent. How can they be induced to promote the social welfare and yet preserve their personal freedom of action? For Rousseau, this was a problem in political theory. As "solution", he postulated a "general will", which each individual in a society must be forced to obey (i.e. he must be "forced to be free"). (This insight was later to be applied by Robespierre, Stalin, and Hitler in their political contributions.) The same problem, however, can be stated in economic terms. Individuals in an economy are interdependent. Insofar as their well-being is thereby affected, externalities may be present. How can they be induced to maximize social welfare and yet maintain their responses, arising from their individual economic motives, to the price system? In other words, how can an economy attain social goals while preserving the free economic interactions of capitalist structures?

To handle this problem, it is not necessary to take refuge in metaphysical entities such as the "general will". We may simply inquire what system of prices is required to induce each economic actor, through his own motives to strive toward social ends. The clash of market forces in laissez-faire capitalist economies does not necessarily elicit this ideal price system. Adam Smith's "Invisible Hand" is often visibly

⁺) I am greatly indebted to Dr. Michael Wagner for his excellent insights which clarified many basic issues underlying this paper.

absent. Just as the individual wills, in Rousseau's theory, do not necessarily coincide with the "general will", individualistic economic activity need not lead to the optimal economic state of society as a whole. The existence of externalities explains why things may go wrong in this respect. Thus, government intervention may be required. The Pigouvian approach to the treatment of externalities shows which form this intervention may take, if prices (to which economic activities are sensitive), but not the decision-making behaviors of economic actors, are tampered with. We do not here specify what the social goals are by which economic activity is to be guided. We assume that they emerge from the political process and our problem is simply that of realizing them through the free price system. (Of course, the goals arising from the political process and the goals required by our analysis may not coincide in manner of formulation or degree of detail. We will consider this problem later). The type of government intervention advocated through the Pigouvian approach depends on the type of externalities to which economic activity gives rise. To begin with, let us examine what, precisely, externalities are and how they can manifest themselves.

Whenever the production activity of one firm affects a real economic variable involved in the activity of another firm or consumer and whenever the activity of one consumer affects a real economic variable of another consumer or firm, an external economic effect is present. If individual A affects individual B in this manner without taking B's welfare gains or losses into account, then an externality is present. In capitalist economies, externalities usually manifest themselves when B is unable to force compensation

from A in an amount equal to the marginal net cost of A's activity on B. (Conversely, if A imposes a boon on B, externalities appear when A is unable to force B to pay for the marginal net benefit.) For example, A, a consumer, may not be in a position to force B, a farmer, to restrict his use of DDT and other chemical insecticides through including A's health damage in B's costs. In short, externalities arise whenever people do not have to pay for the harm they incur or whenever people are not compensated for the boons they provide.

Up to the present day, externalities have been the step-child of economics -- of macro- and micro-economics and of general equilibrium (and disequilibrium) analysis. Economists have restricted externality theory to a small subset of the economic phenomena in which externalities play a prominent part. The externality problems which commonly receive preponderant attention lie in the realm of environmental economics (e.g. lead in the atmosphere, urban slums, highway congestion). It can be shown, however, that unemployment, inflation, stagnation, resource depletion, factor misallocation, and production inefficiency are also externality problems. It is difficult to exaggerate the significance of these problems in today's capitalist economies. As we shall see, conceiving these problems as externalities can permit the achievement of new analytical insights. Let us consider several examples of such problems.

In the Keynesian world, a worker may seek employment at the going wage, but be unable to find it. Were he employed, his effective demand would rise, generating the need for

more consumption goods and (possibly) more capital goods. Thus, if firm A would employ him, not only his level of welfare, but that of other workers and entrepreneurs (all of whom are involved in the consequent multiplier process) as well, may increase. These workers and entrepreneurs cannot extract compensation from firm A for the harm it is incurring by not offering a job to the person above. Hence, this problem may be recognized as one of externalities.

As a second example, let us glance at a problem of factor allocation. A capital good hired by firm A is not available for productive use in firm B, and the comparative social welfare effects of A's and B's activities do not necessarily determine which firm is able to attract the factor. The interfirm distribution of factors is determined by the distribution of factor incentives. Let us suppose that these incentives are offered from profit maximizing motives. Since the contribution to profits of a factor for a given firm need not be equal to its contribution to social welfare, externalities with adverse social welfare effects can arise under these circumstances.

Many more examples along these lines may be adduced without difficulty. Firm A's waste emission does not necessarily take into account consumer B's health damage; firm A's natural resource extraction is not necessarily based on an evaluation of the implicit harm to future generations; and so on. These examples should suffice to illustrate that externalities underlie many of the most significant problems of capitalist economies. A general theory of externalities would clearly provide

the guidelines for an integrated and coherent study of all these problems. A general and implementable theory on the treatment of the adverse social welfare effects of externalities would be tantamount to the solution of all these problems. Clearly, such a theory is not within reach of economists today -- "the end of days" still appears to be distant -- but an analysis of externalities, as we shall see, may be a useful way of approaching these problems.

The Pigouvian approach to the treatment of externalities concentrates exclusively on fiscal measures. The adverse social welfare effects of externalities are to be treated through taxes and subsidies. These fiscal measures are designed so as to force each economic actor, through his reactions to the price system, to take the social welfare consequences of his activity into account. Obviously, this is not the only way in which externalities may be treated. For instance, economic actors may be forced to consider the entire social welfare effects of their activities through direct controls or the reassignment of property rights¹⁾. However, the Pigouvian approach may be particularly convenient and well-suited for capitalist economies. The reason is apparent. In purely capitalist systems, the supplies of and the demands for goods and resources²⁾ are usually responsive to the prices of these goods and resources. In the Pigouvian approach, taxes and subsidies are used to change these prices in such a way that the socially optimal production, consumption, and allocation of goods and resources are achieved. Once the appropriate taxes and subsidies are set, the price system automatically performs the job of attaining the social optimum through the individualistic behavior of the economic actors. Under certain conditions,

the number of taxes and subsidies to be set is small in comparison to the number of special directives (e.g. from a central planning agency) necessary to do the same job. These fiscal measures may also be much more easy to implement -- in political and administrative terms -- than the reassignment of property rights to "internalize" the externalities.

In view of the immense importance of externalities, it is of considerable interest to note that the Pigouvian approach provides a complete fiscal answer to the problem of coping with externality-generated social welfare losses. This outstanding achievement of Pigouvian reasoning does not mean, however, that the problems of externalities have been solved. The difficulties of the Pigouvian approach are inherent in the two basic ingredients required for the formulation of the socially optimal taxes and subsidies. The first ingredient is a summary of each economic actor's decision-making process; the second is a summary of the social welfare effects of his activity.

These prerequisites embody the crucial deficiency of the Pigouvian scheme, since they are theoretically impenetrable and empirically unimplementable. The former ingredient should be gleaned from an examination of each economic actor's role in the market place. The difficulty of this task may be appreciated even if we make the simplifying assumption that consumer and entrepreneurs are the only economic actors and the prices and quantities (of goods and resources) are the only endogenous variables.

Let us make the behavioral hypothesis that these actors are optimizers rather than satisficers. We may treat each

consumer's decision-making process in the standard neoclassical fashion: he maximizes his utility subject to a budget constraint. This approach is acceptable only insofar as our analysis requires few assumptions to be made about the mathematical properties of the utility function. For all practical purposes, utility functions are empirically unobservable³⁾. They are simply that which is maximized when consumption choices are made, i.e. they may be inferred from demand functions but not vice versa.

Each entrepreneur's decision-making process is usually treated as that of profit maximization. Yet there are no compelling reasons to believe that entrepreneurs always single-mindedly maximize profits. Under perfectly competitive conditions, the "natural selection" argument provides strong support for the assumption of profit maximization. Under imperfectly competitive conditions (which prevail in every market in the real world), this argument no longer holds. In fact, Scitovsky has shown⁵⁾ that it is not rational for an entrepreneur to maximize profits unless the level of profits he earns does not affect his subjective trade-off between work and leisure. Thus, other theories of entrepreneurial motivation have been put forth: profit-constrained sales maximization⁶⁾, growth maximization (subject to constraints relating to managerial security)⁷⁾, managerial utility maximization⁸⁾, game-theoretic entrepreneurial interactions⁹⁾, and so on. The empirical evidence in favor of these and other theories of entrepreneurial motivation is inconclusive¹⁰⁾. Of course, entrepreneurial profit maximization may be put on the same tautologous footing as consumer utility maximization. Virtually all entrepreneurial activities may

be interpreted as stochastic profit maximization subject to certain goals which are exclusive to management (as opposed to ownership). Given the past behavior of entrepreneurs, it is probably always possible to find subjective probability distributions and managerial utility functions such that this past behavior may fall into the mold of this interpretation. Once again this approach is acceptable only if our analysis does not require a detailed description of these probability distributions and utility functions.

What emerges from these considerations is that the first ingredient of the Pigouvian scheme -- a summary of each economic actor's decision-making process -- is not easily accessible, both theoretically and empirically. As the state of economic knowledge progresses, it is to be anticipated that this ingredient will come more closely within our reach. Insofar as the formulation of Pigouvian taxes and subsidies depend on this ingredient, our fiscal proposals for the treatment of externalities will improve.

The second ingredient -- a summary of the social welfare effects of each economic actor's activity -- appears to be totally inaccessible. The social welfare effects of economic activities may be derived directly only through a social welfare function. The empirical formulation of such a function requires a comprehensive knowledge of the consequences of production and consumption behavior patterns on the welfare of each individual in the economy and a scheme for making interpersonal comparisons of worth. The prospects of obtaining such knowledge or devising such a scheme are so remote, that it is not even clear what "empirically formulating a social welfare

function" means. We have no methodology for producing such a function and no political experience in reaching a consensus on its underlying value system. Even if it were possible to recognize every social welfare improvement once it manifested itself, it would not, in general, be possible to approach the social optimum through iterative tax setting procedures, on account of nonconvex production sets (which are more likely to be encountered in the presence of externalities than in their absence) and nonconvex utility sets¹¹⁾. Thus, aside from the underlying theoretical conundrums, the empirical implementation of Pigouvian taxes and subsidies appears to be unmanageable.

These difficulties point to the need for replacing the social welfare function by an analytical construct which is more manageable, theoretically and empirically. The most prominent work in this area has been undertaken by W.J.Baumol and W.E.Oates¹²⁾. These authors have restricted their analysis to environmental externalities (i.e. externalities which have noticeable environmental implications) and have substituted environmental standards and an efficiency criteria for the social welfare function. Using entrepreneurial cost-minimization as their first ingredient and the two elements above is their second ingredient, they prove the following theorem: for any given set of outputs produced by the firms in an economy and for the satisfaction of a given environmental standard, the use of unit waste taxes and subsidies permits the production of the above outputs at a minimum cost to society. Naturally, the efficiency criterion and the environmental standards are not perfect substitutes for the social welfare function. Thus, although the Baumol-Oates scheme is (under many circumstances) empirically

implementable, it may not be socially optimal. Presumably, future work in this area will produce empirically implementable schemes which contain closer substitutes of the social welfare function.

In short, the Pigouvian approach to the treatment of externalities calls for research in two areas: (1) empirically testable theories of economic behavior, and (2) empirically manageable substitutes for the social welfare function. The original Pigouvian approach is not implementable. Nevertheless, it will always occupy an essential position in the theory of the fiscal treatment of externalities. The reason why the Pigouvian approach is indispensable in this area is that it specifies the socially optimal fiscal measures, of which any empirically implementable scheme must necessarily be an approximation. In other words, the original Pigouvian approach serves an invaluable function in providing qualitative guidelines to which actual taxes and subsidies for externality treatment must conform. For example, a Pigouvian analysis of externalities may reveal on what goods or resources the fiscal measures should be levied, whether taxes or subsidies should be used alone or in combination, and whether the levels of the tax and subsidy on a given good (or resource) should be of equal magnitude or whether one should be greater than the other. These qualitative properties must be contained by any real-world fiscal scheme for externality treatment, if it is to have a chance at being socially optimal.

In view of the importance of externalities as serious economic problems and the significance of Pigouvian fiscal prescriptions in treating these externalities, it is certainly extraordinary that the economics literature

does not contain a comprehensive exposition of the Pigouvian approach to externalities, in their wide variety of guises (as illustrated by the examples above). What is worse, the commonly accepted view of the Pigouvian approach is plagued by fundamental theoretical confusions. The purpose of this paper is to correct these confusions and to provide the guidelines for a complete statement of the Pigouvian approach. The reasons for undertaking this task have been sketched above -- it is important in its own right.

The Pigouvian approach is not concerned with the identification of externalities; it is concerned solely with the treatment of their adverse social welfare effects¹³⁾. The aim of the Pigouvian approach is to select those goods and resources that may give rise to externalities, examine the price distortions that result from these externalities, and correct these distortions through taxes and subsidies -- thereby establishing the socially optimal flow of the goods and resource services. Thus, it is unnecessary to consider externalities explicitly in the Pigouvian analysis. (e.g. it is unnecessary for the analysis to contain variables that stand for "externalities"). It is the goods and resources that elicit externalities that must be given explicit attention. These goods and resources must not be confused with the externalities themselves. In fact, such a confusion underlies much of the discussion on externalities in the literature -- a confusion whose serious theoretical consequences we will examine here.

So let us not jump ahead of our story. In the next section we describe an economic model which will serve as a concise and rigorous representation of the standard

Pigouvian approach to externalities as it has been popularized in the literature. Although it probably provides a reasonably good picture of the conventional wisdom, it clearly cannot do justice to all economists in this area. It is not an aim here to give a precise characterization of economic thought on the fiscal treatment of externalities. We simply propose to clear up common misconceptions which have gone uncorrected thus far. Our exposition of Section 2 will be the object of our critique in Section 3. This critique will permit a sketch of the Pigouvian approach to several important externality problems in Section 4, thereby providing a foundation for a general theory of Pigouvian externality treatment.

