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Gerhard Rünstler

Institut für Höhere Studien
Institute for Advanced Studies
Vienna

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Gerhard Rünstler

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Gerhard Rünstler
Institut für Höhere Studien
Stumpergasse 56, A-1060 Wien
Phone: +43/1/599 91-151
Fax: +43/1/599 91-163
e-mail: ruenstle@ihs.ac.at

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

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Abstract

The study proposes a multivariate unobserved components model in order to examine relationships at business cycle frequencies among macroeconomic variables. The series are decomposed into non-stationary trends, stationary cycles, and an irregular component. The co-movements among the particular cycles are modelled by a latent factor, whose dynamics is governed by a stochastic cycle. As a consequence of certain symmetry properties of the latter cyclical co-movement can be parametrized in terms of relative variances, phase shifts, and coherence. The model is applied to a U.S. labour market data set.

Keywords
Unobserved components models, business cycles, labour markets

JEL-Classifications
C32, E32

Comments

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1 Introduction

In recent years the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1980) has played the most prominent role for the extraction of cyclical components in macroeconomic time series. Yet several recent works have called its adequacy for this purpose into question. First, King and Rebelo (1993) and Harvey and Jäger (1993) have examined the conditions under which the HP filter is optimal in the sense of minimising the mean square error of the estimated cyclical component and have concluded that 'these are unlikely to be even approximately true in practice (King and Rebelo, 1993: 230)'. It has further been argued that the resulting need to impose rather than estimate the signal-noise ratio might give rise to spurious cycles (Harvey and Jäger, 1993) and seriously distort sample cross correlations among the cycles of the particular series (Cogley and Nason, 1995). Second, Harvey and Jäger (1993) have pointed out the weaknesses in deriving stylized business cycle facts from a pure inspection of cross covariances among the extracted cycles without any statistical inference. These are typically interpreted in terms of the relative variances of the particular cyclical components, their strength of association, and a phase shift (compare, e.g., various chapters of Cooley, 1995). The latter two characteristics are inferred from the cross correlations by some 'eye-ball metrics' (Cogley and Nason, 1995).

A growing number of works (Harvey and Jäger, 1993; Gregory and Smith, 1996; King and Watson, 1996) have therefore put forward alternative methodologies. Specifically, Harvey and Jäger (1993) and Boone and Hall (1995, 1996) have proposed the application of structural time series (STS) models as an approach that encompasses the HP filter and seems capable of overcoming its main deficiencies. STS models are designed to decompose a time series into its trend, cyclical, and irregular components. The relative variances of the respective innovations are estimated and cyclical dynamics is explicitly accounted for. However, the univariate STS approach still requires the need to rely on sample cross correlations for the inspection of cyclical co-movements and is subject to some difficulties with respect to extracting and testing for the presence of cyclical components.

I propose a multivariate version of STS models where the linkages among the particular cycles are assessed by a latent factor. The idea that the business cycle is characterized by high coherence among certain macroeconomic series at business cycle frequencies and that therefore latent factor models are an appropriate tool for describing cyclical relationships has been put forward by many authors.[1] More recent contributions are Stock and Watson (1993), Quah and Sargent (1993), Forni and Reichlin (1995), Kim and Yoo (1996), Norrbin and Schlagenhauf (1996), and Diebold and Rudebusch (1996). What is yet specific to the present approach is the usage of a certain two-dimensional process, i.e., a stochastic cycle (SC) as the main building block for modelling cyclical dynamics. As the spectral density of the SC centers around cyclical frequencies it seems particularly suitable for an assessment of cyclical co-movements. Its very specific properties further allow for introducing phase shifts in a way that symmetrically handles lead and lag relationships among the particular cycles. The inclusion of idiosyncratic cycles finally gives rise to a factor model where the co-movements among the cyclical components of the particular series are explicitly parametrized in terms of relative variances, phase shifts, and coherence.

The plan of the paper is as follows. Section 2 briefly reviews structural time series models. Sections 3 and 4 introduce the generalized common cycles factor model (GCCFM) and discuss estimation and testing procedures. Section 5 presents an application of the model to a U.S. labour market data set. Section 6 concludes.

