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# Unemployment Dynamics: An Unobserved Components Approach

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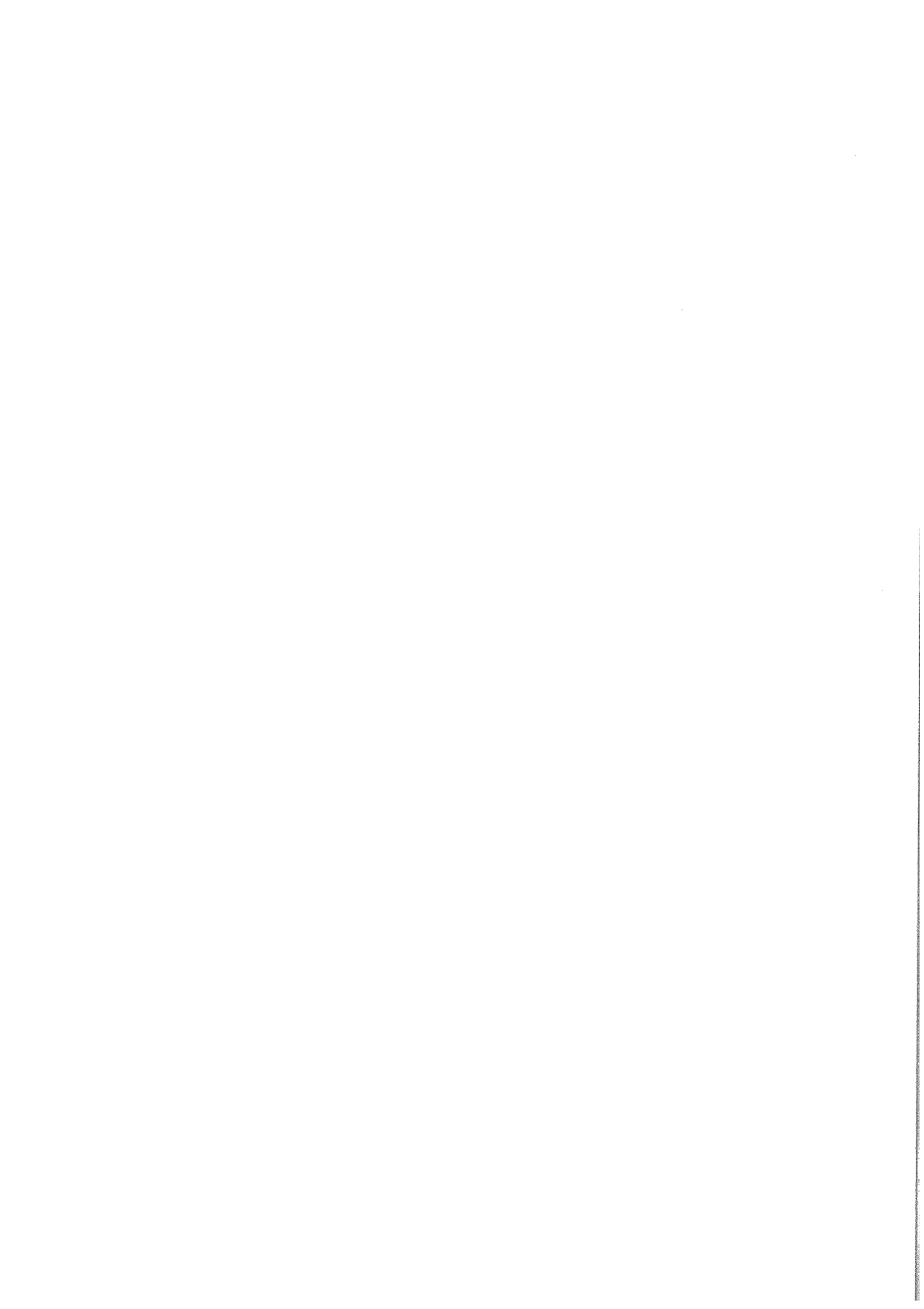
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## **Abstract**

The study uses a bivariate unobserved components model for output and the unemployment rate in order to examine stylised facts of the cyclical behaviour of unemployment and to estimate the size of persistence. The model is applied to the U.S., Canada, and major European economies. Estimates of cyclically adjusted unemployment rates are obtained that account for persistence effects and are subject to considerably smaller confidence bounds compared to approaches based on wage and price setting equations.

## **Keywords**

Trend-and-Cycle decomposition, persistence, multivariate stochastic cycles

## **JEL-Classifications**

C22, E30

**Comments**

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## 1 Introduction

The assessment of the cyclical stance of the major economies has been subject to steady policy concern during recent decades. The matter has been considerably complicated by the fact that unemployment rates have experienced pronounced level shifts thereby making the extraction of their cyclical component a rather difficult task. It has been further argued that some part of the rise in unemployment may be attributable to a longer-term response of unemployment to the cycle (Blanchard and Summers, 1986; Lindbeck and Snower, 1988; Pissarides, 1992; Saint-Paul, 1995). These considerations have given rise to a widely accepted conceptual distinction between steady-state and short-run (quasi-equilibrium) non-accelerating inflation rates of unemployment (NAIRU). As a consequence of persistence the short-run NAIRU depends on both the underlying steady state NAIRU and on past unemployment. It therefore adjusts only sluggishly towards the former in response to cyclical deviations (Layard et al., 1991; Franz and Gordon, 1993).

Empirically, the issue has been addressed in estimates of structural wage setting and labour demand equations, where the steady state NAIRU is identified by relating it to a set of variables aimed at explaining the mark-ups of consumption wages over prices and vice versa (e.g., Alogoskoufis and Manning, 1991; Layard et al., 1991; Franz and Gordon, 1993; Nickell, 1995). While this approach has been highly informative in assessing the impact of institutional labour market features on the outcome of the wage bargaining process it has its limitations as regards estimates of the steady state NAIRU and the size of persistence effects. First, estimates of the steady-state NAIRU have been found to be sensitive to the details of the specification and subject to large confidence bounds (Setterfield et al. 1992; Cromb, 1993; Funke, 1993; Manning, 1993). Second, many studies have used deterministic polynomial trends in order to account for the unexplained rise in unemployment (e.g., Alogoskoufis and Manning, 1988; Layard et al., 1991).

Yet it is not quite clear whether these sufficiently accommodate the behaviour of the trend. In fact, it has been argued that the trend should be better regarded as stochastic (Jäger and Parkinson, 1994; Kösters and Belke, 1995; Roed, 1996) a neglect of which being likely to seriously distort estimates of the NAIRU and the persistence coefficient.

The present study employs a complementary, less structural approach in order to investigate some stylised facts of unemployment dynamics. The basic idea is to model unemployment rate trends and cycles as unobserved components and to identify the latter by their close co-movement with output at business cycle frequencies. For this purpose I construct a bivariate unobserved components (UOC) model for output and the unemployment rate, where both series are decomposed into their trend and cyclical components. Cyclical co-movement is modelled as a bivariate stochastic cycle thereby extracting the specific components of both series characterised by high coherence at business cycle frequencies. I allow for level shifts in the unemployment rate by a random walk without drift and account for persistence effects from an extended version of the partial adjustment equation as typically used in the relevant literature.