2. A Representation of the Conventional Pigouvian Approach to Externalities¹⁴⁾

As we have mentioned above, our definition of externalities is "confined to effects operating through utility or production functions"¹⁵⁾. Pigouvian taxes and subsidies are designed to ensure that individuals take the welfare effects of their activities on each other into account. What the Pigouvian fiscal prescriptions are, depends on the depletability characteristics of the externalities to be treated.¹⁶⁾ Externalities are said to be depletable if their intake by one economic actor reduces their availability to others. Conversely, undepletable externalities may be absorbed or consumed by one without diminishing their availability to others. Lastly, there are two types of externalities which lie between the two groups above²²⁾: (a) "the case of eventual congestion" (for which the depletability property sets in only at high utilization levels), and (b) "mixed externalities" (in which both depletable and undepletable externalities are generated by a single economic activity).

Numerous examples of undepletable externalities may be adduced. One person's intake of smoke does not significantly reduce the amount left for others to inhale. Public gardens, noise in cities, and effluent in rivers usually have the same property. Undepletable externalities may usually be identified as public goods (or bads) -- although public goods must also satisfy the exclusion property, from which undepletable externalities are exempt. By contrast, it is difficult to find examples of depletable externalities. Baumol and Oates provide the following illustration: "In the postwar period, when there was a severe shortage of fuel, it is reported that in

several parts of Europe many persons spent a good part of their time walking along railroad tracks looking for coal that had been dropped by passing trains. It is clear that this is a depletable externality because every additional bit of coal found by gatherer A meant that so much less was available to B¹⁷⁾. Depletable externalities exist only when there are institutional or legal restrictions preventing their elimination through competitive pricing or when "the cost of collecting a price for (them) exceeds the potential gains"¹⁸⁾ (e.g. when they do not satisfy the exclusion property).

From this characterization of externalities, it becomes clear why depletable externalities have not received much attention in the literature. Depletable externalities usually satisfy the exclusion property and thus, if they are economically important (i.e. their potential price is significant), private enterprise will usually succeed in eliminating them. ("... it is hardly an accident that Bator¹⁹⁾ found a dearth of depletable externalities that constitute critical policy issues."²⁰⁾). Thus, Baumol and Oates conclude that "the undepletable and mixed cases will occupy the bulk of our attention and ... encompass most of the significant cases for public policy..."²¹⁾.

Depletable externalities are "cases where institutional impediments make it impossible to impose the appropriate prices."²³⁾ At the social optimum, the marginal social benefit from an externality should be equal both to the compensation received by the supplier of the externality and to the payment made by the recipient of the externality. Then both the supplier and the recipient would be carrying the full social welfare effects of their

"production" and "consumption" decisions (with respect to the externality). In the absence of Pigouvian intervention, however, the price of a depletable external benefit is too low (i.e. lower than its marginal social benefit). The suppliers are not fully compensated for the boon they provide to others and the recipients are not fully penalized for the harm they incur on others (through reducing the availability of the externality). Thus, a tax must be imposed on the recipients (equal to the difference between the marginal social benefit and the price paid) and a subsidy must be imposed on the suppliers (equal to the difference between the marginal social benefit and the price received). Clearly, the tax and the subsidy are of equal magnitude. (An analogous argument can be made for detrimental depletable externalities). In other words, the price of the externality is adjusted in the socially optimal fashion.

" ... an inefficiency or misallocation of resources resulting from the presence of a depletable externality can be corrected simply by changing an ordinary price equal to marginal social cost (benefit)."²⁴)

By contrast, undepletable externalities give rise to an asymmetry between the proportion of the total social benefit absorbed by the suppliers and that absorbed by the recipients of the externalities. As in the case above, suppliers of an external benefit are not fully compensated for their boon to others, but the recipients incur no harm to others -- for the intake of an undepletable externality does not reduce its availability to others -- and hence they should not be penalized. Consequently, a subsidy should be granted to the suppliers (so that their incentive to supply is equal to the marginal social benefit) and no charge should be imposed on the recipients (since the marginal social

cost of "consuming" the externality is zero).

The intermediate cases of externalities may be treated by a combination of the Pigouvian prescriptions above. The case of eventual congestion (i.e. zero marginal congestion costs up to full capacity) should be treated by the symmetric tax-subsidy scheme at full utilization levels and by the asymmetric scheme at lower utilization levels. The case of mixed externalities requires the simultaneous implementation of the two schemes, the former scheme for the depletable externality and the latter scheme for the undepletable externality.

In order to provide an analytical structure which will serve as reference point throughout this paper, let us sketch the mathematical derivations of the Pigouvian prescriptions for depletable and undepletable externalities. We picture an economy inhabited only by entrepreneurs and consumers, both of which are price takers (an assumption commonly associated with perfect competition). The externalities are by-products of the productive activity and they constitute costs to the entrepreneurs and disutility to the consumers.

Let us first derive the Pigouvian prescriptions with respect to an economy that generates only one depletable externality. For this purpose, we employ the following notation:

$\underline{x}_{i,j} = [x_{11}, \dots, x_{ij}, \dots, x_{nm}]$, a vector of goods and resources of types i ($i = 1, \dots, n$) consumed

(positive x_{ij}) or supplied (viz. labor, negative x_{ij}) by individuals j ($j = 1, \dots, m$);

$\underline{x}_{ij} = [x_{i1}, \dots, x_{ij}, \dots, x_{im}]$, a vector of goods and resources of types i consumed or supplied by individual j ;

$\underline{y}_{ik} = [y_{1k}, \dots, y_{ik}, \dots, y_{nk}]$, a vector of goods and resources of types i produced (positive y_{ik}) or used (negative y_{ik}) by firm k (where $k = 1, \dots, m$)

$\underline{z}_j = [z_1, \dots, z_j, \dots, z_m]$, a vector of the amounts of the depletable externality received by individuals j ;

z_k , the amount of the depletable externality absorbed by firm k ;

s_k , the amount of the depletable externality emitted by firm k ;

s_{-1} , the amount of the depletable externality remaining from the last time period;

r_i , the amount of the good or resource i available from the last time period;

$U^j(\underline{x}_{ij}, z_j)$, individual j 's welfare function; and

$U(\underline{x}_{ij}, \underline{z}_j)$, the social welfare function;

$f^k(\underline{y}_{ik}, s_k, z_k) \leq 0$, firm k 's production function.

Computation of the social welfare optimum permits specification of the suppliers' and recipients' prices, with respect to which the Pigouvian taxes and subsidies are set. Thus, we maximize the social welfare function subject to the firms' production function constraints, the product and resource constraints, and the externality constraint:

(1) Maximize $U(\underline{x}_{ij}, \underline{z}_j)$

subject to

$$f^k(\underline{y}_{ik}, s_k, z_k) \leq 0 \quad k = 1, \dots, h$$

$$\sum_{j=1}^m x_{ij} - \sum_{k=1}^h y_{ik} \leq r_i \quad i = 1, \dots, n$$

$$\sum_{j=1}^m z_j + \sum_{k=1}^h z_k \leq \sum_{k=1}^h s_k + s_{-1}.$$

The corresponding Lagrangian expression is

$$\begin{aligned} \mathcal{L} = & U(\underline{x}_{ij}, \underline{z}_j) - \sum_k \mu_k [f^k(\underline{y}_{ik}, s_k, z_k)] \\ & + \sum_i \omega_i [r_i - \sum_j x_{ij} + \sum_k y_{ik}] + \rho [\sum_k s_k + s_{-1} - \sum_j z_j - \sum_k z_k] \end{aligned}$$

where μ_k , ω_i , and ρ are Lagrangian multipliers.

The first-order conditions may be written as follows:

$$(1a) \quad \frac{\partial \mathcal{L}_i}{\partial x_{ij}} = U_{ij} - \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_i}{\partial x_{ij}} \cdot x_{ij} = 0;$$

$$(1b) \quad \frac{\partial \mathcal{L}_i}{\partial y_{ik}} = -\mu_k \cdot f_i^k + \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_i}{\partial y_{ik}} \cdot y_{ik} = 0; \quad (25)$$

$$(1c) \quad \frac{\partial \mathcal{L}_j}{\partial z_j} = U_j - \rho \leq 0, \quad \frac{\partial \mathcal{L}_j}{\partial z_j} \cdot z_j = 0;$$

$$(1d) \quad \frac{\partial \mathcal{L}_k}{\partial s_k} = -\mu_k \cdot f_s^k + \rho \leq 0, \quad \frac{\partial \mathcal{L}_k}{\partial s_k} \cdot s_k = 0;$$

$$(1e) \quad \frac{\partial \mathcal{L}_k}{\partial z_k} = -\mu_k \cdot f_z^k - \rho \leq 0, \quad \frac{\partial \mathcal{L}_k}{\partial z_k} \cdot z_k = 0;$$

where $U_{ij} = (\partial U / \partial x_{ij})$, $U_j = (\partial U / \partial z_j)$, $f_i^k = (\partial f^k / \partial y_{ik})$,
 $f_s^k = (\partial f^k / \partial s_k)$, and $f_z^k = (\partial f^k / \partial z_k)$;

and the constraints and nonnegativity conditions of program (1) must be satisfied as well. We assume that the second-order conditions are satisfied, i.e. that a solution to the social welfare program exists and is unique. (As is commonly known, sufficient conditions for this assumption to hold true are that each firm's production constraints be twice differentiable and define a convex feasible production set and that the social welfare function is a twice differentiable, quasi-concave function monotonically increasing with respect to the variables x_{ij} .)

If we recall, these characteristics of the social optimum constitute the second ingredient in the formulation of Pigouvian taxes and subsidies; they are a summary of the social welfare effects of each entrepreneur's and consumer's economic activity. We now turn to the first ingredient: a summary of each economic actor's decision-making process. Let us follow the usual approach of treating each consumer's activity as the outcome of utility maximization subject to a budget constraint and each firm's activity as the outcome of profit maximization subject to a production function constraint. Our aim is to use taxes and subsidies to influence consumption and production behavior in such a way that the social welfare optimum is attained. For each consumer we impose the taxes t_{ij}^x and t_j^z per unit of the good x_{ij} and externality z_j (respectively) "consumed"; and for each entrepreneur we impose the taxes t_{ik}^y , t_k^z , and t_k^s per unit of the good (y_{ik}) produced or used and per unit of the externality absorbed (z_k) and emitted (s_k), respectively.

The budget constraint of each consumer indicates that his income earned in the present time period ($P_1 \cdot x_{1j}$, where x_{1j} stands for the labor services of the j 'th individual) plus his expendible wealth for this time period²⁶⁾ (W_j) must not fall short of his purchases of goods and resources ($\sum_{i \neq 1} P_i \cdot x_{ij}$) plus his tax payments ($\sum_i t_{ij}^x \cdot x_{ij} + t_j^z \cdot z_j$). The entrepreneur is pictured as maximizing profits after tax. Thus, the individualistic behavior of the actors in our competitive economy may be described as follows:

$$(2) \text{ Maximize } U^j(x_{ij}, z_j)$$

subject to

$$\sum_{i=1}^n P_i \cdot x_{ij} + \sum_{i=1}^n t_{ij}^x \cdot x_{ij} + t_j^z \cdot z_j \leq W_j$$

for $j = 1, \dots, m$;

$$(3) \text{ Maximize } \sum_{i=1}^n P_i \cdot y_{ik} - \sum_{i=1}^n t_{ik}^y \cdot y_{ik} - t_k^z \cdot z_k - t_k^s \cdot s_k$$

subject to

$$f^k(y_{ik}, s_k, z_k) \leq 0$$

for $k = 1, \dots, h$.

The first-order conditions for a general equilibrium may be expressed as follows:

$$(2a) \quad \frac{\partial \mathcal{L}_j}{\partial x_{ij}} = U_i^j - \alpha_j \cdot P_i - \alpha_j \cdot t_{ij}^x \leq 0, \quad \frac{\partial \mathcal{L}_j}{\partial x_{ij}} \cdot x_{ij} = 0;$$

$$(2b) \quad \frac{\partial \mathcal{L}_j}{\partial z_j} = U_z^j - \alpha_j \cdot t_j^z \leq 0, \quad \frac{\partial \mathcal{L}_j}{\partial z_j} \cdot z_j = 0;$$

$$(3a) \quad \frac{\partial \mathcal{L}_k}{\partial y_{ik}} = P_i - \gamma_k \cdot f_i^k - t_{ik}^y \leq 0, \quad \frac{\partial \mathcal{L}_k}{\partial y_{ik}} \cdot y_{ik} = 0;$$

$$(3b) \quad \frac{\partial \mathcal{L}_k}{\partial z_k} = -t_k^z - \gamma_k \cdot f_z^k \leq 0, \quad \frac{\partial \mathcal{L}_k}{\partial z_k} \cdot z_k = 0;$$

$$(3c) \quad \frac{\partial \mathcal{L}_k}{\partial s_k} = -t_k^s - \gamma_k \cdot f_s^k \leq 0, \quad \frac{\partial \mathcal{L}_k}{\partial s_k} \cdot s_k = 0;$$

where \mathcal{L}_1 and \mathcal{L}_2 stand for the Lagrangian expressions corresponding to the two programs above, α_j and γ_k are their respective Lagrangian multipliers, $U_i^j = (\partial U^j / \partial x_{ij})$, and $U_z^j = (\partial U^j / \partial z_j)$. In addition, the constraints and nonnegativity conditions of the two programs must hold. We assume that an equilibrium for this system exists and is unique.³⁷⁾

Let us compare conditions (1a) - (1c) with conditions (2a) - (3c) and inquire how the taxes and subsidies must be set so that the solutions of the two systems are identical. In particular, these fiscal prescriptions will constitute necessary and sufficient conditions for the social optimality of our competitive equilibrium. Social welfare, U , may be understood as a function of the individual welfares of all consumers, U^j ($j = 1, \dots, m$), and these in turn depend on goods and resources consumed and supplied and the externality absorbed. Let us use the inverse of $\alpha_j = (\partial U^j / \partial W_j)$ as the factor whereby the j 'th individual welfare function is weighted in the social welfare function, i.e. as $(\partial U / \partial U^j)$. In other words, the amount by which j 's welfare rises as result of an infinitesimal outward shift of his budget constraint is the yardstick we use to make interpersonal comparisons of worth. This yardstick is merely an arbitrary mathematical convention, because, for any given value of U^j , there always exists a monotonic transformation of the ordinal welfare function U^j such that $(\partial U^j / \partial W_j)$ is equal to an independently - chosen weight. Thus, we set α_j in such a way that $U_{ij} = (U_i^j / \alpha_j)$. (To be specific, take any social welfare function $U = U(\underline{U}^j)$, where \underline{U}^j is a vector of the utilities of all individuals j , and find $(\partial U / \partial U^j)$ for all j . Then take any individual welfare function U^j

and, in the neighborhood of the optimum of program 2, transform its slope such that $1/(\partial U^j / \partial W_j) = (\partial U / \partial U^j)$. In this way any value of U^j (as determined by program (2)) may be made compatible with the interpersonal-comparisons weight as given by the social welfare function).