[1] Sargent (1987: 282) offers the following definition: 'The business cycle is a phenomenon of a number of important aggregates (such as GNP, unemployment, and layoffs) being characterized by high pairwise coherences at low business cycle frequencies, the same frequencies at which most aggregates have most of their spectral power'.

2 Structural time series (STS) models

The following model has been proposed by Harvey (1985, 1989) for the decomposition of a macroeconomic time series $x_{i,t}$ into a non-stationary trend $x_{i,t}^{tr}$, a stationary cycle $x_{i,t}^{C}$, and an irregular white-noise component $\nu_{i,t}$.

$$x_{i,t} = x_{i,t}^{tr} + x_{i,t}^{C} + \nu_{i,t} \tag{1}$$

The trend component follows a so-called local linear trend, that is, a random walk with a stochastic slope term $\mu_{i,t}$, which, in turn, again is specified as random walk, i.e.,

$$\begin{aligned} \Delta x_{i,t}^{tr} &= \mu_{i,t-1} + \eta_{i,t} \\ \Delta \mu_{i,t} &= \zeta_{i,t} \end{aligned} \tag{2}$$

where Δ denotes the difference operator and level and slope innovations $\eta_{i,t}$ and $\zeta_{i,t}$ are both white noise. If $\sigma_{i,\zeta}^2 = 0$ the process reduces to a random walk with drift, while for the case of $\sigma_{i,\eta}^2 = 0$, but $\sigma_{i,\zeta}^2 > 0$, it represents a second-order random walk, i.e., $\Delta^2 x_{i,t}^{tr} = \zeta_{i,t}$. In the latter case $x_{i,t}^{tr}$ tends to evolve smoothly over time.

The cyclical component $x_{i,t}^{C}$ is specified as a stochastic cycle, $x_{i,t}^{C} = \varphi_{i,t}$, with the stochastic process governing $\varphi_{i,t}$ being represented by

$$\begin{bmatrix} \varphi_{i,t} \\ \varphi_{i,t}^{*} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \varphi_{i,t-1} \\ \varphi_{i,t-1}^{*} \end{bmatrix} + \begin{bmatrix} \kappa_{i,t} \\ \kappa_{i,t}^{*} \end{bmatrix} \tag{3}$$

or, in short,

$$\widetilde{\varphi}_{i,t} = \rho C(\lambda) \widetilde{\varphi}_{i,t-1} + \widetilde{\kappa}_{i,t}$$

with decay $0 < \rho < 1$ and frequency $0 < \lambda < \pi$. This represents a special case of a first-order autoregressive process with a conjugate complex root thereby generating a cyclical impulse response to both innovations (Harvey, 1993). Throughout the paper the usually imposed assumption of $E\widetilde{\kappa}_{i,t}\widetilde{\kappa}_{i,t}' = \sigma_i^2 I_2$ will be maintained. As outlined in appendix A.1 the auto covariance function (ACF) $\Gamma(s)$ for $\widetilde{\varphi}_{i,t}$ is then given by dampened cosine and sine waves of length $2\pi/\lambda$, respectively, i.e.,

$$\Gamma(s) = \rho^{|s|} \frac{\sigma_i^2}{(1-\rho^2)} \begin{bmatrix} \cos(s\lambda) & \sin(s\lambda) \\ -\sin(s\lambda) & \cos(s\lambda) \end{bmatrix} \tag{4}$$

Note that $\Gamma(s)$ is skew-symmetric which will be of crucial importance for various symmetry properties of the latent factor model presented below.

It is finally noteworthy that the HP-filter represents the optimal Wiener filter for the model

$$\begin{aligned} x_{i,t} &= x_{i,t}^{tr} + \nu_{i,t} \\ \Delta^2 x_{i,t}^{tr} &= \zeta_{i,t} \end{aligned} \tag{5}$$

where the smoothing parameter corresponds to the signal-noise ratio $\sigma_\varsigma^2/\sigma_\nu^2$ (King and Rebelo, 1993; Harvey and Jäger, 1993). The HP filter thus emerges as the optimal estimator of a very restricted version of the STS model neglecting both random walk components and cyclical dynamics. In fact, from an STS modelling point of view, the need to impose the signal-noise ratio rather than estimating it is a direct consequence of the limited nature of model (5).