The model thereby provides an estimate of persistence based on the longer-term response of unemployment to the output cycle. As a further outcome an estimate of a cyclically adjusted unemployment rate is obtained that accounts for persistence effects and will turn out to be subject to considerably smaller confidence bounds as compared to NAIRU estimates based on price and wage equations. It should be noted yet that the decomposition is based on output rather than wage and price dynamics and does not make use of the property of inflation neutrality, which is essential to the definition of the NAIRU. I will test for the NAIRU property of the extracted trend and return to the issue in the discussion.

Sections 2 and 3 present the methodology used and empirical results for U.S., U.K., Canadian, German, and French data from 1960 to 1996. Section

4 concludes the paper.

## 2 Methodology

I follow the majority of empirical works (e.g., Layard et al., 1991, Franz and Gordon, 1993) in applying a first-order partial adjustment mechanism for modelling persistence in unemployment. The unemployment rate  $u_t$  is composed of the short-run NAIRU  $u_t^{QE}$  and the cyclical component  $u_t^C$ . The former depends on the underlying steady state NAIRU  $u_t^*$  and on past unemployment. The partial adjustment equation is embedded in a bivariate UOC model for output and unemployment. The basic setup is given by equations

$$\begin{aligned} y_t &= y_t^T + y_t^C + \nu_t^{(y)} \\ u_t &= u_t^{QE} + u_t^C + \nu_t^{(u)} \\ u_t^{QE} &= (1 - \omega)u_t^* + \omega u_{t-1}, \quad 0 \leq \omega \leq 1 \end{aligned} \tag{1}$$

Output is decomposed along the lines of a structural time series (STS) model (Harvey, 1985; 1989) into a non-stationary trend ( $y_t^T$ ), a stationary cyclical component ( $y_t^C$ ) and an irregular white noise term  $\nu_t^{(y)}$ . Joint cyclical dynamics is modelled as a bivariate stochastic cycle while  $u_t^*$  is assumed to follow a random walk without drift. The model requires assumptions on the dynamics underlying the particular components and certain orthogonality restrictions on the respective innovations in order to achieve identifiability. I start with discussing the unemployment equation and subsequently present the model for cyclical dynamics.

### 2.1 Unemployment equation

I take the presence of a stochastic trend component in unemployment as given and assume that  $u_t^*$  follows a random walk without drift thereby ac-

counting for long-term shifts in the unemployment level.<sup>1</sup> Combining the partial adjustment equation with the random walk model, i.e.,  $\Delta u_t^* = \eta_t$ , inserting for  $u_{t-1}$ , and taking first differences gives the following expressions for  $u_t^{QE}$  and  $u_t$ , respectively (ignoring the irregular term).

$$\begin{aligned}(1 - \omega L)\Delta u_t^{QE} &= \omega \Delta u_{t-1}^C + (1 - \omega)\eta_t \\ (1 - \omega L)\Delta u_t &= \Delta u_t^C + (1 - \omega)\eta_t\end{aligned}$$

The latter equation reveals that the partial adjustment equation essentially amounts to an autoregressive term in  $u_t$ . It is quite interesting to note that the univariate equation is, in principle, identifiable if a stochastic cycle is used for modelling cyclical dynamics and cyclical and trend innovations are assumed to be orthogonal. This holds though both components share a common autoregressive root.<sup>2</sup> However, for the purpose of disentangling business cycle dynamics and persistence effects the univariate equation seems of limited use. The multivariate approach further allows for higher flexibility by including a polynomial  $\beta(L) = \beta_1 L + \dots + \beta_k L^k$  in the cycle which gives

$$(1 - \omega L)\Delta u_t = (1 + \beta(L))\Delta u_t^C + (1 - \omega)\eta_t \quad (2)$$

## 2.2 Cyclical dynamics

The GDP trend component  $y_t^T$  is assumed to follow a so-called local linear trend, that is, a random walk with a stochastic slope term  $\mu_t$ ,

$$\begin{aligned}\Delta y_t^T &= \mu_{t-1} + \zeta_t^{(1)} \\ \Delta \mu_t &= \zeta_t^{(2)}\end{aligned} \quad (3)$$

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<sup>1</sup>This assumption has been made by several recent studies (Gordon, 1997; Faruqee et al., 1997) in estimating Phillips curve type equations. It has been argued that the random walk model is not data-admissible as it does not restrict forecast confidence bounds to remain within the unit interval. Yet the essential property for the current purpose is its high capability of accommodating a variety of different trend patterns (Garbade, 1977). It might be noted that more severe difficulties arise with deterministic polynomial trends.

<sup>2</sup>To see this note that the reduced form equation for  $\Delta u_t$  is given by an ARMA(3,2) process and that the only unknown parameters in the moving average part are the variances of cyclical and trend innovations (see Harvey, 1989: 206ff).

where  $\zeta_t^{(1)}$  and  $\zeta_t^{(2)}$  are both white noise. If  $\sigma_2^2 = \text{var}(\zeta_t^{(2)}) = 0$  this reduces to a random walk with drift, while for the case of  $\sigma_1^2 = \text{var}(\zeta_t^{(1)}) = 0$ , but  $\sigma_2^2 > 0$ , the trend is represented by a second-order random walk. Such a trend tends to be relatively smooth compared to a random walk (see, e.g., Harvey and Jäger, 1993).

For modelling joint cyclical dynamics I use a generalisation of the common cycle restriction (Harvey and Koopmans, 1996) proposed by Rünstler (1997) the basic building block of which is the stochastic cycle (SC)  $\tilde{\varphi}_{i,t} = (\varphi_{i,t}, \varphi_{i,t}^*)'$ . This is given by equation

$$\begin{bmatrix} \varphi_{i,t} \\ \varphi_{i,t}^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} \varphi_{i,t-1} \\ \varphi_{i,t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_{i,t} \\ \kappa_{i,t}^* \end{bmatrix} \quad (4)$$

with decay  $0 < \rho < 1$  and frequency  $0 \leq \lambda < \pi$ . Usually  $\kappa_{i,t}$  and  $\kappa_{i,t}^*$  are assumed to be orthogonal and of same variance, i.e.,  $\Sigma_\kappa = \sigma_\kappa^2 I$  (Harvey, 1989). In this case, the SC exhibits certain symmetry properties with  $\varphi_{i,t}$  and  $\varphi_{i,t}^*$  sharing identical auto spectra while the real part of the cross spectrum is identical to zero.