Next, we note that our assumption of only one externality in our system, $\sum_k s_k$, implies that all other goods and resources must be k priced at their socially optimal levels. (As we have seen above, socially suboptimal prices give rise to externalities). In other words, $P_i = \omega_i = (\partial U / \partial r_i)$ for all goods and resources i . A glance at conditions (1a) and (2a) -- $(\partial \mathcal{L} / \partial x_{ij})$ -- reveals that if $U_{ij} = (U^j / \alpha_j)$ and $P_i = \omega_i$, then the tax t_{ij}^x must be set equal to zero if both conditions are always to be satisfied simultaneously.

We now turn to conditions (1b) and (3a) -- $(\partial \mathcal{L} / \partial y_{ik})$. Since $P_i = \omega_i$, these conditions may be rewritten as follows:

$$(1b) \quad P_i \leq \mu_k \cdot f_i^k \quad (\text{for social optimality})$$

$$(3a) \quad P_i \leq \gamma_k \cdot f_i^k - t_{ik}^y \quad (\text{for competitive equilibrium}).$$

Now the tax t_{ik}^y must be set in such a way that, for any given values of P_i and f_i^k (as determined by the competitive equilibrium), whenever condition (3a) holds, (1b) must hold as well. It is clear upon inspection that this can only happen when $t_{ik}^y = 0$ and $\mu_k = \gamma_k$. (If $\mu_k \neq \gamma_k$, then there does not exist a constant tax rate

t_{ik}^y such that the two conditions above hold simultaneously for any given P_i and f_i^k . If $t_{ik}^y \neq 0$, then there do not exist a constant μ_k and a constant γ_k such that the conditions above are satisfied for any P_i and f_i^k . In either case, our assumption of the existence of solutions to the social welfare program and the competitive equilibrium system would be violated).

The rest of the Pigouvian taxes may be derived in an analogous manner. Comparing conditions (1c) and (2b) -- $(\partial \mathcal{L} / \partial z_j)$ -- if $U_j = (U_j^j / \alpha_j)$, for which α_j is set appropriately, then $t_j^z = 0$. Comparing conditions (1d) and (3c) -- $(\partial \mathcal{L} / \partial s_k)$ -- $\mu_k = \gamma_k$ means that $t_k^s = -0$. Lastly, comparing conditions (1e) and (3b) -- $(\partial \mathcal{L} / \partial z_k)$ -- $t_k^z = 0$.

The Pigouvian prescriptions for the treatment of depletable externalities, which were explained intuitively above, have now been derived rigorously. Production and consumption of all the goods and resources except for the externality are not taxed or subsidized, because these goods and resources are already priced at their socially optimal levels:

$$(4) \quad t_{ij}^x = 0 \quad \text{and} \quad t_{ik}^y = 0 \quad \text{for all } i \text{ and } k.$$

The externality, however, has no price in the absence of Pigouvian intervention and taxes and subsidies are imposed on it so that its new price is equal to its marginal social benefit:

$$(5) \quad t_j^z = t_k^z = 0 \quad \text{and} \quad t_k^s = -0.$$

In short, the users of the externality pay in accordance with its marginal welfare effect, which is the amount by which the suppliers of the externality are compensated. Now the externality, along with all other goods and resources, is priced in the socially optimal manner.

Pigouvian taxes and subsidies in response to undepletable externalities may be derived analogously. The two ingredients of these fiscal measures are, once again, the social welfare maximization program on the one hand and the consumer and firm maximization programs on the other. The economy is assumed to generate only one undepletable externality (which is a by-product of production activity). Each consumer's intake of the externality (say, smoke²⁸) is independent of any other economic actor's intake (otherwise the externality would not be undepletable). The total amount of the externality would not be undepletable). The total amount of the externality in the system (z) is equal to the amount left over from last time period (s_{-1}) plus the amount generated in the present time period ($\sum_k s_k$). The j 'th consumer inhales $a_j \cdot z$ of smoke. (For the sake of simplicity, we suppose that the smoke content of the atmosphere is uniform over all geographic locations). a_j is a function of the goods and resources purchased by j : $a_j = a_j(x_{ij})$. For example, individual j may reduce his intake of smoke by installing an air-purification machine in his home. Similarly, the k 'th firm is affected by $b_k \cdot z$ of smoke, where $b_k = b_k(y_{ik})$.

As in the case of undepletable externalities, the consumer may be taxed or subsidized both with respect to the goods and resources he consumes (t_{ij}^x) and with respect to the

externality he absorbs (t_j^z). (The Coasian possibility of taxing the victims of externalities²⁹⁾ is thus allowed for.) Each of these taxes can influence both his purchases x_{ij} and his externality intake $a_j(x_{ij})$. Firms may be taxed or subsidized for the goods they produce (t_{ik}^y) and for the externality they absorb (t_k^z) and emit (t_k^s). Once again, each of these taxes can affect y_{ik} , $b_k(y_{ik})$, z , and s_k .

The social welfare program and the programs of individualistic behavior may be presented as follows:

(6) Maximize $U(\frac{x_{ij}}{i,j}, z)$

subject to

$$f^k(\frac{y_{ik}}{i}, s_k, z) \leq 0 \quad \forall k \quad \mu_k$$

$$\sum_j x_{ij} - \sum_k y_{ik} \leq r_i \quad \forall i \quad \omega_i$$

$$\sum_k s_k + s_{-1} = z \quad \rho$$

(7) Maximize $U^j(\frac{x_{ij}}{i}, z)$

subject to

$$\sum_i P_i \cdot x_{ij} + \sum_i t_{ij}^x \cdot x_{ij} + t_j^z \cdot a_j \cdot z \leq W_j \quad \alpha_j$$

(8) Maximize $\sum_i P_i \cdot y_{ik} - \sum_i t_{ik}^y \cdot y_{ik} - t_k^z \cdot b_k \cdot z - t_k^s \cdot s_k$

subject to

$$f^k(\frac{y_{ik}}{i}, s_k, z) \leq 0 \quad \forall k \quad \gamma_k$$

(The greek letter next to each constraint is its Lagrangian multiplier).

We assume that solutions to (4) and to (5) and (6) exist and are unique. The first-order conditions (in addition to the satisfaction of the constraints and the nonnegativity conditions) may be written in corresponding pairs:

$$\frac{\partial \mathcal{L}_1}{\partial x_{ij}} = U_{ij} - \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_1}{\partial x_{ij}} = U_i^j - \alpha_j \cdot [P_i + t_{ij}^x + a_j^i \cdot t_j^z] \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial x_{ij}} \cdot x_{ij} = 0, \quad \frac{\partial \mathcal{L}_1}{\partial x_{ij}} \cdot x_{ij} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial y_{ik}} = -\mu_k \cdot f_i^k + \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial y_{ik}} = P_i - t_{ik}^y - t_k^z \cdot b_k^i \cdot z - \gamma_k \cdot f_i^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial y_{ik}} \cdot y_{ik} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial y_{ik}} \cdot y_{ik} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial s_k} = -\mu_k \cdot f_s^k - \rho \leq 0, \quad \frac{\partial \mathcal{L}_3}{\partial s_k} = -t_k^s - \gamma_k \cdot f_s^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial s_k} \cdot s_k = 0, \quad \frac{\partial \mathcal{L}_3}{\partial s_k} \cdot s_k = 0;$$

where $a_j^i = (\partial a_j / \partial x_{ij})$ and $b_k^i = (\partial b_k / \partial y_{ik})$.

The Pigouvian fiscal prescriptions that emerge from the simultaneous fulfillment of these conditions may be given as

$$t_{ij}^x + t_j^z \cdot a_j^i \cdot z = 0$$

$$t_{ik}^y + t_k^z \cdot b_k^i \cdot z = 0$$

$$t_k^s = \rho;$$

and since, in general, $a_j^i \cdot z$ and $b_k^i \cdot z$ need not be equal to zero, we find

$$(9) \quad t_{ij}^x = t_j^z = t_{ik}^y = t_k^z = 0$$

$$(10) \quad t_k^s = \rho.$$

Thus, only one tax should be imposed: a tax on the emitter of the externality. The tax rate is to be set equal to the marginal welfare cost of the externality ($\rho = \partial U / \partial z$). The intake of the externality is to be neither taxed nor subsidized, since each individual absorbs the full social cost of his intake.

In conclusion, the Pigouvian approach to the treatment of externalities yields important qualitative conclusions. A depletable externality is to be treated like any other private good; its price should be set equal to its marginal social welfare. If the pretax price of a depletable external benefit is zero, its socially optimal price may be attained through the imposition of a tax on the receiver and a subsidy -- of equal magnitude -- on the supplier. An undepletable externality requires

asymmetric treatment of receivers and suppliers. Receivers should not be taxed or compensated, and suppliers should be reimbursed in proportion to the marginal social benefit (or cost) of their externality. The intermediate cases of externalities require a combination of the fiscal prescriptions for the two polar extremes above. The case of eventual congestion (i.e. zero congestion costs until capacity is reached³⁰⁾) should be treated as undepletable externality below capacity and as depletable externality once capacity has been reached. The mixed case³¹⁾ requires a tax (subsidy) on the receiver equal to the marginal social benefit (cost) of the depletable part and a subsidy (tax) on the supplier equal to the marginal social cost (benefit) of the depletable part plus the marginal social cost (benefit) of the undepletable part.

These are the conventional Pigouvian prescriptions as they have been popularized in the literature. In the next section, we will give this analysis a second thought and find several serious confusions in it. Once these confusions have been eliminated, it will become clear that externalities comprise a much broader class of economic phenomena than the discussion of this section would indicate. Furthermore, it will be possible to present the general methodology for deriving the Pigouvian fiscal prescriptions for all of these phenomena. The analysis presented in this section will serve us well in this endeavour: it will provide the theoretical foundation upon which our general statement of the Pigouvian approach to the treatment of externalities will be built.

3. Some Fundamental Conceptions, Misconceptions, and Reconceptions on Externality Treatment

The fundamental ambiguity of the analysis presented in the last section lies in its failure to distinguish between externalities and the goods or resources which elicit them. Once this ambiguity has been overcome, the Pigouvian approach to externality treatment appears in a new light -- one which has policy implications for virtually every troublesome economic problem. As we mentioned in the opening sentence of the last section, externalities are "effects operating through utility or production functions". Externalities arise as the result of a relation between two or more economic actors. They arise only when the activity of one individual (or economic entity) influences the well-being of another individual and when the former is not forced to take this welfare effect into account. If we consider a world of greedy individualists -- i.e. a world in which altruism is absent -- we find that externalities appear when one individual influences another's welfare without being compensated for this effect.

The standard analytical specifications of externalities -- as typified in the last section -- do not agree with this conception, however. These "externalities" enter utility functions and production functions as independent variables. For instance, Baumol and Oates write that "... the variable z in each utility and production function represents the possibility that the utility (production) of the corresponding individual (firm) is affected by the output of the externality in the community."³³) Clearly, if an externality is generated

through a relation between two or more economic actors, then it cannot enter the utility or production function of any one actor. An externality can be determined only once the utility or production functions of two or more actors are known (so that the nature of the inter-individual economic interaction is specified); it cannot be specified with reference to just one actor's behavioral relations. Interindividual welfare effects are not meaningful as independent variables of utility and production functions. Besides, calling the variable z (above) the "output of an externality" leads one to believe that a producer can generate an externality out of his own accord, through the productive services of his factors and resources. Interindividual welfare effects cannot be designated as outputs, without doing violence to the accepted definitions of "welfare effects" or "outputs".

Although externalities are identified as inter-individual welfare effects in countless introductory statements of the literature on this subject, we can see that the "externalities" we have encountered in the last section do not fit this description. Rather, they may be identified as the goods or resources generating an uncompensated interaction between economic actors.³⁴⁾ The appropriateness of this definition becomes clear once we consider what "depletable externalities" and "undepletable externalities" are. External effects cannot be described as "depletable" or "undepletable": it does not make sense to state that the intake of an inter-individual welfare effect by one individual reduces (or does not reduce) its availability to others. Only goods and resources can be characterized by depletability. Goods and resources can enter utility and production

functions; they can be taxed and subsidized.