The properties of the STS model are discussed in detail by Harvey (1989). Harvey and Koopmans (1996) propose multivariate versions for n series $x_{i,t}$ where the particular trend and cyclical components are modelled as in equations (2) and (3) and certain covariance structures among trend and cyclical components, respectively, might be imposed. As a special case, several cyclical components $x_{i,t}^C$ might be represented by one common SC. The following section introduces a generalized version of the common stochastic cycles model where the particular cyclical components $x_{i,t}^C$ participate at the common SC with certain phase shifts.

3 A generalised common cycles factor model

The basic idea for introducing a phase shift into a common cycle $\widetilde{\varphi}_t$ is the exploitation of the information contained in φ_t^*. While any linear combination $\theta\varphi_t + \theta^*\varphi_t^*$ shares the same autocorrelation function as from (4) the cross correlation between φ_t and $\theta\varphi_t + \theta^*\varphi_t^*$ is given by a dampened cosine wave subject to a certain phase shift. The inclusion of a further idiosyncratic cycle allows for modelling the strength of association.

Specifically, let $\widetilde{\varphi}_{0,t}$ and $\widetilde{\varphi}_{i,t}$ represent stochastic cycles of the same decay ρ and frequency λ, the innovations $\widetilde{\kappa}_{0,t}$ and $\widetilde{\kappa}_{i,t}$ of which have the variances σ_0^2 and σ_i^2 and are mutually uncorrelated. Let $\widetilde{f}_t^C = \widetilde{\varphi}_{0,t}$ represent the latent factor, while $x_{i,t}^C$ is linked to both $\varphi_{0,t}$ and $\varphi_{0,t}^*$ and a further idiosyncratic cycle $\varphi_{i,t}$.

$$\begin{bmatrix} f_t^C \\ x_{i,t}^C \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \theta_i & \theta_i^* \end{bmatrix} \begin{bmatrix} \varphi_{0,t} \\ \varphi_{0,t}^* \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varphi_{i,t} \tag{6}$$

As shown in appendix A.1, the ACF $\Gamma_x(s)$ of f_t^C and $x_{i,t}^C$ is given by

$$\Gamma_x(s) = \rho^{|s|} \frac{\sigma_0^2}{(1-\rho^2)} \begin{bmatrix} \cos(s\lambda) & \alpha_i \vartheta_i \cos(\lambda(s-\xi_i)) \\ \alpha_i \vartheta_i \cos(\lambda(s+\xi_i)) & \vartheta_i{}^2 \cos(s\lambda) \end{bmatrix}$$

where

$$\begin{aligned} \vartheta_i &= \sqrt{\theta_i^2 + \theta_i^{*2} + (\sigma_i/\sigma_0)^2} \\ \alpha_i &= \operatorname{sign}(\theta_i)\vartheta_i^{-1}\sqrt{\theta_i^2 + \theta_i^{*2}} \\ \xi_i &= \lambda^{-1}\tan^{-1}(\theta_i^*/\theta_i) \end{aligned} \tag{7}$$

Hence, $x_{i,t}^C$ and f_t^C are subject to the same autocorrelation function. The cross correlation function

$$\operatorname{corr}(x_{i,t-s}^C, f_t^C) = \alpha_i \rho^{|s|} \cos(\lambda(s-\xi_i)) \tag{7a}$$

is given by a dampened cosine wave with a certain phase shift ξ_i which is normalized to lie within the range of one quarter of the cycle length in absolute value. It therefore provides a direct measure for the phase shift between the two series. The variance of $x^C_{i,t}$ relative to f^C_t is given by ϑ^2_i. The multiple correlation $|\alpha_i| \leq 1$ of $x^C_{i,t}$ with $\widetilde{\varphi}_{0,t}$ finally gives rise to a measure for the strength of association and might be given an interpretation in terms of a factor loading. Note that the size of the contemporaneous correlation $\text{corr}(x^C_{i,t}, f^C_t) = \alpha_i \cos(\lambda\xi_i)$ is limited both by the factor loading and the phase shift. As the borderline case of $\xi_i = \pi/2\lambda$ emerges a phase shift of one quarter of the cycle length which yields a contemporaneous correlation of zero regardless of the strength of association.[2]