Let the cyclical component in output be represented by a stochastic cycle, i.e.,  $y_t^C = \varphi_{1,t}$ , while  $u_t^C$  is linked to both  $\varphi_{1,t}$  and  $\varphi_{1,t}^*$  from equation (4) and contains a further SC  $\varphi_{2,t}$  of same decay and frequency.

$$\begin{bmatrix} y_t^C \\ u_t^C \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \theta_1 & \theta_1^* \end{bmatrix} \begin{bmatrix} \varphi_{1,t} \\ \varphi_{1,t}^* \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \theta_2 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{2,t} \\ \varphi_{2,t}^* \end{bmatrix} \quad (5)$$

The related innovations  $\tilde{\kappa}_{1,t}$  and  $\tilde{\kappa}_{2,t}$  are assumed to have the same covariance matrix and to be uncorrelated, i.e.,  $E\tilde{\kappa}_t\tilde{\kappa}_t' = \Sigma_\kappa \otimes I_2$  where  $\tilde{\kappa}_t' = (\tilde{\kappa}_{1,t}', \tilde{\kappa}_{2,t}')$ . For the case of  $\Sigma_\kappa = \sigma_\kappa^2 I$ , as shown in appendix A.1, the autocovariance function (ACF)  $\Gamma_x(s)$  of  $y_t^C$  and  $u_t^C$  is given by

$$\Gamma_x(s) = \rho^{|s|} \frac{\sigma_\kappa^2}{(1 - \rho^2)} \begin{bmatrix} \cos(s\lambda) & \alpha\vartheta \cos(\lambda(s - \xi)) \\ \alpha\vartheta \cos(\lambda(s + \xi)) & \vartheta^2 \cos(s\lambda) \end{bmatrix} \quad (6)$$

where

$$\begin{aligned}
\vartheta &= \sqrt{\theta_1^2 + \theta_1^{*2} + \theta_2^2} \\
\alpha &= \text{sign}(\theta_1) \vartheta^{-1} \sqrt{\theta_1^2 + \theta_1^{*2}} \\
\xi &= \lambda^{-1} \tan^{-1}(\theta_1^*/\theta_1)
\end{aligned} \tag{7}$$

The ACF gives rise to an interpretation in terms of the relative variance  $\vartheta^2$ , coherence  $\alpha$ , and a phase shift  $\xi$  between the two cycles. Both cycles are subject to the same autocorrelation function. The cross correlation function is given by a dampened cosine wave subject to a phase shift  $\xi$  which is normalized to lie within the range of one quarter of the cycle length in absolute value. i.e.,  $|\lambda\xi| \leq \pi/2$ . The multiple correlation  $|\alpha| \leq 1$  of  $u_t^C$  with  $\tilde{\varphi}_{1,t}$  finally gives a measure for the strength of association between the two cyclical components. As the special case of  $\theta_2 = 0$  or, equivalently,  $|\alpha| = 1$  emerges one generalised common cycle. The Wald test for the latter case follows an  $0.5(1 + \chi_1^2)$  distribution (see appendix A.1). The test for the presence of a phase shift, i.e., the null of  $\theta_1^* = \xi = 0$  is standard.

The SC has the attraction that its spectral density centers around cyclical frequencies (Harvey, 1993). It seems therefore particularly suitable for extracting the specific components in output and unemployment that are typically regarded as the business cycle. The model might be extended by using a more general  $\Sigma_\kappa$  allowing for a correlation among cyclical innovations. In this case, however, the specific interpretation of the ACF is lost which hinges on the above mentioned symmetry properties of the SC for the case of  $\Sigma_\kappa = \sigma_\kappa^2 I$ .<sup>3</sup>

Various applications of univariate (Harvey, 1985; Harvey and Jäger, 1993) as well as multivariate STS models (Rünstler, 1997) indicate that these sufficiently capture the dynamics of output and other macroeconomic series. Further, inspecting bivariate VARMA models for the U.S. GDP and unemployment rates Hofer et al. (1998) have found low order VARMA

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<sup>3</sup>The properties of the model, i.e., coherence and the phase shift might yet be inspected in the frequency domain. The respective expressions are obtainable from the author.

models to be superior to the VAR approach. The autoregressive part invariably contained a conjugate complex root which compares favourably to the reduced form of STS models.

### 2.3 Estimation

The model is given by equations (1), (2), (3), and (5). It comprises three types of innovations, i.e., innovations  $\zeta_t^{(1)}$  and  $\zeta_t^{(2)}$  to the output level and slope, innovations  $\eta_t$  to the unemployment level, and cyclical innovations  $\tilde{\kappa}_{i,t}$  as from equation (5). Identifiability is achieved by imposing blockwise orthogonality among slope, level, and cyclical innovations and the irregular components, respectively. Yet a correlation among output and unemployment level innovations might be introduced (Harvey and Koopmans, 1996). For estimation the model is cast in state-space form

$$\begin{aligned} x_t &= Z\delta_t + \nu_t \\ \delta_t &= T\delta_{t-1} + \varepsilon_t \end{aligned} \tag{8}$$

with the vector  $x_t = (y_t, u_t)'$  of the observations at time  $t$  of GDP and the unemployment rate, the state vector  $\delta_t$  comprising the unobserved state variables, and  $\varepsilon_t$  and  $\nu_t$  representing the vectors of trend and cyclical innovations and irregular components, respectively. The set of hyperparameters is estimated by maximum likelihood using the prediction error decomposition provided by the Kalman filter. The subsequent application of a fixed interval smoother provides efficient estimates of the particular trend and cyclical components (Harvey, 1989). The state-space representation is outlined in appendix A.2.

## 3 Empirical results

The empirical analysis uses seasonally adjusted quarterly data for the U.S., the U.K., Canada, Western Germany, and France from 1960(1) to 1996(4)

(for Germany 1962(1) to 1994(4) and France 1970(1) to 1996(4)).<sup>4</sup>

I started with estimating models that allow for both correlations among cyclical innovations and output and unemployment level components and tested for the order of the lag polynomial  $\beta(L)$  by a sequence of LR tests. There is no indication for including any lag for the European countries while one lag appeared to be sufficient for the U.S. and Canada (see Table 1). Further, the restriction of orthogonal cyclical innovations is generally accepted with the exception of the U.K. In neither case it affects any of the subsequent results and is therefore imposed for the results presented.

Despite the stylised nature of the model there are little signs of auto-correlation in standardised prediction errors with the exception of Canadian and French output. However, there arise several sharp outliers in U.S., Canadian, and French unemployment rate trend innovations at around 1975 and 1982 indicating that the rise in unemployment in these two periods might have been sharper than what can be accommodated by the random walk model. I therefore included innovation dummies in the unemployment trend equations of these countries.<sup>5</sup> Bianchi and Zoega (1994a, 1994b) have argued that European unemployment dynamics should be represented by models that allow for sudden level shifts in the unemployment rate trend. Curiously, if one adjusts for the cyclical dynamics these models may be rather appropriate for the U.S. and Canada.

Table 1 sets out the main estimation results the top panel showing the parameters related to the cycle. Estimates of the cycle length range from

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<sup>4</sup>The data, i.e. real GDP (in logs) and standardised unemployment rates, are taken from the OECD Main Economic Indicator (MEI) data base. The German series were provided by the DIW (Berlin) and have been seasonally adjusted by Census X-11. Estimation was done in GAUSS using the algorithm by Rosenberg (1973).

<sup>5</sup>More specifically, I included one dummy in unemployment trend equations for 74(4), 75(1), and 81(4) for the U.S., 75(1), 80(1), and 80(2) for Canada, and 74(4) and 84(1) for France, respectively. For the U.K. I further added one additive dummy to output with opposite signs in 74(1) and 75(1). The dummies have little effect on the parameter estimates. The only noteworthy difference is a somewhat higher estimate of  $\vartheta^{-1}$  for the U.S which would amount to 3.6 otherwise.