If we now reconsider the analysis of section 2 in this light, it becomes evident that the Pigouvian approach to the treatment of externalities is concerned with the pricing of goods and resources. Taxes and subsidies are set with a view to the inter-individual welfare effects that arise from the production and use of these goods and resources. It is unnecessary to identify and evaluate the welfare effects explicitly. Simply, after-tax prices must be set in such a way that economic actors produce and utilize goods and resources in the best interests of society as a whole. In other words, they must be set so as to activate Adam Smith's "Invisible Hand": each actor, through his individualistic responses to these prices, will maximize social welfare.

If every uncompensated inter-individual welfare effect is to be generated by "goods and resources", these must be interpreted broadly. "Goods" are here identified as the result of production or consumption activity; they are real (i.e. nonmonetary) economic entities which could not come into being without the expenditure of economic activity. "Resources" are real economic entities which are used for (or absorbed in) production or consumption activities but which are not directly generated by these activities. Tractors, industrial effluent, and roads are thus called "goods", whereas coal, soil, and labor are called "resources". Not all goods and resources give rise to externalities. A good or resource which concerns only one individual (or firm) cannot elicit externalities, because the individual,

when he makes use of it, always takes its entire social welfare effect into account (viz. its effect on him). Only a good or resource which influences or is influenced by more than one individual can generate externalities. We call such a good (or resource) a common good (or resource). A good (or resource) which enters the production or utility function of only one individual (or firm) we call an individual good (or resource).

If we drop the word "externality" from the analysis of Section 2 and substitute "common goods and resources" instead, this analysis acquires novel and far-reaching implications. Once the Pigouvian approach is formulated in terms of depletable and undepletable common goods and resources, it is evident that both types of goods and resources are abundant in the real world. Whereas Baumol and Oates write that "it is not easy to provide a convincing example of a depletable externality"³⁵) -- a term whose meaning is inherently ambiguous -- it is certainly not hard to find a convincing example of a depletable common good or resource. The goods and resources which have received most attention in economics -- consumption goods such as tobacco products, apparel, and newspapers, capital goods such as valves, turbines, and computers, and resources such as labor, petroleum, and wood -- are all depletable. The goods most commonly associated with externalities -- solid wastes (such as DDT is food products), liquid wastes (e.g. high BOD effluent in fishing waters), and atmospheric wastes (such as lead or sulphur dioxide) -- are undepletable. Nevertheless, both depletable and undepletable goods (and resources) can generate externalities, as long as they are common goods (and resources).

In the last section we mentioned that depletable externalities have received scant attention because they are hard to find in the real world. As long as their potential price is sufficiently high and the cost of changing a price is not prohibitive (because they do not satisfy the exclusion property or because of institutional impediments to competitive pricing), it will be profitable for private enterprise to eliminate them (by demanding compensation for them). In the light of our discussion above, this idea translates into the following proposition: in the absence of institutional restriction, competitive pricing eliminates all externalities generated by depletable goods and resources which satisfy the exclusion property. Let us examine this proposition closely and decide whether economists are justified in ignoring externalities elicited by depletable goods and resources for this reason.

The assumption that there are no institutional restrictions to competitive pricing whenever interpersonal welfare effects are present, is extremely unrealistic. Institutional restrictions to competitive pricing are so commonplace, that an economic proposition which assumes them away is of questionable relevance. I may be greatly irritated that my neighbor, Mr. Smith, wears pink pyjamas, yet I am unable to extract payment from him whenever he does so (the case of private decisions in a liberal democracy). The purchase of aircraft by the airline industry gives rise to expanded output and employment activity in other industries, but the entrepreneurs of the airline industry are unable to receive full compensation for this multiplier effect

(the absence of institutions for the compensation of current interpersonal welfare effects). I may be willing to buy a pencil sharpener (of a specified type) at the price of 50 cents on April 1, 2000 A.D., yet I cannot find a seller who will seriously consider my offer -- not to speak of the myriad buyers and sellers necessary for competitive pricing (the absence of perfect futures markets). Future generations cannot force us to take their welfare into account when we exhaust our natural resources and destroy our environment (the absence of institutions for the compensation of intertemporal interpersonal welfare effects). In sum, institutional restrictions to competitive pricing are certainly not hard to find; in fact, they are so common that they threaten to turn our proposition above into a theoretical curiosity.

For the sake of argument, we will nevertheless assume these institutional restrictions away and consider whether competitive pricing eliminates the externalities generated by depletable goods and resources. In fact, it will be necessary to consider only whether socially harmful externalities generated by depletable goods and resources are eliminated. Externalities of the latter type are the only ones that warrant Pigouvian fiscal treatment. If they exist in the face of competitive pricing, then clearly externalities in the wider sense exist as well. We have mentioned above that the externalities will not be eliminated if the goods and resources do not satisfy the exclusion property. Clearly, prices can be charged on depletable goods only if it is possible to exclude potential recipients who have not paid these prices. The difference between excludable and nonexcludable goods is often a difference

of degree rather than of kind. A retailer of jewelry must install a burglar-alarm system and a steal-reinforced show-window in order to exclude customers who have not paid the appropriate price from consuming his product. The costs of exclusion necessary to extract compensation for the use of roads are, for example, the costs of building and maintaining tollbooths and the remuneration of officials to man them. The goods (and resources) required for this exclusion process we will call exclusion goods (and resources). The costs of these goods will be identified as exclusion costs. The higher the exclusion costs necessary to impose a price on a depletable good (or resource) relative to the potential compensation receivable from that good (or resource), the more likely are externalities to remain in the face of competitive pricing. In the example above, cost of the burglar alarm system and the show window as proportion of the value of the jewelry sold is often less than the cost of the toll booths and their officials as proportion of the toll revenue collected. The sale of every good or resource implies positive exclusion costs. The popular examples of goods which do not satisfy the exclusion property may be viewed as goods with prohibitively high exclusion costs. The costs of exclusion with respect to the services of National defence comprise, among others, the cost of printing admission tickets to fall-out shelters and the cost of rounding up non-contributors and flying them to the Falkland Islands in time of war. (There seem to be several "institutional impediments" to such a scheme, as well).

Exclusion costs are the costs of identifying the recipient of a good (or resource). These are not the only costs

necessary to impose a price on the good, however. The costs of identifying the emitter (or supplier) of a good (or resource) must be considered as well. We will call such costs "metering costs". These costs are particularly important with respect to a variety of undepletable goods (e.g. the costs of monetaring fly-ash from coal-fired power plants), but they are present for depletable goods as well. Externalities from dumping garbage on someone's private property may arise either because the exclusion costs (e.g. the costs of building a fence) or the metering costs (e.g. the costs of guarding the property so as to identify the dumper and extract a fine from him) are high. The activity of Irish bomb manufacturers has interpersonal welfare effects for which adequate compensation would not be demanded even in the absence of "institutional impediments", because this activity is difficult to detect. (A bomb has some properties of a depletable good). The magnitude of metering costs plus exclusion costs -- relative to the value of the goods or resources to which these costs are directed -- is a determinant of whether private enterprise finds it profitable to eliminate externalities generated by depletable goods or resources under a perfect competitive price system.

In Section 1 we considered the problem of allocating depletable and excludable factors among firms in an economy. The interfirm allocation of labor depends, say, on relative real wages, which -- in the absence of institutional impediments -- we may take to depend on entrepreneurial opportunities for profit maximization. Since there is no reason to believe that labor's relative contributions to the profits of different firms mirror

its relative contributions to social welfare, externalities with adverse social welfare effects may be generated by this depletable resource. It is evident that if the free price system permits final outputs to be produced and used in a socially suboptimal manner then the common input services embodied in these outputs generate socially harmful externalities. For the sake of methodological simplicity, we may avoid consideration of input externalities entirely. Elsewhere³⁷⁾ I have shown how input externalities may be expressed as components of final-output externalities and hence it is necessary to consider only the latter.

As for final outputs, there exists an extensive literature on the conditions under which competitive pricing of these outputs gives rise to a general equilibrium which is unique and socially optimal. A review of this literature lies beyond the scope of our discussion here. For our purposes, let it suffice to consider just one assumption -- albeit a very important one -- necessary for the social optimality of competitive equilibrium: the assumption that consumers have perfect knowledge on how to maximize their welfare and that entrepreneurs have perfect foresight with respect to the production and sale of their goods. This assumption cannot be upheld in general. It appears to be commonly accepted that consumers do not always know what is best for themselves and that paternalistic judgements -- imposed by politicians on society -- are therefore necessary. A consumer may be unduly myopic when he buys his cartons of cigarettes or his bottles of whiskey and thus the relative profitability of producing these items -- versus that of, say, growing carrots -- may exceed their relative welfare contributions. Taxes and subsidies

may be necessary to force the consumer to take the possibility of lung cancer or liver malfunction into account. Furthermore, entrepreneurs do not have perfect foresight. If an entrepreneur invests under the expectation that consumers will buy the output of his capital purchase, and if consumers - who know how to maximize their welfare -- decide otherwise, then the expenditure on the capital good may exceed the amount which ex post social welfare considerations warranted. In an ex ante sense, however, the capital purchase may generate no externalities. Once again, there is room for paternalistic judgements. If the government has better foresight than the entrepreneurs, fiscal measures may be needed to make the entrepreneurs behave as if they had a more accurate view of the future.

From these various considerations, we must conclude that competitive pricing will not eliminate all externalities generated by depletable goods and resources

- (a) if the goods and resources are associated with sufficiently high exclusion costs or metering costs,
- (b) if there are institutional impediments to competitive pricing, or
- (c) if it is appropriate to make paternalistic judgement with respect to the free decisions of economic actors.

This is certainly not a complete list of conditions, but it will suffice for our purpose. It shows that externalities engendered by depletable goods and resources are a common occurrence in the real world.

No relevant theory on the treatment of externalities can afford to ignore them.

Thus, the scant attention which these externalities have received in the literature is not understandable, as economists thus far would have us believe. It is a grave omission, which has restricted externality theory to a small subset of the economic phenomena in which externalities play a prominent part.

For instance, Baumol and Oates provide the following list of serious externality problems:

- "(a) Sulphur dioxide, lead, and other contaminants of the atmosphere,
- (b) Various degradable and nondegradable wastes that pollute the world's waterways,
- (c) DDT and other pesticides, which, through various routes, become imbedded in food products,
- (d) Deterioration of neighborhoods into slums,
- (e) Congestion along urban highways,
- (f) High noise levels in metropolitan areas." 38)

These are all problems caused by undepletable goods (with the possible exception of (e), which might contain depletable elements). Once the externalities caused by depletable goods and resources are given rigorous theoretical consideration, the list above may be expanded to include other problems -- problems which are as serious as any treated in economics: unemployment, inflation, stagnation, resource depletion, factor misallocation, and production inefficiency. The Pigouvian

approach yields qualitative prescriptions for the fiscal treatment of these externality problems. Not only does it provide guidelines for the fiscal treatment of one of these problems in isolation from the others, but also for fiscal responses to their simultaneous occurrence. In view of economist's general helplessness in analyzing -- not to speak of curing -- a combination of these problems, this is certainly an analytical avenue worth pursuing.

As we have indicated above, both externalities generated by undepletable goods and resources and those generated by depletable goods and resources are prevalent in the real world and have serious economic implications. Consequently, the Pigouvian approach must be formalized with respect to both types of externalities. The failure of the literature on this subject to distinguish between externalities and the goods and resources which give rise to them -- as illustrated in the previous section -- has prevented an accurate derivation of the Pigouvian prescriptions. We now proceed to examine a small sample of such prescriptions in the light of the thoughts above. It is surely superfluous to note that our sample of externality problems and corresponding Pigouvian prescriptions will be neither representative nor comprehensive, for we are here dealing with a vast area -- the fiscal treatment of virtually every serious economic problem. The few problems explored in the next section are simply meant to clarify the methodology necessary for the development of a general theory of Pigouvian externality treatment.

4. Towards a General Theory of Pigouvian Externality Treatment

Let us begin by considering a model in which adverse social welfare effects can arise from externalities generated by only one depletable resource: labor³⁹⁾. To ensure that adverse social welfare effects cannot arise from any other source, we assume that all other resources and goods are priced in the socially optimal manner. There are j consumers in the economy ($j=1, \dots, m$) and, for the sake of generality, we let each consumer represent a different type of labor (that is, the productive services of each individual in the economy enter separately into the production functions of the firms). The amount of labor supplied by each individual may be positive or zero. We assume that labor is employed only by firms, not by households.⁴⁰⁾ There are k firms in the economy ($k=1, \dots, h$). The production function of the k 'th firm is $f^k(y_{ik}, L_{jk}) \leq 0$, where a positive y_{ik} stands for the i 'th output of that firm, a negative y_{ik} stands for its input of the i 'th good or resource (not including labor), and L_{jk} is the amount of labor of the j 'th type working for the firm. Each type of labor in each firm receives a wage which is set at a fixed, arbitrary level, w_{jk} (the wage of the j 'th type of labor in the k 'th firm). This level need not be socially optimal, and thus adverse social welfare effects from labor-generated externalities may arise. These adverse effects may be treated by Pigouvian taxes and subsidies.