Auto and cross spectra for f^C_t and $x^C_{i,t}$ are derived in appendix A.2 and plotted in Fig.1. The auto spectra for $\varphi_{i,t}$ and $\varphi^*_{i,t}$ are identical and center around cyclical frequencies while the real part of the cross spectrum is identical to zero. As a result, $x^C_{i,t}$ again exhibits the same auto spectrum, subject to the scaling factor $\vartheta_i{}^2$. The expressions for coherence and the phase spectrum are found with

$$\begin{aligned}
\text{Coh}(\omega) &= \vartheta_i{}^{-2}\left[\theta^2_i + \theta^{*2}_i\left(\frac{\text{Im}(g_{12}(\omega))}{g_1(\omega)}\right)^2\right] \\
\text{Ph}(\omega) &= \tan^{-1}\left[-\frac{\theta^*_i}{\theta_i}\frac{\text{Im}(g_{12}(\omega))}{g_1(\omega)}\right].
\end{aligned} \tag{8}$$

where

$$\frac{\text{Im}(g_{12}(\omega))}{g_1(\omega)} = \frac{-2\rho\sin\lambda\sin\omega}{1+\rho^2-2\rho\cos\lambda\cos\omega}$$

represents the ratio of the imaginary part of the cross spectrum and the auto spectrum. It is particularly noteworthy that, first, the SC centers around cyclical frequencies and therefore measures coherence and the phase shift just at these. Second, $\text{Coh}(\omega)$ does not depend on the signs of θ_i and θ^*_i while, due to the skew-symmetry of $\tan^{-1}(\omega)$ around $\omega = 0$, the phase

[2] In turn, for an integer phase shift ξ_i it would hold $\text{corr}(x^C_{i,t-\xi_i}, f^C_t) = \alpha_i\rho^{\xi_i}$. The phase shift thus imposes an upper bound on cross correlations. This compares to the proposition that, for a vector autoregressive process $A(L)x_t = \varepsilon_t$, the correlation between $x_{i,t}$ and $x_{j,t-s}$ is limited by $|\rho_{\max}|^s$ where $\rho_{\max}$ denotes the largest root in absolute value of $\det(A(z))$ (see Lütkepohl, 1994: 23). Yet, as long as ρ is not too far below one, this does not impose a severe constraint.

simply reverts with the sign of θ_i^*/θ_i. Thus lead and lag relationships are handled entirely symmetrically![3]

Fig. 1a: Auto spectrum of the SC for $\rho = 0.9$ and $2\pi/\lambda = 20$

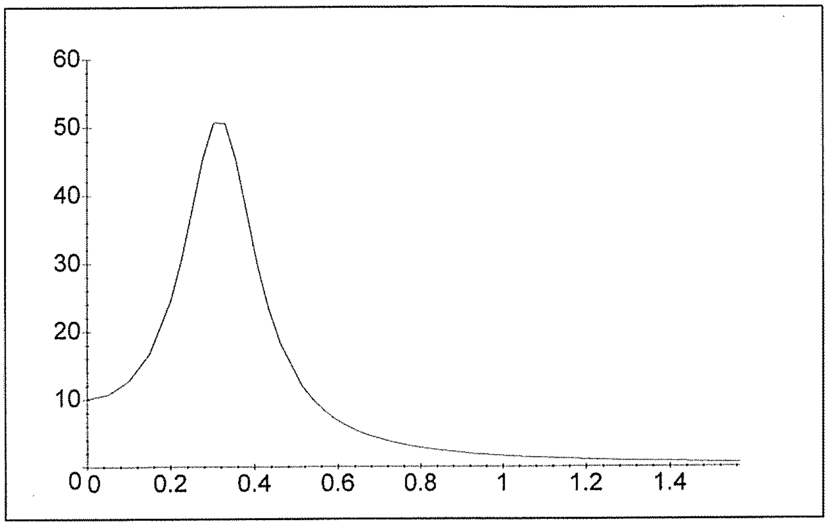
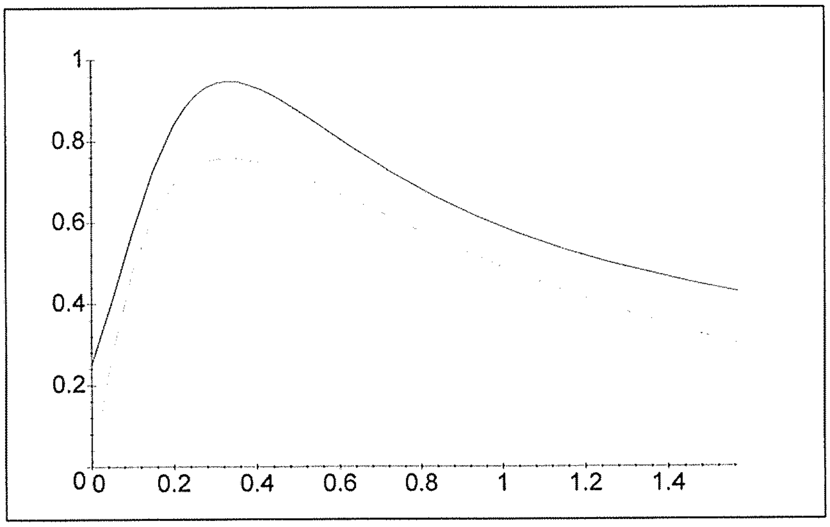