Table 1: Parameter estimates

		U.S.	Canada	Germany	U.K.	France
Cycles						
decay	$\rho$	.957	.948	.956	.973	.955
cycle length	$2\pi/\lambda$	28.450	37.248	24.326	33.550	42.834
std.dev. $y^C$		2.218	2.187	2.110	2.300	1.173
rel. std.dev.	$\vartheta^{-1}$	4.475	3.990	6.457	4.715	8.392
phase	$\xi$	-.663	-.051	1.474	-2.454	-1.081
mult.corr.	$\alpha$	-1.000	-1.000	-1.000	-1.000	-1.000
Unemployment trend						
	$\omega$	.155	.065	.707	.594	.507
	$\beta_1$	.538	.526	—	—	—
Innovations						
	$\sigma_\nu^{(y)}$	.2776	.1829	.5076	.5659	.0000
	$\sigma_1$	.0000	.4145	.0000	.0091	.2551
	$\sigma_2$	.0318	.0389	.0559	.0369	.0218
	$\sigma_\eta$	.1616	.1586	.1076	.1349	.1293
	$\sigma_\kappa$	.6377	.6958	.6170	.5239	.5151
Likelihood		543.569	490.996	486.821	531.138	451.384
LR <sub>1</sub>	$\omega = \beta_1 = 0$	**36.91	**23.97	**106.20	**90.25	**52.33
LR <sub>2</sub>	$\beta_1 = 0$	**9.18	**12.00	.16	2.36	2.12
LR <sub>3</sub>	$\theta_2 = 0$	1.97	.00	.00	.00	.00
LR <sub>4</sub>	$\theta_1^* = 0$	*4.24	.01	*5.16	**6.97	.02
S.E. ( $\hat{u}_T^*$ )		.19	.21	.23	.34	.09
Q(20) GDP		24.52	*29.89	25.90	20.91	**34.47
Q(20) UR		16.49	23.63	17.52	20.61	21.93

Standard deviations related to output ( $\sigma_\nu, \sigma_1, \sigma_2$ , and  $\sigma_\kappa$ ) are multiplied by 100. There was no irregular component found in unemployment rates. S.E. ( $\hat{u}_T^*$ ) denotes the standard error of the final sample point estimate of  $u_t^*$ .

Critical values for the likelihood ratio tests are given from  $\chi^2$ -distributions with the exception of test LR<sub>3</sub>. The 5% critical value of the latter is given by Kodde and Palm (1986) with 2.71. Q(20) denotes the Ljung-Box statistics for serial correlation in prediction errors. Its 5%-critical value is given from a  $\chi_{16}^2$  distributions with 26.29. \* and \*\* indicate significance at the 5% and 1% level, respectively.

28 to 42 quarters and are somewhat higher compared to the conventional view about the duration of the business cycle. However, the period under consideration has been subject to particularly long and deep cycles related to oil price shocks and periods of disinflation. For all countries estimates of  $\theta_2$  are very close to zero and insignificant (test LR<sub>3</sub>). Hence, cyclical dynamics is represented by one common cycle though the co-movement is subject to certain phase shifts (test LR<sub>2</sub>). With the exception of Germany, the unemployment cycle tends to lag the output one up to 2.4 quarters. The inverse of the relative standard deviation of the unemployment rate to the output cycle,  $\vartheta^{-1}$ , is closely related to Okun's coefficient. For the U.S., Canada and the U.K the estimates range from 4.0 to 4.7. They take somewhat higher values of 6.5 and 8.4 for Germany and France, respectively, indicating a lower cyclical volatility of unemployment compared to the former countries. Some evidence in this direction has been provided by Elmeskov and McFarlan (1993).

Turning to the persistence equation (2), the estimates indicate a rather different response of unemployment to the cycle for the European countries compared to the U.S. and Canada. For the latter two countries persistence is found to be very small and insignificant while there appears to be some short-run impact of the cycle on the trend. In contrast, for the European economies persistence is found to be highly significant (see test LR<sub>1</sub>) with estimates of the persistence coefficient ranging from 0.51 to 0.71.

Fig. 1 to 5 show estimates of the output cycle and unemployment rates together with the component due to trend innovations. In order to visualise the effect of the GCC model estimates of the output cycle from a univariate STS model as from equation (4) and (3) are included. The unemployment rate trend component rose steadily in the European countries between the seventies and mid-eighties. In contrast, in the U.S. and Canada, this increase seems to have occurred more sharply in response to the 1974 and 1979 oil price shocks. The trend reverted in the U.S. and the U.K. around 1983 and came

to a halt at around 1983 to 1986 in Canada and the remaining European economies. Since then it seems to have remained constant for Canada and Germany, but continued to rise in France from 1990 on. Standard deviations of the final sample point estimate of  $u_t^*$  range from 0.09 to 0.34 and are considerably lower than those found from wage and price setting equations (Staiger et al., 1997; European Commission, 1995).<sup>6</sup>

Table 2 displays the results of a simple test for the NAIRU property of the extracted trend. From a reduced form equation for changes in price inflation derived from structural wage and price setting equations (e.g., Layard et al., 1991: 378ff) it follows with some re-arrangement that  $\Delta^2 p = -\gamma u^C$ .

**Table 2: Test for the NAIRU property**

	U.S.	Canada	Germany	U.K.	France
const	.0001 (.01)	.0005 (.01)	.0016 (.85)	-.0002 (.04)	-.0031 (.52)
$u_t^C$	-.0301 (4.29)	-.0170 (2.62)	-.0313 (4.09)	-.0212 (2.33)	-.0304 (2.32)
$\Delta u_t^C$	.0049 (.55)	.0083 (.87)	-.0105 (1.11)	-.0138 (1.04)	-.0082 (.26)
$\Delta u_t^*$	.0019 (.83)	-.0087 (.83)	-.0067 (1.22)	.0029 (.85)	-.0061 (.46)

Dependent variable is the consumption deflator in second differences (yearly data). For Germany  $u_{t-1}^C$  is included in the place of  $u_t^C$ . t-values are shown in parentheses. As instrumental variables past capacity utilisation (UK: index of coincident indicators), and past changes in unemployment rates, wage and price inflation, price wedges, and unit labour costs have been used.

The NAIRU property therefore amounts to the absence of an influence on inflation changes of changes in the unemployment cycle and the trend component. A test can thus be performed from a regression of the former on the unemployment cycle in levels and first differences, and changes in the

<sup>6</sup>It should be noted that the confidence bounds do not reflect the uncertainty of the parameter estimates (see Hamilton, 1986).

trend. I base the test on yearly data of the consumption deflator. As can be inferred from Table 2, the cycle is significantly related to inflation changes for all countries while the NAIRU property is accepted.