We assume that each good and resource in the economy is depletable. (The undepletable case is treated in the

last part of this section). Thus, if x_{ij} is the amount of the i 'th good or resource consumed by the j 'th individual, y_{ik} is the amount of the i 'th input or output of the k 'th firm, and r_i is the amount of the good or resource available from the last time period, we may write $\sum_j x_{ij} + \sum_k y_{ik} \leq r_i$ for $i=1, \dots, n$. Similarly, for each labor type j , we have $\sum_k L_{jk} \leq \sum_k^s L_{jk}$ (where L_{jk} is labor j demanded by firm k and $^s L_{jk}$ is labor j supplied to firm k). Exclusion and metering costs are taken to be negligible⁴¹; in other words, we ignore the costs of attaching a price P_i , to a good or resource. Both consumers and firms are price takers.

As in Section 2, each consumer maximizes his welfare subject to a budget constraint. The j 'th individual's welfare, U^j , depends on the goods and resources he consumes, x_{ij} , and on the labor he performs. Since we are hereby describing his decision-making process, we must use an ex ante welfare function. To maximize his welfare (ex ante), he has two sets of decision variables, x_{ij} and $^s L_{jk}$. Whereas his ex post welfare depends on L_{jk} , this labor demand is out of his control and not known in advance. What is crucial for his decision-making process is his subjective evaluation of the demand for his services, and this evaluation will determine the services he supplies. For the moment, we assume that each working individual expects all the labor services he supplies to be demanded. Thus, the j 'th individual's welfare function may be written as $U^j(x_{ij}, ^s L_{jk})$.

Each firm maximizes his profits subject to a production function constraint. The Pigouvian taxes and subsidies (to ensure the optimal production and use of every good and resource) may be restricted to labor, because -- by assumption -- all other goods and resources are optimally priced. The per unit tax on the j 'th individual working in the k 'th firm is $^s t_{jk}$, and that on the k 'th firm hiring the j 'th individual is t_{jk} .

Given the above analytical terrain, the two ingredients for the Pigouvian fiscal prescriptions -- the social welfare programs and the programs of individualistic decision-making -- may be presented as follows:

$$(11) \text{ Maximize } U\left(\frac{x_{ij}}{i,j}, \frac{^s L_{jk}}{j,k}\right)$$

subject to

$$f^k\left(\frac{y_{ik}}{i}, \frac{j_k}{j}\right) \leq 0 \quad \forall k \quad \mu_k$$

$$\sum_j x_{ij} - \sum_k y_{ik} \leq r_i \quad \forall i \quad \omega_i$$

$$\sum_k L_{jk} \leq \sum_k ^s L_{jk} \quad \forall j \quad \rho_j$$

$$(12) \text{ Maximize } U^j\left(\frac{x_{ij}}{i}, \frac{^s L_{jk}}{k}\right)$$

subject to

$$\sum_i p_i \cdot x_{ij} - \sum_k w_{jk} \cdot ^s L_{jk} + \sum_k ^s t_{jk} \cdot ^s L_{jk} \leq w_j \quad \alpha_j$$

$$(13) \text{ Maximize } \sum_i p_i \cdot y_{ik} - \sum_j w_{jk} \cdot L_{jk} - \sum_j t_{jk} \cdot L_{jk}$$

subject to

$$f^k\left(\frac{y_{ik}}{i}, \frac{L_{jk}}{j}\right) \leq 0 \quad \gamma_k$$

(W_j is the wealth of the j 'th consumer carried over from the last time period and the greek letter beside each constraint is its Lagrangian multiplier).

It is important to note that these programs are analogous to those presented in Section 2 for "depletable externalities". (All we have done is identified the "depletable externality" z_j as the depletable resource, labor, and let it be supplied by the consumers rather than by the firms). The advantage of our approach over that of Section 2 is to show that externalities can be generated by any common depletable goods and resources and thus to open the way to an analysis of the variety of problems to which a suboptimal production or use of these goods and resources can give rise. For the simple model described above, the Pigouvian fiscal prescriptions are analogous to those derived for the depletable case of Section 2:

$$(14) \quad w_{jk} - s_{t_{jk}} = \rho_j$$

$$w_{jk} + t_{jk} = \rho_j, \quad \text{for all individuals } j.$$

(The method of deriving these prescriptions was presented in Section 2). Thus, in the absence of any other externalities, the adverse social welfare effects caused by labor-generated externalities arise because the wage is set inappropriately. The Pigouvian taxes and subsidies simply serve to set the wage at its socially optimal level (ρ_j instead of w_{jk}). (Besides, we have arrived at the standard neoclassical result that, for each type of labor, the wage rate should be the same for all firms).

As we have noted in Section 2, discussions on Pigouvian externality treatment commonly conclude that "depletable externalities" always arise from inappropriate prices. The Pigouvian prescriptions for these cases are thus always equivalent to price changes. In terms of our analysis, the question arises whether Pigouvian taxes and subsidies, imposed in response to externalities generated by a depletable good or resource, always have the function of a price change for this good and resource. This significant question must be answered in the negative. Thus, an important and conspicuous result in the literature on this subject -- namely, that these externalities could be eliminated by competitive pricing, in the absence of institutional restrictions, sufficiently low exclusion and metering costs, and so on -- cannot be upheld. Let us examine a simple example of externalities generated by a depletable resource -- labor, as above -- which cannot be optimally treated through taxes and subsidies of equal magnitude, equivalent to a change in the wage rate.

For this purpose, we relax the dubious assumption made above, that each worker expects all the services he supplies to be demanded. We replace this assumption by the following, more plausible, one: the i 'th individual expects that, of the services he supplies to firm k ($^sL_{jk}$), a fraction will be demanded: $L_{jk}^e = A_{jk} \cdot ^sL_{jk}$, where $0 \leq A_{jk} \leq 1$. In other words, the more labor the i 'th individual supplies, the more he expects will be demanded, although the proportionality constant relating supply and demand may be less than unity. If we suppose that a firm hires and fires its work force randomly, then the more labor an individual j makes available to that firm (ceteris paribus), the more likely is he to be hired (i.e. $A_{jk} \cdot ^sL_{jk}$ may be interpreted as the expected value

of being hired by the k 'th firm⁴²⁾. Clearly, the realism of this hypothesis may be criticized, but this issue is of no consequence to our analytical conclusions. We will show that whenever an individual does not expect all his available labor services to be demanded automatically, the Pigouvian fiscal prescriptions are not equivalent to changes in the wage rate -- regardless of what, precisely, the individual's expectations are. There are two cases to be considered: either the individual expects that his services demanded are related to his services supplied or he does not. The former case can be handled by setting $0 < A_{jk} \leq 1$, the latter by setting $A_{jk} = 0$ and $A_{jk} = 1$ on either side of a back-off point. (It makes no difference to our qualitative conclusions if L_{jk}^e is zero or some positive constant).

The mathematical program describing the behavior of the j 'th individual may be amended accordingly. The labor variables that are relevant to his consumption and labor-supply decisions, ex ante, are not L_{jk} -- the amounts of his services actually utilized \bar{k} -- but L_{jk}^e -- the amounts of his services he expects to be utilized. Thus, his utility function and his budget constraint both contain the term $A_{jk} \cdot {}^sL_{jk}$.

Let us suppose that the Pigouvian taxes (or, if negative, subsidies) on the use and supply of labor (t_{jk} and ${}^st_{jk}$, respectively) are imposed on labor services actually performed. Thus, each entrepreneur knows that for L_{jk} hired, he must pay $t_{jk} \cdot L_{jk}$; consequently, this term enters his profit function. The tax ${}^st_{jk}$, however, effects the behavior of individual j only through his expectation of productive services to be rendered; therefore the term

$s_{t_{jk}} \cdot A_{jk} \cdot s_{L_{jk}}$ enters his budget constraint.

The Pigouvian prescriptions for this scenario may be derived from the following programs:

$$(15) \text{ Maximize } U\left(\frac{x_{ij}}{i}, \frac{A_{jk} \cdot s_{L_{jk}}}{j, k}\right)$$

subject to

$$f^k\left(\frac{y_{ik}}{i}, \frac{L_{jk}}{j}\right) \leq 0 \quad \forall k \quad \mu_k$$

$$\sum_j x_{ij} - \sum_k y_{ik} \leq r_i \quad \forall i \quad \omega_i$$

$$\sum_k L_{jk} - \sum_k s_{L_{jk}} \leq 0 \quad \forall j \quad \rho_j$$

$$(16) \text{ Maximize } U^j\left(\frac{x_{ij}}{i}, \frac{A_{jk} \cdot s_{L_{jk}}}{k}\right)$$

subject to

$$\sum_i P_i \cdot x_{ij} - \sum_k p_{jk} \cdot A_{jk} \cdot s_{L_{jk}} + \sum_k s_{t_{jk}} \cdot A_{jk} \cdot s_{L_{jk}} \leq W_j$$

α_j

$$(17) \text{ Maximize } \sum_i P_i \cdot y_{ik} - \sum_j w_{jk} \cdot j_k - \sum_j t_{jk} \cdot L_{jk}$$

subject to

$$f^k\left(\frac{y_{ik}}{i}, \frac{L_{jk}}{j}\right) \leq 0 \quad \forall k \quad \gamma_k$$

The first-order conditions for these programs (in addition to the satisfaction of the constraints and the nonnegativity conditions) may be presented in corresponding pairs:

$$\frac{\partial \mathcal{L}_1}{\partial x_{ij}} = u_{ij} - \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial x_{ij}} = u_i^j - \alpha_j \cdot P_i \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial x_{ij}} \cdot x_{ij} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial x_{ij}} \cdot x_{ij} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial y_{ik}} = -\mu_k \cdot f_i^k + \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial y_{ik}} = P_i - \alpha_k \cdot f_i^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial y_{ik}} \cdot y_{ik} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial y_{ik}} \cdot y_{ik} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial {}^s L_{jk}} = u_{jk} + \rho_j \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial {}^s L_{jk}} = u_k^j + \alpha_j \cdot [w_{jk} \cdot A_{jk} - {}^s t_{jk} \cdot A_{jk}] \leq 0$$

$$\frac{\partial \mathcal{L}_1}{\partial {}^s L_{jk}} \cdot {}^s L_{jk} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial {}^s L_{jk}} \cdot {}^s L_{jk} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial L_{jk}} = -\mu_k \cdot f_j^k - \rho_j \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial L_{jk}} = -w_{jk} - t_{jk} - \alpha_k \cdot f_j^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial L_{jk}} \cdot L_{jk} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial L_{jk}} \cdot L_{jk} = 0.$$

We now apply the reasoning outlined in Section 2 to derive the Pigouvian taxes and subsidies which make both sets of conditions identical. Obviously, the

Lagrangian conditions with respect to x_{ij} and y_{ik} are identical, since $U_{ij} = (U_i^j/\alpha_j)$, $\mu_k = \gamma_k$, and $P_i = \omega_i$ (since all goods and resources, except labor, are priced in the socially optimal manner). For the Lagrangian conditions with respect to $s_{L_{jk}}$ and L_{jk} , we note that $U_{jk} = (U_k^j/\alpha_j)$ and (as above) $\mu_k = \gamma_k$, and thus we are left with the Pigouvian fiscal prescriptions for the treatment of labor-generated externalities:

$$(18) \quad w_{jk} - s_{t_{jk}} = (\rho_j/A_{jk})$$

$$w_{jk} + t_{jk} = \rho_j.$$

As may be expected, if the j 'th individual anticipates that all his services supplied to the k 'th firm will be demanded, $A_{jk} = 1$, and these prescriptions are the same as prescriptions (14). For $A_{jk} < 1$, however, the magnitude of the optimal subsidy (tax) on the supply of the j 'th labor services to the k 'th firm is not equal to the magnitude of the optimal tax (subsidy) on the use of the j 'th labor services by the k 'th firm. Hence, the externalities elicited by this depletable resource cannot be optimally treated through a price readjustment. (Furthermore, whereas the standard neo-classical result that the cost of each type of labor should be the same to all firms holds, the compensation for labor services may vary both by labor type and by firm.)

The reason for this asymmetry between optimal taxes and subsidies is self-evident. Suppose that a tax and subsidy of equal magnitude had been imposed ($t_{jk} = -s_{t_{jk}}$). These fiscal measures are levied only on labor services

actually performed. Thus, while the cost of hiring the j 'th individual would be $(w_{jk} + t_{jk}) \cdot L_{jk}$ for the k 'th firm, the compensation expected by the j 'th individual for supplying his services to firm k would only be $(w_{jk} + t_{jk}) \cdot A_{jk} \cdot {}^sL_{jk}$. The wage plus symmetric fiscal measures give rise to asymmetric incentives to the demanders and suppliers of labor. Of his total services supplied to firm k , ${}^sL_{jk}$, the j 'th individual does not expect $(1 - A_{jk}) \cdot {}^sL_{jk}$ to be rewarded with the wage and the subsidy. Of his total services used, L_{jk} , the k 'th firm expects all to be compensated by the wage and the tax. Consequently, the wage and the subsidy are a weaker incentive to supply labor than the wage and the tax are a disincentive to demand labor. At the social optimum, the marginal social benefit from the services of each type of labor should be equal to the expected compensation received by the j 'th individual and the expected payment made by the recipient of his services. (Then both the supplier and the demander of labor are carrying the full social welfare effects of their respective decisions). If taxes and subsidies of equal magnitude are imposed, labor's expected compensation would fall short of entrepreneurs' expected payment. Thus, too little labor would be supplied. This phenomenon is commonly known as disguised unemployment caused by the "discouraged worker" effect. To remedy this problem in the socially optimal fashion, the subsidy to labor (${}^st_{jk}$) must be larger than the tax to the entrepreneur (t_{jk}). How large this difference must be clearly depends on the magnitude of A_{jk} .