Fig. 1b: Coherence (—) and phase shift (- - -) between f_t^C and $x_{i,t}^C$
($\theta_i = \theta_i^* = 1/\sqrt{2}$ and $\alpha_i = 1$)



[3]Both coherence and the phase peak at $\omega_0 = \arccos\left[2\rho(1+\rho^2)^{-1}\cos\lambda\right]$ which, for values of ρ not too far below 1, is close to λ. The phase at ω_0 is then close to ξ_i.

Turning to the multivariate case of n cycles $x_t^C = (x_{1,t}^C, ..., x_{n,t}^C)'$ let, for $i = 1, ..., n$, the dynamics of $x_{i,t}^C$ be given by

$$x_{i,t}^C = \theta_i \varphi_{0,t} + \theta_i^* \varphi_{0,t}^* + \varphi_{i,t} \tag{9}$$

$\varphi_t = (\varphi_{0,t}, ..., \varphi_{n,t})'$ is a vector of $n+1$ stochastic cycles, whose decays ρ_i and cycle lengths λ_i are assumed to be equal. The innovations $\widetilde{\kappa}_{i,t}$ of $\widetilde{\varphi}_{i,t}$ are of variance σ_i^2 and are assumed to be both mutually orthogonal,i.e.,

$$\Sigma_\kappa = \text{diag}(\sigma_0^2, \sigma_1^2, ..., \sigma_n^2) \otimes I_2.$$

The ACF of the particular $x_{i,t}^C$ with the latent factor $\widetilde{f}_t^C = \widetilde{\varphi}_{0,t}$ is then given as from equations (7). Further, as shown in appendix A.1, phase shifts turn out to be additive. The auto correlation function of x_t^C is therefore given by the expression

$$\rho^{|s|} \begin{bmatrix} c_s(0) & \alpha_1\alpha_2\, c_s(\xi_2 - \xi_1) & .... & \alpha_1\alpha_n\, c_s(\xi_n - \xi_1) \\ \alpha_1\alpha_2\, c_s(\xi_1 - \xi_2) & c_s(0) & .... & \alpha_2\alpha_n\, c_s(\xi_n - \xi_2) \\ ..... & ..... & .... & .... \\ \alpha_1\alpha_n\, c_s(\xi_1 - \xi_n) & \alpha_2\alpha_n\, c_s(\xi_2 - \xi_n) & .... & c_s(0) \end{bmatrix} \tag{10}$$

where $c_s(z)$ denotes $\cos(\lambda(s-z))$. It is immediately evident from equation (10) that identifiability requires a normalization of both the variance and the phase shift of the latent factor. This might be achieved by setting $\theta_i = 1$ and $\theta_i^* = 0$ for some i. Note further that the system comprises $n+1$ cycles. As with any latent factor model, identifiability of the factor loadings α_i therefore requires the dimensionality of the model to be $n \geq 3$.

Various works applying STS models to macroeconomic data (Harvey, 1985, 1989; Harvey and Jäger, 1993) indicate that the SC is, in fact, sufficient for modelling business cycle dynamics. Further evidence in this direction has been provided by Hofer et al. (1998). Inspecting bivariate VAR and VARMA models for GDP and various cyclical indicators they found low order VARMA models to be superior compared to the VAR approach. Further, the autoregressive part of the VARMA specifications invariably contained a complex conjugate root which compares favourably to the reduced ARIMA form of STS models (see Harvey, 1989).