## 4 Concluding remarks

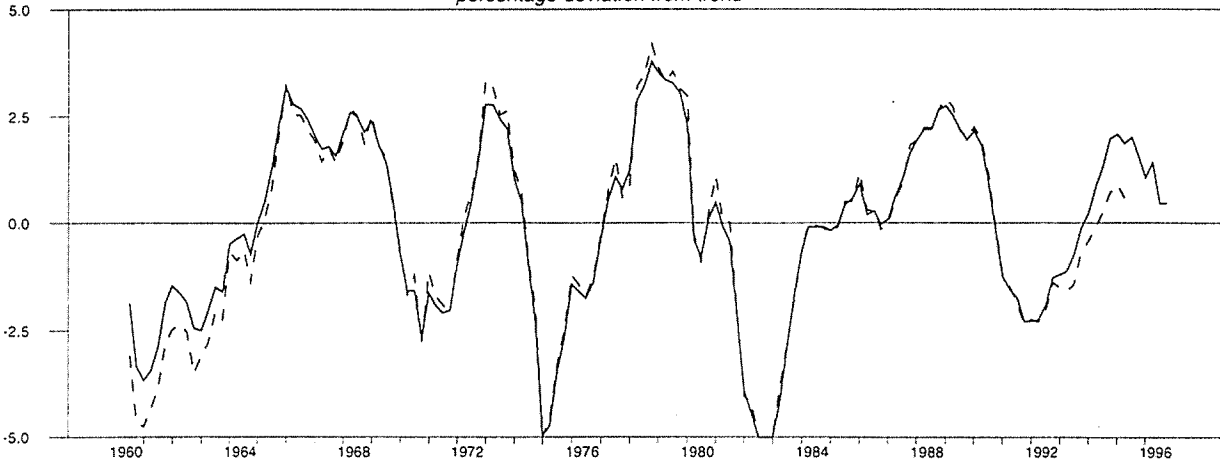
The present study applied a bivariate unobserved components model for the decomposition of output and unemployment into their trend and cyclical components the main purpose of which has been an assessment of stylised facts with regard to the cyclical behaviour of unemployment rates and the extraction of cyclically adjusted unemployment rates. It is found that a very close cyclical co-movement combined with persistence effects in unemployment provides a reasonable description of the joint dynamics of the two series. Most important, the results point to a sharp distinction between the U.S. and Canada compared to the European economies as regards the size of persistence effects. Many studies based on price and wage equations have found a considerable degree of persistence also in Canada while the evidence for the U.S. is mixed (Layard et al., 1991; Alogoskoufis and Maninngs, 1998; Franz and Gordon, 1993). The current results suggest that for both countries the finding of persistence might be rather attributable to cyclical dynamics than to a longer-term response of unemployment to the cycle.

Estimates of the unemployment rate trend are subject to considerably smaller confidence bounds compared to estimates based on wage and price equations. While they are based on output dynamics and therefore do not make explicit use of the NAIRU property there is no indication for a violation of the latter. In fact, Franz and Gordon (1993) and Staiger et al. (1997) have argued that price inflation might exhibit a more stable relationship with the output gap compared to cyclical unemployment. The former authors have therefore proposed to use the concept of a mean capacity utilisation rate of unemployment as a measure for the quasi-equilibrium rate. Given the inherent difficulties in finding reliable NAIRU estimates from wage and price

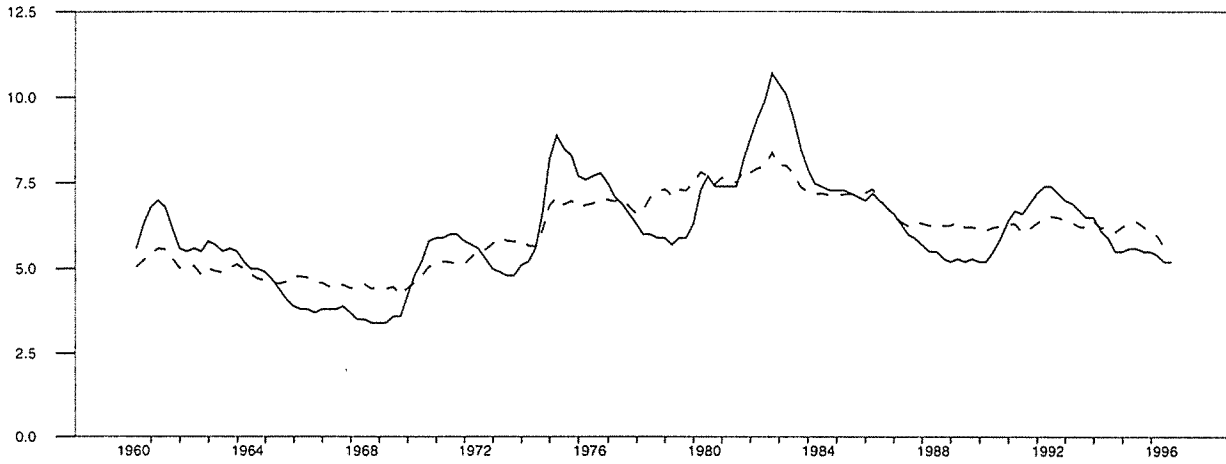
setting equations (Setterfield et al., 1992) a measure based on the output gap may provide a more useful guide for an assessment of the current cyclical position of unemployment.

It should be noted yet that a structural interpretation of the particular innovations has its limitations. While cyclical movements in output might stem from both supply and demand shocks this does not seem to pose a major problem for the estimate of persistence as the literature does not emphasize the operation of different persistence mechanisms in response to either type of shock. Yet some difficulties might stem from possible negative effects of unemployment persistence on capital accumulation (Dreze and Bean, 1990) thereby giving rise to a corresponding permanent loss in output. This would violate the orthogonality conditions required for identification. To the extent at which mechanisms of that kind have been operating the persistence coefficient might be underestimated.

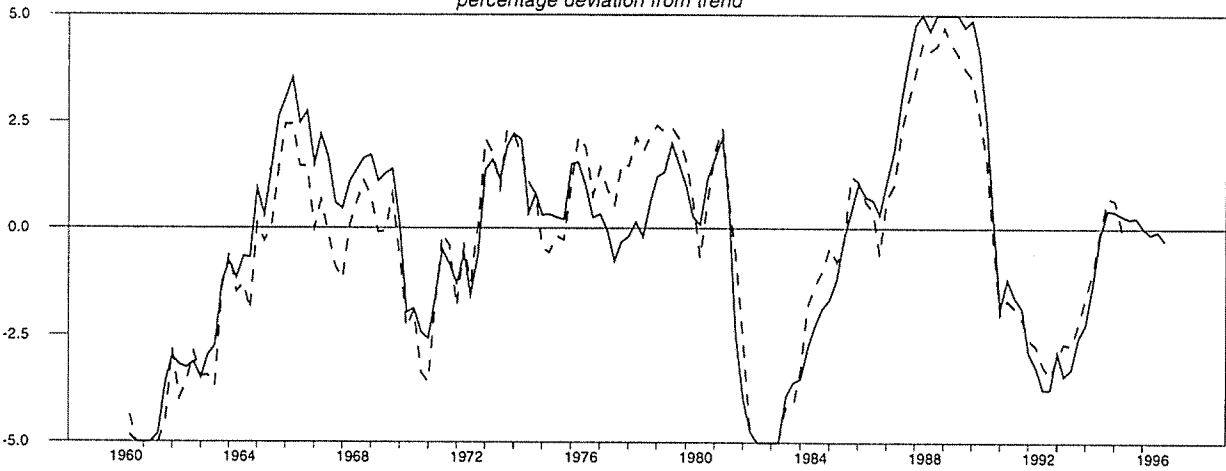
**Fig. 1a: USA GDP cycle**  
*percentage deviation from trend*