To arrive at our conclusion above -- that externalities elicited by depletable goods or resources cannot

always be treated optimally through a price readjustment -- it is unnecessary to assume that employment expectations are governed by $L_{jk}^e = A_{jk} \cdot {}^sL_{jk}$; all expectations which are not characterized by $L_{jk}^e = {}^sL_{jk}$ give rise to the same result. As we have noted above, either the j 'th individual believes that his labor supply is relevant to the demand for his services, or he does not. If he does, the relationship may be pictured in the most general way: $L_{jk}^e = g_{jk}({}^sL_{jk})$. Through program (15), the optimal supply of labor services, ${}^sL_{jk}^*$, may be derived. The corresponding value for L_{jk}^e is L_{jk}^{e*} (given by the function g_{jk}). We thus find $A_{jk}^* = (L_{jk}^{e*}/{}^sL_{jk}^*)$, and this proportionality constant is to be used in the Pigouvian prescriptions (18).

On the other hand, suppose that the j 'th individual believes that the demand for his services is independent of their supply. Then L_{jk}^e may be set equal to some constant, ψ_{jk} . (If L_{jk}^e depends on variables endogenous to program (15), then the socially optimal values of these variables must be chosen to determine ψ_{jk}). If $\psi_{jk} - {}^sL_{jk} = \sigma_{jk}$, where σ_{jk} is a positive constant, then any increase in ${}^sL_{jk}$ up to the magnitude of σ_{jk} will be met by an equal increase in the use of these labor services. In this case, $A_{jk} = 1$ and the symmetric Pigouvian fiscal measures are appropriate. If $\sigma_{jk} \leq 0$, any increase in ${}^sL_{jk}$ will not lead to a changed use of these labor services. Here $A_{jk} = 0$ and the Pigouvian prescriptions (18) break down. Since the j 'th individual does not expect that any additional offering of his services will lead to additional employment, the wage rate and the subsidy -- both of

which are conditional upon employment -- cannot act as incentives to supply labor. Thus, if the j 'th individual's employment expectations are misguided and more than ψ_{jk} of his services to the k 'th firm could be optimally employed, the Pigouvian subsidy is powerless to influence his behavior.

The Pigouvian approach may be reinstated, however, if the subsidy is made conditional on the offering of labor services, rather than their use. For instance, every individual who is involuntarily unemployed may register his condition at an employment office. There he may contract to take the first suitable job which the office manages to find for him and, in return, the office gives him a subsidy proportional to the services he offers. Now the j 'th individual has an incentive to offer his services even if he (mistakenly) believes that they will not be utilized. The program describing his behavior may now be written as follows:

$$(19) \text{ Maximize } U_j^j(x_{ij}, \frac{A_{jk} \cdot s_{L_{jk}}}{k}, \frac{(1-A_{jk}) \cdot s_{L_{jk}}}{k})$$

subject to

$$\sum_i P_i \cdot x_{ij} - \sum_k w_{jk} \cdot A_{jk} \cdot s_{L_{jk}} + s_{t_{jk}} \cdot s_{L_{jk}} \leq w_j$$

In other words, his ex ante welfare depends on his consumption pattern, the amount of work he expects to do, and the amount of services he offers without anticipation of work. In his budget constraint, the wages he is offered appear only in conjunction with his expected employment opportunities, while his subsidies appear in conjunction with his labor supply.

Once the first order conditions for this program, together with those for programs (15) and (17), are examined, the following Pigouvian prescriptions emerge:

$$(20) \quad A_{jk} \cdot w_{jk} + {}^s t_{jk} = \rho_j$$

$$w_{jk} + t_{jk} = \rho_j.$$

If, as we have hypothesized, $A_{jk} = 0$, then ${}^s t_{jk} = \rho_j$. Since the wage rate cannot act as incentive to supply labor whenever an individual does not believe that he has a chance of employment, the subsidy (granted in response to his labor offering) is left as the only remaining incentive. At the social optimum, $\rho_j = {}^s t_{jk} = t_{jk} + w_{jk}$, (i.e. the marginal social benefit from the services of the j 'th individual is set equal to the expected remuneration for offering these services and to the expected payment for using them).

It is evident that the proposal above -- which makes the subsidy contingent on the offering of labor services -- may be implemented also when $A_{jk} > 0$. If $A_{jk} = 1$, prescriptions (20) are equivalent to prescriptions (14). If $0 < A_{jk} < 1$, the magnitude of the subsidy (tax) to the worker must be greater than the magnitude of the tax (subsidy) to the entrepreneur. Clearly, the subsidy to the worker granted under this proposal must be less than the subsidy conditional upon his employment (considered previously). In the former case the subsidy is a stronger incentive to offer labor services.

(Comparing the subsidies prescribed in condition (18) and (20), we find that $[\rho_j - A_{jk} \cdot w_{jk}] - [(\rho_j / A_{jk}) - w_{jk}] = \rho_j \cdot [1 - (1/A_{jk})] + w_{jk} \cdot [1 - A_{jk}] > 0$ if $0 < A_{jk} < 1$.)

In sum, the proposal underlying program (19) gives rise to Pigouvian prescriptions which are applicable in response to all types of employment expectations.

What we have done thus far is to use the phenomenon of disguised unemployment to examine whether externalities generated by depletable goods or resources can always be optimally treated through taxes and subsidies which are equivalent to price readjustments. We have found this proposition to be false. In exploring the optimal fiscal response to disguised unemployment, we discovered that the common Pigouvian practice -- apparently ubiquitous in the literature on this subject -- of making taxes and subsidies conditional upon work performed cannot deal with all types of employment expectations in the socially optimal manner. We then described a different proposal which can handle all employment expectations. There is one major idea underlying this analysis which constitutes a basic message of this paper. Not only undepletable goods and resources -- to which externality theory thus far has attributed such preponderant importance -- but depletable goods and resources, as well, may give rise to significant and serious externality problems. Disguised unemployment is only one example of an externality problem elicited by a depletable resource. As we have seen, the Pigouvian approach to the treatment of disguised unemployment gives rise to prescriptions which may serve as valuable policy guidelines in this area. Let us now turn to another significant example of externalities generated by depletable goods and resources: unemployment caused by adverse entrepreneurial expectations in the goods

market. This phenomenon is intimately related to the Keynesian unemployment problem.

In our models of competitive activity above, entrepreneurial behavior was described in terms of profit maximization subject to a production function constraint. Just as the j 'th consumer's welfare function was specified in ex ante terms in order to describe his decision-making process, so the k 'th entrepreneur's objective function (his profit function) must be specified in ex ante terms as well: expected revenues minus expected costs. We assume that the buyers and sellers in our economy are all price takers and that all the relevant prices are known in advance. Thus, the entrepreneur's expected revenues are left to depend on the quantities of his product he expects to sell. His expected costs are left to depend on his purchases of intermediate goods and resources, as derived from his production decision. What the entrepreneur expects to sell is not entirely within his control. He can determine only what he will produce; his customers determine what of his product will be sold. Hence, whereas his costs are given once his production decision is made, his ex ante revenues are left to the vicissitudes of his anticipations concerning the demand he faces.

In our models above, we implicitly assumed that each entrepreneur expects to sell all the goods he supplies. (Given that y_{ik} is the k 'th entrepreneur's output of the i 'th good. $P_i \cdot y_{ik}$ appeared as expected revenue in his objective function). This is an assumption of doubtful realism. Even when the entrepreneur is a price-taker, the prices he accepts need not be the market-

clearing prices. Many real-world situations may be adduced in which entrepreneurs do not have much latitude in setting their prices and yet do not always expect the supply of their goods to be met by an equal demand. Let us consider the k 'th entrepreneur's sales expectations with respect to a capital good, s_k , which he produces. Analogously to the example above, we replace our assumption of "perfectly harmonious" sales expectations by the following, more general one: the entrepreneur expects that a fraction of the capital good which he produces will be demanded: $K_k^e = B_k \cdot s_k$, where K_k^e is his expected demand and $0 \leq B_k \leq 1$. The more of the good he supplies, the more he expects will be demanded, although the increase in his production may exceed the demand response. If the process of buying this capital good from its various suppliers has random elements, then K_k^e may be understood as an expected value: the more of the capital good an entrepreneur puts up for sale (ceteris paribus) the more of it he is likely to sell. The proportionality constant may be taken to depend on the inventory - to - production ratios the entrepreneur has encountered in the past.

Once again, this expectational hypothesis may be generalized to all types of sales expectations, those for which expected sales depend on quantities supplied and those for which they do not. Such generalization does not affect the conclusions of our analysis. The generalization procedure is analogous to that described before with reference to employment expectations. The cases for which $B_k = 0$ is applicable, call for Pigouvian

proposals in which the taxes and subsidies on the entrepreneur's output of the capital good are not contingent on the sale of that good. Such proposals have been discussed above; to avoid undue repetition, we let $B_k > 0$ in the following discussion. Furthermore, to differentiate our model of entrepreneurial behavior from those presented above, we let $B_k < 1$. Thus, we will conduct our discussion in terms of the basic hypothesis, $K_k^e = B_k \cdot K_k$, where $0 < B_k < 1$; yet it will be understood implicitly that a more general formulation could have been used as well.

Imagine an economy, composed of welfare-maximizing consumers and profit-maximizing entrepreneurs, of the sort described in the models of this section. We assume, for the moment, that it contains only depletable goods and resources. At its initial position, we suppose that there are no adverse employment expectations (and thus no disguised unemployment as in the case above) and no adverse sales expectations. Consequently, any adverse social welfare effects arising from inter-individual economic dependence can be treated through price adjustments. Furthermore, we assume that prices are set at their socially optimal levels. As above, we take it for granted that a competitive equilibrium exists for this price vector. Clearly, it is not necessary to impose Pigouvian taxes and subsidies on the economic activities of this system.

Now imagine that a change takes place: entrepreneurial sales expectations turn adverse. In order to bring the effect of these expectations on unemployment into sharp relief, we assume that such expectations hold only with

respect to one good, the capital good K: $0 < B_k < 1$. For all other goods and resources, entrepreneurs anticipate demand sufficient to absorb their supply.⁴³⁾ We may now calculate the Pigouvian taxes and subsidies necessary to restore the social optimum. It can be shown that these fiscal measures are to be imposed only on K; the prices of all other goods and resources should remain unchanged. The reason for this prescription is intuitively apparent. In the initial situation, every individual was compensated for the social welfare effects of his actions. After the expectational swing, the expected compensation to the producers of K falls short of the social value of K (since entrepreneurs now expect deficient demand). No other change has taken place in the economy. Consequently, the optimal policy response is to increase the supply price of K (through a per unit subsidy) and leave the demand price of K and the prices of all other goods and services unchanged.

Under the free price system, the demand price is equal to the supply price for each good and resource, and hence this system cannot achieve the social optimum. Government intervention is necessary. Returning to the initial competitive equilibrium, let us examine the effects of the expectational swing on the production and use of the goods and resources in the absence of government intervention. As a heuristic expedient, suppose that prices react with a discrete lag to changes in excess demand conditions and that these reactions are well-behaved (i.e. prices rise in the face of Positive excess demand and fall in the presence of positive excess supply). If entrepreneurs begin to expect that only a fraction of what they supply will be demanded, the price of K becomes a weaker incentive to supply K.

Thus, the supply of the capital good will fall. In the first instance, the demand for the good and the demands for and supplies of all other goods and resources remain unchanged. The fall in K means that the demand for inputs used in the production of K falls as well. Let us concentrate on just one input: labor. Since wage rates remain unchanged, labor supplies remain the same, and the drop in the demand for inputs elicits unemployment.

This chain of events is a typical case of the Keynesian unemployment phenomenon. If prices in the economy are inflexible, then the appearance of adverse entrepreneurial sales expectations will call for unemployment. (For example, an adverse swing in investment expectations -- leading to unemployment in the Keynesian system -- can be interpreted as a fall in expected sales for a number of time periods in the future. The externality model presented here can easily be restated in intertemporal terms in order to take this possibility into account.) Now suppose that prices react (with a lag). It can be shown that if the price of capital falls to its market-clearing level and all other prices remain unchanged, there will still be unemployment. If input prices react fully (including the wage rates of the labor employed in the production of K), then unemployment may be eliminated.⁴⁴⁾ It is clear, however, that these price changes cannot lead to the achievement of the social optimum, for we know that the socially optimal input prices are equal to their initial values. Thus, even if prices are flexible, they should not be allowed to change. The case for government intervention in the presence of adverse sales expectations need not be

founded on the assumption of price inflexibility. The only optimal course to follow is for the government to grant a subsidy to the producees of the capital good. The subsidy should be sufficiently high to prevent deficient supply of this good and thus all consequent price changes.

We must conclude that the free price system reacts in a socially suboptimal way to an adverse swing in sales expectations and that government intervention is necessary to ensure the stability of all prices except the seller price of K. Not only is intervention necessary to overcome the Keynesian unemployment phenomenon in the face of inflexible prices; but price responsiveness in this situation is undesirable and intervention is necessary to prevent it. Of course, this conclusion rests on the assumption that the initial state of the economy was socially optimal. If the initial state was suboptimal and then adverse sales expectations set in, a combination of two distinct fiscal packages may be implemented:

- (1) Pigouvian prescriptions for the attainment of a socially optimal initial state, and
- (2) Pigouvian prescriptions for the externality generated by the expectational swing.

The first package does not concern us here; it should be implemented regardless of the state of expectations. It is important to note that the second package consists of a very detailed policy proposal. This proposal is not simply expansionary fiscal policy in a general sense -- a common Keynesian policy prescription -- but a special type of expansionary fiscal policy: a subsidy per unit of output to the producers suffering from adverse expectations, of a magnitude sufficient to

to eliminate the supply shortage.