4 Estimation and testing

The model comprises equations (1) for n series $x_t = (x_{1,t}, ..., x_{n,t})'$ where the particular trend components are modelled as from equations (2) and the cycles as from the GCCFM (9). The conditions for the identifiability of multivariate STS models as discussed by Harvey and Koopmans (1996) carry over straightforwardly to the GCCFM. Denote with η_t, ζ_t, and ν_t the $n \times 1$ vectors of level and slope innovations, and irregular components, respectively. $\widetilde{\kappa}_t$ is the vector of cyclical innovations of length $2(n+1)$ as from equations (9). The vector ε_t comprises the particular innovations, i.e., $\varepsilon_t' = (\eta_t', \zeta_t', \widetilde{\kappa}_t')$. Both ν_t and ε_t are assumed to be identically independently normally distributed. If the innovations are blockwise orthogonal, i.e.,

$$\begin{aligned} E\varepsilon_t\varepsilon_t' &= \text{diag}(\Sigma_\eta, \Sigma_\zeta, \Sigma\widetilde{\kappa}) \\ E\varepsilon_t\nu_t' &= 0 \end{aligned}$$

then the system is identifiable (see Harvey and Koopmans, 1996). For estimation the model is cast in state-space form

$$\begin{aligned} x_t &= Za_t + \nu_t \\ a_t &= Ta_{t-1} + \varepsilon_t \end{aligned} \qquad (11)$$

with the state vector a_t comprising the unobserved state variables. The structure of the state-space form is presented in appendix A.3. The parameters are estimated by maximum likelihood using the prediction error decomposition provided by the Kalman filter (Harvey, 1989).

Testing and diagnostics in the context of STS models have been discussed by Harvey (1989) and Harvey and Koopmans (1992, 1996). As pointed out by Harvey (1989: 251) the test for the presence of an SC is in general subject to difficulties related to a lack of identifiability under the null. I therefore focus on Wald tests with regard to the cyclical structure taking the presence of the latent cyclical factor as given. The test for $H_0 : (\theta_i, \theta_i^*) = (0,0)$, i.e., the presence of an association of $x_{i,t}^C$ with the latent factor requires at least three pairs of parameters (θ_j, θ_j^*) to be left unrestricted in order to maintain identifiability of the covariance structure under the null. The test for the presence of an idiosyncratic cycle $\varphi_{i,t}$, in turn, amounts to the null of $H_0 : \sigma_i^2 = 0$. As the null lies at the boundary of the admissible parameter space the distribution of the Wald test is given by $\frac{1}{2}(1 + \chi_1^2)$ (Kodde and Palm, 1986; Gill and Lewbel, 1992).

If $(\theta_i, \theta_i^*) = (1,0)$ is chosen for normalization and (θ_j, θ_j^*) differs from zero, the test for $H_0 : \theta_j^* = 0$ amounts to $\xi_j - \xi_i = 0$ and involves no complications. Alternatively one might test for $H_0 : \xi_j - \xi_i = \pi/2\lambda$, or, equivalently, $\theta_j = 0$. More general hypotheses on relative phase shifts among the particular cycles yet require likelihood ratio tests due to non-linearities in the involved transformations. Finally, given $\sigma_i^2 > 0$, model misspecification tests might be based on the equality of cycle lengths and decays, i.e., the hypotheses $\lambda_i = \lambda_0$ and $\rho_i = \rho_0$.

5 Application to labour market data

I apply the model to a labour market data set that has played a central role in the calibration of real business cycle models, i.e., quarterly data for the U.S. business sector of output (y_t), total employment (e_t), hours per worker (h_t), the output deflator (p_t), and real compensation per hour (w_t). The data stem from the establishment survey and range from 1959:1 to 1994:4. They are taken in logs.[4]

[4] The Citibase labels are LBOUT, LBEMP, LBHP, LBCP and LBCP7. Maximum likelihood estimation was done in GAUSS using the algorithm by Rosenberg (1973).