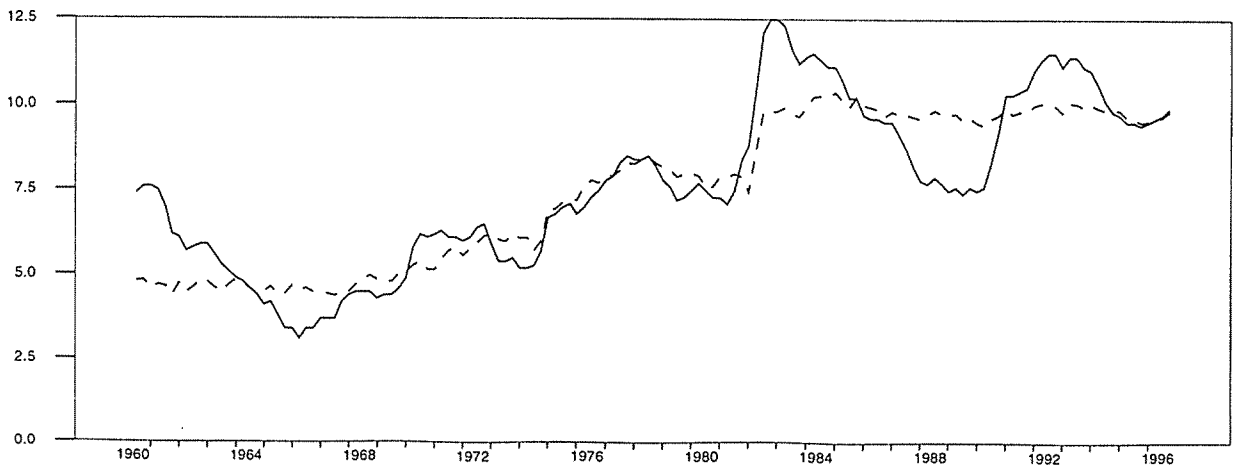
**Fig. 1b: USA UR trend**



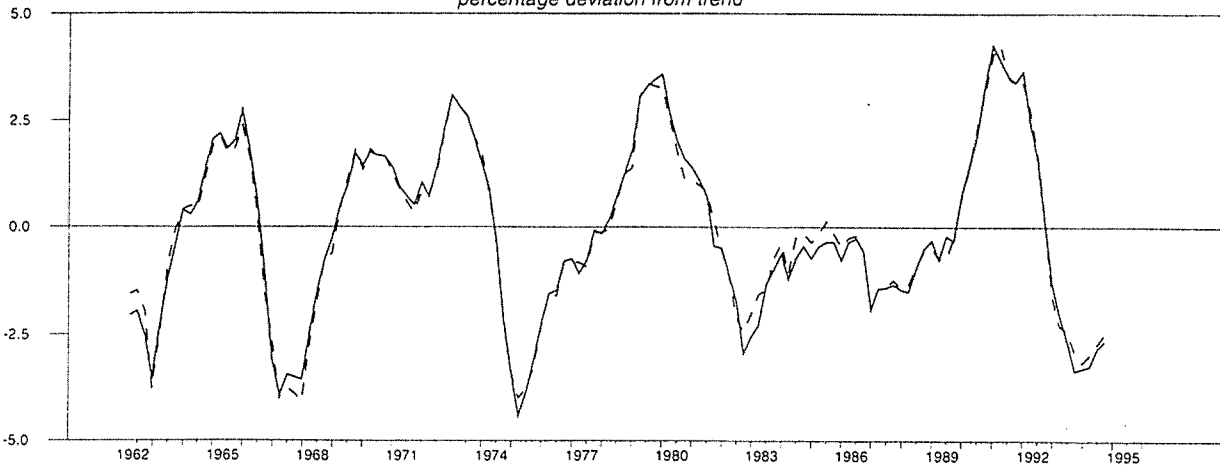
**Fig. 2a: CAN GDP cycle**  
*percentage deviation from trend*



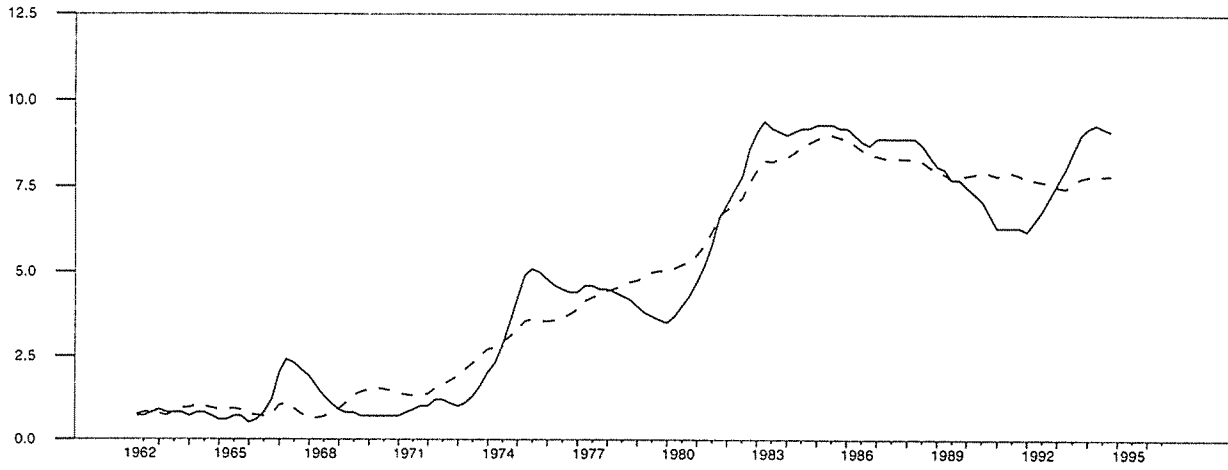
**Fig. 2b: CAN UR trend**



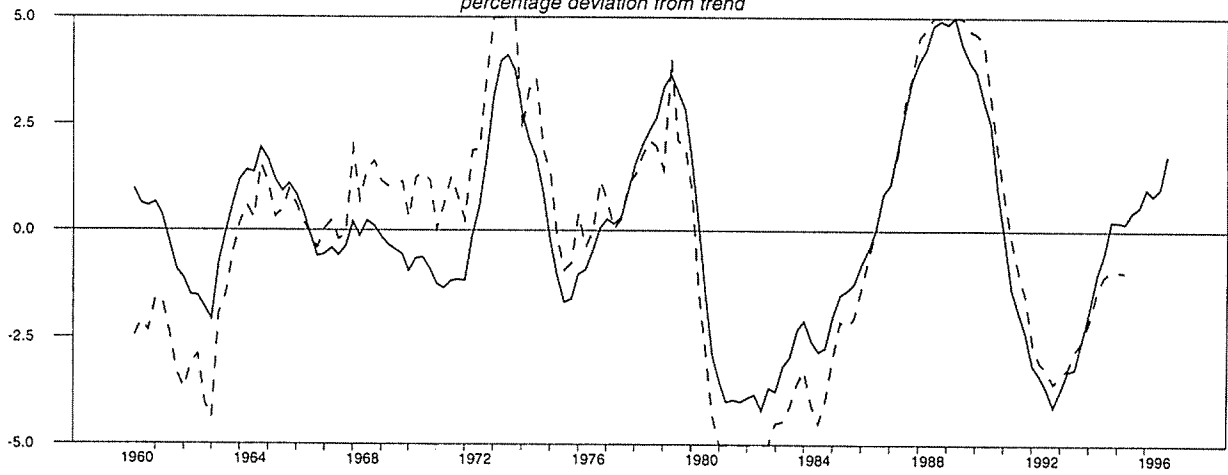
**Fig. 3a: Germany GDP cycle**  
*percentage deviation from trend*