As we can see, there is a considerable advantage to perceiving the problem of unemployment caused by adverse sales expectations as one of externalities. Economists have long complained that the Keynesian fiscal policy prescriptions are too general. It is not enough to know that expansionary fiscal policy should be used to fight unemployment; it is necessary to describe what type of fiscal policy should be employed. Clearly, not all fiscal instruments have equivalent economic repercussions. If Keynesian unemployment is seen as an externality effect, the Pigouvian approach to the treatment of externalities may be used to give a precise description of the type of fiscal stimulus that is needed.

The argument above consists of a number of propositions which we must now examine more rigorously. Assume an economy with the following characteristics: (1) each of its consumers maximizes his own welfare subject to a budget constraint; (2) each of its producers maximizes his own profits subject to a production function constraint; (3) it is in competitive equilibrium; (4) it produces and uses only depletable goods and resources; (5) each consumer expects to sell the labor services he supplies (at the given wage rates); (6) each entrepreneur expects to sell the goods he supplies (at the given prices), (7) the price of each good and resource is at its unique, socially optimal level; (8) labor is needed to produce the capital good K ; and (9) the price of K , P_K , is positive. This is a picture of the economy in its initial position. We now impose a

comparative static shock on the system: the producers of K expect to sell a constant fraction (greater than zero and less than one) of the quantity of K supplied. Then the following propositions may be shown to hold: (a) the socially optimal vector of goods produced and the socially optimal vector of goods and resources used is the same before and after the expectational change; (b) the socially optimal vector of prices, except P_K , is the same before and after the expectational change; (c) if the initial vector of prices (including P_K) remains unchanged, there will be unemployment; (d) if all prices, except P_K , are unchanged, and P_K is set at its market-clearing level, there will be unemployment; (e) there exists no price vector that permits the attainment of the social optimum; and (f) the social optimum can only be achieved through a per unit subsidy on the sale (or production) of K .⁴⁵⁾ These proportions can be handled quite simply on the basis of the externality analysis presented in this section.

Let sK_k be the amount of the capital good produced by the k 'th firm; KS be the stock of this good inherited from the last time period, and K_j and K_k be the amounts of this good used by the j 'th consumer and the k 'th firm, respectively⁴⁶⁾. As in the two previous models of this section, $^sL_{jk}$ and L_{jk} stand for the supply of and the demand for the services of the j 'th individual to the k 'th firm, respectively. x_{ij} and y_{ik} are the amounts of good (or resource) i -- not including labor or the capital good above -- used by individual j and firm k , respectively. Since all these goods and resources, and labor, are assumed to be optimally priced, they do not require Pigouvian fiscal treatment. Pigouvian taxes and subsidies will only be levied on the capital good K .

The initial state of the economy -- in socially optimal competitive equilibrium -- may be identified through a social welfare program and a set of programs describing the individualistic behavior of consumers and entrepreneurs. The welfare implications of the expectational change may be evaluated by means of the same programs.

$$(21) \text{ Maximize } U\left(\frac{x_{ij}}{i,j}, \frac{{}^sL_{jk}}{j,k}, K_j\right)$$

subject to

$$F^k\left(\frac{y_{ik}}{i}, \frac{L_{jk}}{j}, K_k, {}^sK_k\right) \leq 0 \quad \forall k \quad \mu_k$$

$$\sum_j x_{ij} - \sum_k y_{ik} \leq r_i \quad \forall i \quad \omega_i$$

$$\sum_k L_{jk} \leq \sum_k {}^sL_{jk} \quad \forall j \quad \rho_j$$

$$\sum_j K_j + \sum_k K_k \leq \sum_k {}^sK_k + K_S \quad \eta$$

$$(22) \text{ Maximize } U^j\left(\frac{x_{ij}}{i}, \frac{{}^sL_{jk}}{k}, K_j\right) \quad \forall j$$

subject to

$$\sum_i P_i \cdot x_{ij} - \sum_k w_{jk} \cdot {}^sL_{jk}$$

$$+ PK \cdot K_j + t_j \cdot K_j \leq W_j \quad \alpha_j$$

$$\begin{aligned}
 (23) \text{ Maximize } & \sum_i P_i \cdot y_{ik} - \sum_j w_{jk} \cdot L_{jk} - PK \cdot K_k - t_k \cdot K_k \\
 & + PK \cdot B_k \cdot {}^s K_k - {}^s t_k \cdot B_k \cdot {}^s K_k \quad \forall k \\
 \text{subject to } & f^k\left(\frac{y_{ik}}{i}, \frac{L_{jk}}{j}, K_k, {}^s K_k\right) \leq 0 \quad \delta_k
 \end{aligned}$$

The first-order conditions (in addition to the constraints and the nonnegativity conditions) may be written in the following corresponding pairs:

$$\frac{\partial \mathcal{L}_1}{\partial x_{ij}} = u_{ij} - \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial x_{ij}} = u_i^j - \alpha_j \cdot P_i \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial x_{ij}} \cdot x_{ij} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial x_{ij}} \cdot x_{ij} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial y_{ik}} = -\mu_k \cdot f_i^k + \omega_i \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial y_{ik}} = P_i - \delta_k \cdot f_i^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial y_{ik}} \cdot y_{ik} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial y_{ik}} \cdot y_{ik} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial {}^s L_{jk}} = u_{jk} + \rho_j \leq 0, \quad \frac{\partial \mathcal{L}_2}{\partial {}^s L_{jk}} = u_k^j + \alpha_j \cdot w_{jk} \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial {}^s L_{jk}} \cdot {}^s L_{jk} = 0, \quad \frac{\partial \mathcal{L}_2}{\partial {}^s L_{jk}} \cdot {}^s L_{jk} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial L_{jk}} = -\mu_k \cdot f_j^k - \rho_j \leq 0, \quad \frac{\partial \mathcal{L}_3}{\partial L_{jk}} = -w_{jk} - \alpha_k \cdot f_j^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial L_{jk}} \cdot L_{jk} = 0, \quad \frac{\partial \mathcal{L}_3}{\partial L_{jk}} \cdot L_{jk} = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial s_k} = -\mu_k \cdot f_{s_k}^k + \eta \leq 0, \quad \frac{\partial \mathcal{L}_3}{\partial s_k} = PK \cdot B_k - t_k \cdot B_k - \alpha_k \cdot f_{s_k}^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial s_k} \cdot s_k = 0, \quad \frac{\partial \mathcal{L}_3}{\partial s_k} \cdot s_k = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial K_k} = -\mu_k \cdot f_K^k - \eta \leq 0, \quad \frac{\partial \mathcal{L}_3}{\partial K_k} = -PK - t_k - \alpha_k \cdot f_K^k \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial K_k} \cdot K_k = 0, \quad \frac{\partial \mathcal{L}_3}{\partial K_k} \cdot K_k = 0;$$

$$\frac{\partial \mathcal{L}_1}{\partial K_j} = u_j - \eta \leq 0, \quad \frac{\partial \mathcal{L}_3}{\partial K_j} = u_j^i - \alpha_j \cdot PK - \alpha_j \cdot t_j \leq 0,$$

$$\frac{\partial \mathcal{L}_1}{\partial K_j} \cdot K_j = 0, \quad \frac{\partial \mathcal{L}_3}{\partial K_j} \cdot K_j = 0.$$

From a comparison of these two sets of conditions, the socially optimal prices may be identified:

$$(24) \quad P_i = \omega_i \quad \text{for all } i$$

$$w_{jk} = \rho_{ij} \quad \text{for all } j \text{ and } k$$

$$PK + t_j = \eta, \quad PK + t_k = \eta, \quad \text{and } PK - s t_k = (\eta/B_k) \\ \text{for all } j \text{ and } k.$$

In the initial state of the economy $B_k = 1$, for all k , and all prices are at their socially optimal levels. Thus, $P_i = \omega_i$ ($\forall i$), $w_{jk} = \rho_j$ ($\forall j, k$), and $PK = \eta$. Now the expectational change occurs: $0 < B_k < 1$, for all k . The constant B_k appears only in the entrepreneurial programs (23), it does not appear in the social welfare program (21), which determines the socially optimal quantities of goods and resources produced and used. Consequently, a change in B_k cannot affect these optimal quantities. (The optimal quantities are those which maximize the social welfare function subject to the production function constraints and the good and resource constraints. Obviously, entrepreneurial expectations are not relevant here). Thus, proposition (a) holds. Furthermore, it is clear from equations (24) that the socially optimal levels of all prices, except PK , are the same before and after the expectational swing. (The expectational swing changes the relation between the social value of K and the expected private returns from production K ; but it leaves the private and social benefits (costs) of every other good and resource in equality. Consequently, the socially optimal prices of these latter goods and resources remain unchanged). Thus, proposition (b) holds.

Suppose that prices do not react to the expectational swing. Then the supply price of the capital good lies

below its socially optimal level (and all other prices are socially optimal). As we can see through equations (24), the socially optimal supply price of K is (η/B_K) , which is greater than η (provided that $\eta > 0$, according to assumption (9)). Under the assumption that the production functions are all twice differentiable and strictly concave, less K will be supplied at the price η than at (η/B_K) . If labor is used in the production of K , the demand for this labor falls whereas the supply remains unchanged (since prices are inflexible and employment expectations are not influenced by the fall in demand). Thus, in accordance with proposition (c), there is unemployment.

Proposition (d) can be handled quite simply. We have assumed that the socially optimal price of the capital good at the initial state of the economy is positive (assumption 9); thus this good must be fully employed at the initial social optimum. (If there is deficient demand for K , $\eta = 0$). After the expectational swing, the optimal demand price remains $P_K = \eta > 0$, and the optimal supply price becomes $P_K = (\eta/B_K) > 0$ (since $0 < B_K < 1$). Thus, the demand for and the supply of K must be equal at the final social optimum as well. Now suppose the price of the capital good is set equal to (η/B_K) . Then the supply of this good will be at its socially optimal level, but the demand be deficient. Hence, η must lie above the market-clearing price. However, if all goods and resources remain priced at their socially optimal levels, no price lower than (η/B_K) is sufficient to elicit enough production of K so as to prevent unemployment. Thus, there exists unemployment at the market-clearing price for K .

Propositions (e) and (f) may be verified by inspection of equations (24). If $P_i = \omega_i (\forall i)$, $w_{jk} = \rho_i (\forall j,k)$, and $PK = \eta$, then -- given sales expectations characterized by $0 < B_k < 1$ -- the social optimum can only be achieved through a per unit subsidy, $-^s t_k = (\eta - \eta \cdot (B_k)) / B_k$, on the sale of K. Since the demand price and the supply price of each good are identical under the free price system, ordinary prices cannot perform this function.

Thus, our contention that the Keynesian unemployment phenomenon may be understood as an externality problem has been substantiated. If the economy is initially in a state of full employment and a socially acceptable status quo (e.g. an acceptable distribution of income), then an adverse swing in sales expectations should be handled through expansionary fiscal policy. The price inflexibilities -- which Keynesians take for granted in deriving their policy proposals -- are desirable under these circumstances; the free price system cannot treat expectational swings in the socially optimal manner. From the model of Keynesian unemployment above it becomes clear that externalities generated by depletable goods and resources can constitute economic phenomena of great importance. The scant attention which these externalities have received in the literature thus far means that many of the most pressing current economic problems still remain to be explored in this light. Not only may unemployment be seen as a problem of externalities, but -- as we have mentioned above -- inflation, stagnation, resource depletion, factor misallocation, and production inefficiency may be seen in these terms as well.

The advantage of seeing these problems as externalities are apparent. First, the Pigouvian approach permits the treatment of these problems even when they occur simultaneously. This is indeed a considerable promise, because, whereas economists have made progress in treating the problems above in isolation from each other, they have made little headway in treating combinations of them (such as inflation and unemployment). What needs to be done is to amend the social welfare program and the programs of individual economic behavior in such a way as to generate more than one of these problems at once. The Pigouvian prescriptions may then be derived from this new set of programs. To be sure, such a task lies beyond the scope of this paper. Here we are only concerned with the methodology, not the implementation, of such an undertaking. It must be noted that the Pigouvian prescriptions can only be as good as the model of economic behavior underlying them. As economists abilities to describe economic activity improve, so will these prescriptions.

Second, the Pigouvian approach to the treatment of externalities permits the derivation of detailed qualitative policy prescriptions -- not the common theoretical prescriptions which are usually too general for "fine tuning". Nevertheless, it must be emphasized that the Pigouvian prescriptions must remain only qualitative. The Pigouvian approach rests on the specification of a social welfare function, and the empirical infeasibility (impossibility?) of devising such a function prevents the quantitative formulation of these prescriptions. However, this difficulty does not mean that the Pigouvian approach is of little value in drafting economic policy. Qualitative prescriptions

are important. Besides, as we have mentioned in Section 1, several endeavors have been made -- and are still being made -- to replace the social welfare function by an economic construct which can be given empirical substance. What these endeavors all have in common, to my knowledge, is the use of standards -- unemployment standards, inflation standards, environmental standards -- in setting the Pigouvian fiscal measures. The Pigouvian approach presented here represents the foundations upon which these endeavors are made.

To conclude our discussion of the methodology of constructing a general theory of Pigouvian externality treatment, we consider externalities generated by undepletable goods or resources. This area has received abundant attention and the qualitative results presented in Section 2 will be left unamended here. However, as we have noted before, a confusion of externalities with the goods and resources generating them has been prevalent and thus we will restate the Pigouvian position in the light of the reconceptions offered in Section 3. To use an analytically concrete example, a waste -- generated by production activities -- will serve as our undepletable good. (We define "waste" here as any produced good which incurs potential harm to at least one individual. Thus, sulphur dioxide, nuclear refuse, and noise all qualify as wastes). We employ the notation of Section 2. The total waste stock in the economy (z) is equal to the waste generated in the present time period ($\sum_k s_k$) plus the stock inherited from last time period (z_{-1}). $a_j \cdot z$ and $b_k \cdot z$ are the

j 'th consumers and the k 'th firm's intake of the waste (respectively), where $a_j = a_j(k_{ij})$ and $b_k = b_k(y_{ik})$. Aside from the waste, all i other goods and i resources in the economy are depletable and are priced in the socially optimal manner. The waste receives no price in the market place. We examine the Pigouvian taxes and subsidies on goods and resources consumed and produced (t_{ij}^* and t_{ik}^y), on wastes emitted by firms (t_k^s), and on wastes absorbed by consumers and firms (t_j^z and t_k^z). With the variables of Section 2 redefined in these ways, we may return to programs (6) - (8) to derive the familiar Pigouvian results:

$$t_{ij}^* = t_j^z = t_{ik}^y = t_k^z = 0$$

$$t_k^s = \rho.$$

The Pigouvian approach to the treatment of externalities certainly covers a far broader set of economic problems than is commonly recognized. In fact, as we have shown above, virtually every troublesome economic problem can be understood as one of externalities. Externalities generated by undepletable goods and resources have been the favorite child of economists. The most popular externalities of this category have been those generated by waste by-products of consumption and production activities. That is probably the reason why externality analysis has been so closely allied with environmental economics. We have shown here that the realm of externality analysis extends far beyond this subject matter. Externalities generated by depletable goods and resources represent a comparatively

unexplored set of phenomena. We have given two examples of such externalities (both concerning unemployment) and examined the appropriate Pigouvian fiscal treatment for each. The common result that such externalities may be optimally treated by ordinary price adjustments was found inapplicable in all but a restricted set of economic circumstances. This discovery reveals an enhanced area of economic problems for which the Pigouvian prescriptions may serve as valuable guidelines of fiscal policy. The two examples we have considered barely represent the top of the iceberg. The challenge of devising Pigouvian externality treatment measures remains open for the large number of economic models founded on optimizing behavior and dealing with the problems above.

Footnotes

- 1) See, for example, Frank Knight, "Some Fallacies in the Interpretation of Social Cost," Quarterly Journal of Economics, vol.38, 1924, pp.582-606.
- 2) "Goods and resources" are here interpreted broadly. See Section 3, p.32.
- 3) The theory of revealed preference outlines a method of observing utility functions, but this method is impracticable. It is usually impossible to perform sufficient market experiments to derive an indifference map from the two axioms of revealed preference in a time period short enough to ensure constancy of tastes.
- 4) A clear exposition of this argument is given by Armen A.Alchian in "Uncertainty, Evolution, and Economic Theory," Journal of Political Economy, 58, June 1950, pp.211-21. Briefly, the argument is that firms which do not maximize profits will not survive in the long run; they will be driven out of the market by their competitions.
- 5) Tibor Scitovsky, "A Note on Profit Maximization and its Implications," Review of Economic Studies, 11, 1943-44, pp.57-60.
- 6) This theory is most intimately associated with Baumol. W.J.Baumol, Business Behavior, Value and Growth, New York: Harcourt Brace Jovanovich, Inc., 1967.
- 7) See Rabin Marris, "A Model of 'Managerial' Enterprise," Quarterly Journal of Economics, 77, May 1963, pp.185-209.

- 8) See, for example, A.A.Berle and G.C.Means, The Modern Corporation and Private Property, New York: Commerce Clearing House, 1932; and O.E.Williamson, Economics of Discretionary Behavior: Managerial Objectives in a Theory of the Firm, Chicago: Markham Publishing Co., 1967.
- 9) For example, Martin Shubik, "A Curmudgeon's Guide to Microeconomics," Journal of Economic Literature, 8, June 1970.
- 10) The following is a very small sample of such evidence. Marshall Hall (in "Sales Revenue Maximization: An Empirical Investigation," Journal of Industrial Economics, 15, April 1967, pp. 143-56) finds no empirical support for the profit-constrained sales maximization hypothesis. His conclusions have been partially challenged by Robert J. Saunders (in "The Sales Maximization Hypothesis and the Behavior of Commercial Banks," Mississippi Valley Review of Business and Economics, 5, Fall 1970, pp.21-32. David R.Kamerschen (in "The Influence of Ownership and Control on Profit Rates," American Economic Review, 58, June 1968, pp.432-47) finds no relation between profit rates and the manager-owner status of large nonfinancial corporations; and his results have been partially challenged by R.I.Monsen, J.S.Chiu, and D.E.Cooley (in "The Effect of Separation of Ownership and Control on the Performance of the Large Firm," Quarterly Journal of Economics, 82, August 1968, pp.435-51.
- 11) The argument that nonconvexities in production sets can arise on account of externalities has been made, for example, by W.Baumol and D.Bradford in "Detrimental Externalities and Non-Convexity of the Production Set," Economica, XXXIX, May 1972, pp.160-76.
This argument deals exclusively with production-to-production externalities. It is clear that an analogous argument can be made for nonconvexities in utility sets with reference to consumption-to-consumption externalities.

- 12) Their first contribution in this area was "The Use of Standards and Prices for Protection of the Environment," (in Swedish Journal of Economics, LXXIII, March 1971, pp.42-54.
- 13) A rigorous distinction between these two issues is made by the author in A Proposal for Evaluating Production and Consumption Externalities in an Economic-Environmental Context, Section 7, pp.72-93, Department of Economics, University of Maryland, May 1976, unpublished manuscript.
- 14) The theoretical analysis of this section is methodologically similar to that offered by W.Baumol and W.Oates in The Theory of Environmental Policy (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1975, ch.4). This inspiring study of externalities has gained a prominent place in the literature on this subject. Economists have shown no inclination to quarrel with the authors' derivation of Pigouvian taxes and subsidies. Thus it appears to be a suitable representation of the Pigouvian approach as commonly described in the literature.
In the name of strict scholarly justice, it is appropriate to indicate several conspicuous differences between our analytical exposition and that of Baumol and Oates. First, Baumol and Oates derive the proposition that the Pigouvian taxes and subsidies are necessary and sufficient for the social optimality of the competitive equilibrium from two very restrictive assumptions: (1) "there exists one item, some of which is consumed by every individual" (Ibid, p.41), and (2) there exists "some item that is potentially either an output or an input for every firm" (Ibid., p.42). The authors adduce "labor-leisure" as an example of such an item. Certainly, this example becomes implausible once "labor" is disaggregated. These assumptions appear to be unnecessary. The above proposition is derived without them in this section. Second, Baumol and Oates use the Pareto welfare criterion to examine the welfare implications of each economic actor's activity, whereas we employ a regular social welfare function. Thus, our framework is somewhat narrower since it necessitates a larger number of social welfare comparisons. However, our analysis could have been conducted in terms of the Pareto framework

without effecting our conclusions. We make use of a social welfare function here only for the sake of notational economy and algebraic simplicity. Third, the consumer of Baumol and Oates minimizes the expenditure necessary to attain a given level of utility, whereas our consumer maximizes his utility subject to a budget constraint. This difference is immaterial. Ours is simply the usual formulation of consumer behavior. Lastly, each of the Pigouvian taxes on the consumer (t_{ij}^x and t_j^z) and on the firm (t_{ik}^y , t_k^z , and t_k^s) are specified explicitly in our analysis, while Baumol and Oates simply use t^j for the consumer and t^k for the producer. Hopefully, our degree of detail helps to clarify the workings of the Pigouvian fiscal prescriptions.

- 15) Ibid., p.17, footnote 8.
- 16) Ibid., p.19-24.
- 17) Ibid., p.20.
- 18) Ibid., p.20.
- 19) F.M.Bator, "The Anatomy of Market Failure," Quarterly Journal of Economics, LXXII, August 1958, pp.351-79.
- 20) Baumol and Oates, op.cit., p.20.
- 21) Ibid., p.23.
- 22) This characterization is described in Baumol and Oates, Ibid., p.27.
- 23) Ibid., p.23.
- 24) Ibid., p.23.

- 24A) In general, the social welfare function may be understood to depend on the welfare of each individual. Since the individual welfare functions depend on the externalities and the goods and resources consumed, the social welfare function may be written in terms of these variables.
- 25) Some goods can be produced and used by firms and for these goods y_{ik} is unrestricted in sign. Then, $\frac{\partial L_i}{\partial y_{ik}} = -\mu_k \cdot f_i^k + \omega_i = 0$. However, certain resources, like coal and labor, cannot be produced by firms; thus $y_{ik} \leq 0$ in such cases. The formulation of conditions (16) is completely general.
- 26) We ignore the intertemporal problem of determining what part of his wealth he makes available for expenditure in the present time period. This problem would call for an unnecessary -- but not unexecutable -- complication of our analysis. The variable W_j may be described as that part of the j 'th consumer's initial holdings which he sells in the present time period plus his share of profits. (The profits gleaned by the firms of the economy may be distributed to the consumers.) A thus defined W_j is identified as "wealth by K.D.Arrow and G.Debreu (in "Existence of an Equilibrium for a Competitive Economy," Econometrica, 22, 1954, pp.265-290). In addition, the consumer's ability and willingness to acquire credit for his expenditures may be incorporated in W_j .
- 27) See, for example, K.D.Arrow and F.H.Hahn, General Competitive Analysis, San Francisco, California: Holden-Day, Inc., 1971, pp.132-136. The externality analysis presented in this work is applicable to both depletable and undepletable externalities.
- 28) Baumol and Oates, op.cit., p.35.
- 29) R.H.Coase, "The Problem of Social Cost," Journal of Law and Economics, III, Oct. 1960, pp. 1-44.

- 30) This case may be exemplified by seats on a Metroliner train. The social cost of occupying a seat when there are unoccupied seats available is usually zero; when all seats are used, however, one person's seat reservation prevents another person from taking the ride.
- 31) Baumol and Oates (op.cit., p.27) take as example "an expansion in output that increases the availability of on-the-job training (a depletable externality) and simultaneously improves the social climate (reduces crime rates, and so on) in impoverished areas."
- 32) Ibid., p.17, footnote 8.
- 33) Baumol and Oates, op.cit., p.35, I
- 34) The confusion between externalities and the goods and resources which generate them are quite prevalent in the literature on this subject. Here is an example of this confusion occurring in the course of a single sentence: "One might argue that the threat or presence of government intervention can force the polluter to concern himself with the effects of his emissions on those whom he harms, but we would not want to say that his newly awakened concerns disqualify his emissions as an externality." Ibid., p.17. Italics mine.
- 35) Ibid., p.20.
- 36) Ibid., p.17. Itali
- 37) D.Snower, op.cit., Section 4.
- 38) Baumol and Oates, op.cit., p.12.

- 39) It should be clear that labor may be regarded as a depletable resource regardless of whether it is fully employed or underemployed. An entrepreneur's opportunities for hiring labor are limited by the size of the total labor force minus labor used by other economic actors. However, in the case of smoke -- an undepletable resource -- my inhaling of it depends on the total amount of smoke in the atmosphere, and not on the total amount of smoke minus everyone else's intake of it.
- 40) This assumption serves to simplify our notation, but it implies no loss of generality. If the demand for labor services appears in the utility functions as well as the production functions, conclusions may be derived which are qualitatively equivalent to those of our analysis.
- 41) The case of significant exclusion and metering costs is considered in a separate paper. See D.Snowder, "Extensions of the Pigouvian Approach to Externality Treatment", unpublished manuscript.
- 42) A_{jk} may be taken as a function of the j 'th individual's employment history with the k 'th firm. If his employment expectations are formed analogous to the evaluation of his spending power along the lines of the permanent income hypothesis, then the function may be written as follows:

$$A_{jk}^t = \frac{1}{T} \left(\frac{L_{jk}^{t-1}}{s_{L_{jk}}^{t-1}} + \theta \cdot \frac{L_{jk}^{t-2}}{s_{L_{jk}}^{t-2}} + \theta^2 \cdot \frac{L_{jk}^{t-3}}{s_{L_{jk}}^{t-3}} \right. \\ \left. + \theta^{T-1} \cdot \frac{L_{jk}^{t-T}}{s_{L_{jk}}^{t-T}} \right),$$

where $t-i$ is a time subscript, T is the length of the individual's "memory", and θ is a constant (between zero and unity) which indicates how fast his memory "fades".

- 43) Restricting adverse sales expectations only to the capital good K implies no loss of generality. If several goods and resources were characterized by such expectations, our conclusions would remain unaffected -- although the argument leading to these conclusions would be somewhat more complicated.
- 44) Only under special conditions is price inflexibility necessary for the existence of Keynesian unemployment. Suppose that entrepreneurs expect aggregate demand in the economy to be 3Q . Furthermore, suppose that, given such sales expectations, unemployment is present. A fall in wages (cet.par.) will lead entrepreneurs to hire more labor (if their expected aggregate demand remains unchanged) if their production technology permits sufficient substitution of labor for other factors.
- 45) This is the only fiscal means of achieving the social optimum. Naturally, other measures -- such as direct controls -- could, conceptually, achieve this goal as well.
- 46) This formulation is quite general. Naturally, a consumer's utility function may be such that $K_j = 0$, or a firm's production function may make $K_k^j = 0$. Thus, any produced good which is used by entrepreneurs or consumers, or both, fits this description. For example, intermediate goods fall into this category as well.
- 47) The socially optimal per unit subsidy on the production of K is $-s_t = (\eta - \eta \cdot B_K)$.