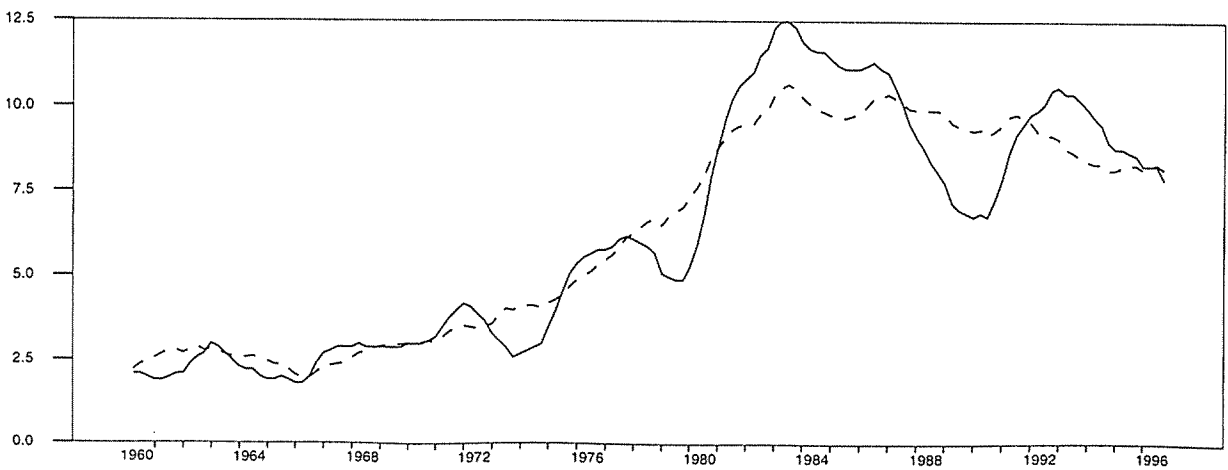
**Fig. 3b: Germany UR trend**



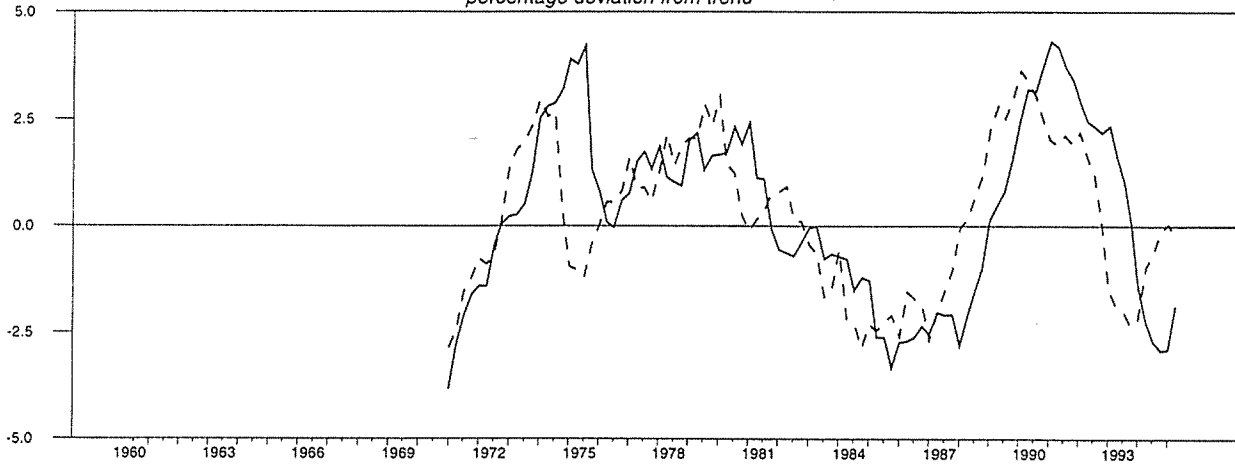
**Fig. 4a: UK GDP cycle**  
*percentage deviation from trend*



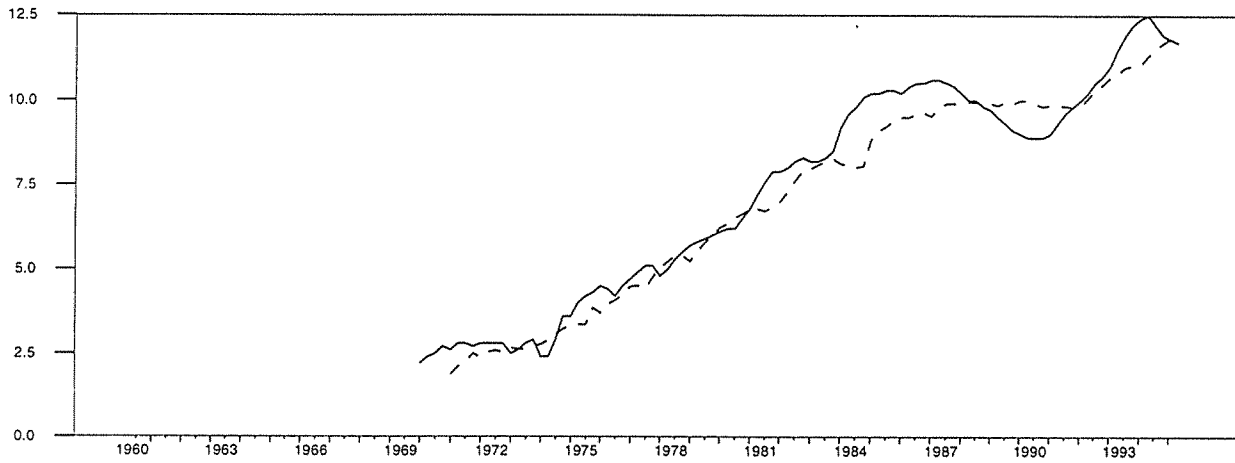
**Fig. 4b: UK UR trend**



**Fig. 5a: France GDP cycle**  
*percentage deviation from trend*



**Fig. 5b: France UR trend**





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## A.1. GCC restriction

Equations (4) and (5) are rewritten as

$$\begin{aligned}\tilde{\varphi}_{i,t} &= \rho C(\lambda) \tilde{\varphi}_{i,t-1} + \tilde{\kappa}_{i,t} \\ x_t^C &= \Theta \tilde{\varphi}_t\end{aligned}$$

where  $x_t^{C'} = (y_t^C, u_t^C)$ ,  $\tilde{\varphi}_t' = (\tilde{\varphi}'_{1,t}, \tilde{\varphi}'_{2,t})$ , and  $\Theta$  is defined accordingly. The autocovariance function  $\Gamma(s)$  of  $\tilde{\varphi}_{i,t}$  is found from the set of equations

$$\begin{aligned}\Gamma(s) &= [\rho C(\lambda)]^s \Gamma(0) \\ \Gamma(0) &= \rho^2 C(\lambda) \Gamma(0) C(\lambda)' + \Sigma_\kappa\end{aligned}$$

For the case of  $\Sigma_\kappa = \sigma_\kappa^2 I$  it is easily verified that  $\Gamma(0) = \sigma_\kappa^2 (1 - \rho^2)^{-1} I$ . Moreover, it holds  $[C(\lambda)]^s = C(s\lambda)$ . Hence, the ACF for  $\tilde{\varphi}_{i,t}$  is given by

$$\Gamma(s) = \rho^s \frac{\sigma_\kappa^2}{(1 - \rho^2)} C(s\lambda)$$

Note that  $\Gamma(s)$  is skew-symmetric while  $\varphi_{i,t}$  and  $\varphi_{i,t}^*$  share the same auto correlation function. The autocovariance function  $\Gamma_x(s)$  for  $x_t^C$  as from equation (7) then follows from  $\Gamma_x(s) = \Theta [\Gamma(s) \otimes I_2] \Theta'$  as

$$\Gamma_x(s) = \rho^s \frac{\sigma_\kappa^2}{(1 - \rho^2)} \begin{bmatrix} \cos(s\lambda) & \theta_1 \cos(s\lambda) + \theta_1^* \sin(s\lambda) \\ \theta_1 \cos(s\lambda) - \theta_1^* \sin(s\lambda) & (\theta_1^2 + \theta_1^{*2} + \theta_2^2) \cos(s\lambda) \end{bmatrix}$$

From application of the trigonometric identity

$$\begin{aligned}\theta_1 \cos(s\lambda) \pm \theta_1^* \sin(s\lambda) &= r \cos(\lambda(s \mp \xi)) \\ r &= \sqrt{\theta_1^2 + \theta_1^{*2}} \\ \xi &= \tan^{-1}(\theta_1^*/\theta_1)\end{aligned}$$

(e.g., Harvey, 1993: 227) follow the expressions in the text. The test for  $H_0 : \theta_2 = 0$  involves no complications apart from the fact that  $\theta_2$  lies at the boundary of the admissible parameter space as identifiability requires  $\theta_2 \geq 0$ . From Kodde and Palm (1986) it follows that the appropriate distribution of the Wald test is given by  $0.5(1 + \chi_1^2)$ . For a discussion of the spectral properties see Rünstler (1997).

## A.2. State-space representation

For estimation, equations (1) to (5) are written in state-space form

$$\begin{aligned}x_t &= Z\delta_t + \nu_t \\ \delta_t &= T\delta_{t-1} + \varepsilon_t\end{aligned}$$

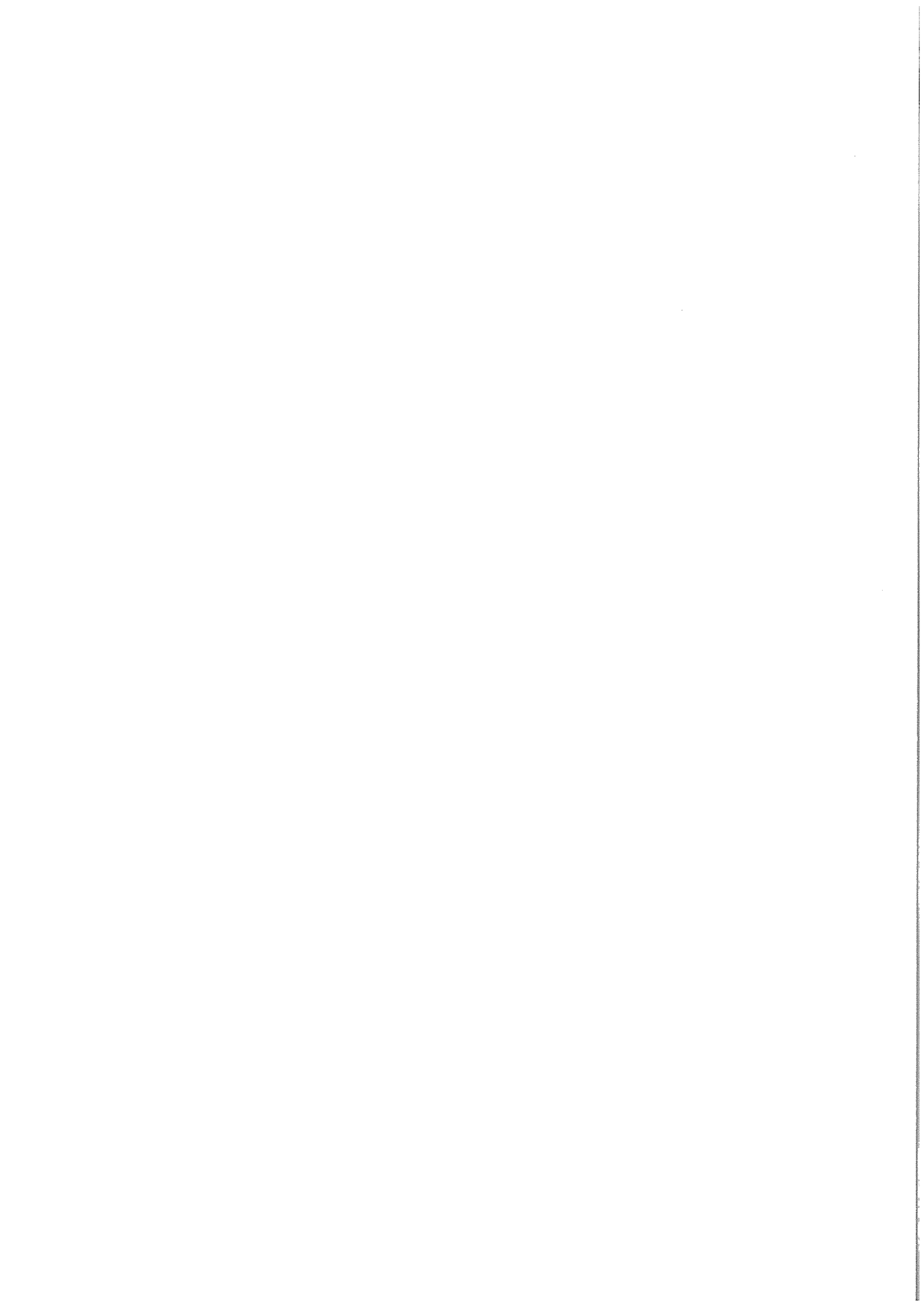
with the vector  $x_t = (y_t, u_t)'$  of the observations at time  $t$  of GDP and the unemployment rate, the state vector  $\delta_t$  comprising the unobserved state variables, and  $\nu_t$  and  $\varepsilon_t$  representing the irregular components and innovations respectively. I outline the state space form for the case of  $\tilde{\varphi}_{2,t} = 0$  and a first order polynomial  $\beta(L)$ . The vectors of state variables and innovations are given by

$$\begin{aligned}\delta_t &= (y_t^T, \mu_t, u_t^{QE}, u_{t-1}^{QE}, \varphi_{1,t}, \varphi_{1,t}^*, \varphi_{1,t-1}, \varphi_{1,t-1}^*)', \text{ and} \\ \varepsilon_t &= (\zeta_t^{(1)}, \zeta_t^{(2)}, \eta_t, 0, \kappa_{1,t}, \kappa_{1,t}^*, 0, 0)'\end{aligned}$$

The matrices  $Z$  and  $T$  are correspondingly of the form

$$\begin{aligned}&\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \theta_1 & \theta_1^* & 0 & 0 \end{bmatrix} \\ &\text{and} \\ &\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 + \omega & -\omega & \theta_1 a & \theta_1^* a & -\theta_1 a & -\theta_1^* a \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho \cos \lambda & \rho \sin \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho \sin \lambda & \rho \cos \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}\end{aligned}$$

where  $a = \omega + \beta_1$ . The covariance matrix of innovations  $\varepsilon_t$  is set up accordingly allowing for correlations between  $\zeta_t^{(1)}$  and  $\eta_t$  and cyclical innovations, respectively. In order to obtain estimates of  $u_t^*$  an extended version that separates  $u_t^{QE}$  from  $u_t^*$  has been used.



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