5.1 Trends

I transform the trends in accordance with conventional balanced-growth assumptions and impose orthogonality restrictions on the respective innovations. The trend in output is assumed to be composed of trends in labour productivity (π_t^{tr}), employment (e_t^{tr}), and hours per worker (h_t^{tr}) while the trend in real compensation per hour is composed of π_t^{tr} plus a further component (s_t^{tr}) representing the labour share of income.

$$\begin{bmatrix} y_t^{tr} \\ e_t^{tr} \\ h_t^{tr} \\ w_t^{tr} \\ p_t^{tr} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t^{tr} \\ e_t^{tr} \\ h_t^{tr} \\ s_t^{tr} \\ p_t^{tr} \end{bmatrix}$$

The particular trends on the right hand side might be modelled as local linear trends. Yet it should be noted that this renders them to be integrated of order 2 ($I(2)$) and therefore has been questioned (e.g., Nelson, 1987). In particular, h_t^{tr} and s_t^{tr} should exhibit no systematic drift though level shifts seem to have occured in the sample period. In turn, there is evidence for prices to be $I(2)$ (e.g., Schwert, 1987). The presence of a stochastic slope is yet less clear for employment and productivity.[5] Since the focus of the study is on cyclical co-movements the modelling strategy should tend to leave the trends unrestricted. I will therefore use, as a benchmark, a model that restricts h_t^{tr} and s_t^{tr} to follow random walks without drifts and allows for stochastic slopes in the remaining trend components. Yet I will comment on the stability of the results with respect to alternative specifications. The covariance matrix Σ_ν of the vector of irregular components is estimated freely.

[5] Harvey and Jäger (1993) and Leybourne and McCabe (1994) have argued that the size of tests for a second unit root is likely to be seriously distorted. Using a test that takes stationarity as the null the latter provide evidence for the view that some macro series might be I(2).

5.2 Results

Table 1 presents estimates for the benchmark model. The transformed parameters and the implied cross correlations among the particular cyclical components are set out in Tables 2 and 3. The latter also contain the implied results for some combinations of interest, i.e., total hours, output per hour, and the nominal wage. Fig. 2 shows the GCCFM and HP filter estimates of the output cycle. Fig. 3a to 3h present the particular estimated cyclical components as extracted from the GCCFM.

The cycle length is estimated with 20.3 quarters which corresponds to the usual characterization of business cycles as being represented by frequencies from 3 to 8 years. The idiosyncratic cycles for output, employment and hours are found to be very small and, with the exception of the employment one, insignificant from the Wald test (W_1 in Table 1). The co-movement of output, employment and hours cycles is thus close to being deterministic in the sense that they follow one common SC. The employment cycle follows the output one with a phase shift of 2.1 quarters, while the cycle in hours per worker moves simultaneously with the latter. From the results presented in Table 2, the cycle in total hours lags the output one by 1.4 quarters, while output per hour is found to be pro-cyclical with a lead of 2.3 quarters. The standard deviation of the cycle in total hours relative to the output one is estimated with 0.73 which results in a respective value for output per hour of 0.48.

The results for real compensation per hour indicate the presence of a cyclical component with a somewhat smaller factor loading of 0.67 and a relative variance of 0.29. Interestingly, the phase shift relative to the output cycle is found to be very close to the borderline case of one quarter of the cycle length. As a consequence, though a significant association with the latent factor is found (W_3 in Table 1), the contemporaneous correlation with the output cycle is virtually zero (Table 3).

Table 1: Parameter estimates of the GCCFM

Cycles		y_t^C	e_t^C	h_t^C	w_t^C	p_t^C
decay	ρ	0.95				
cycle length	$2\pi/\lambda$	20.27				
	θ_i	1.000	.439	.214	-.011	-.164
	θ_i^*	.000	-.327	.019	.196	-.298
Innovations (std.dev*100)						
common cycle	σ_0	.624				
idiosyncr. cycles		y_t^C	e_t^C	h_t^C	w_t^C	p_t^C
	σ_i	.000	.088	.020	.134	.141
irreg. comp.		y_t^ν	e_t^ν	h_t^ν	w_t^ν	p_t^ν
	$\sigma_{\nu,i}$	.519	.000	.118	.000	.000
		π_t^{tr}	e_t^{tr}	h_t^{tr}	s_t^{tr}	p_t^{tr}
level	$\sigma_{\eta,i}$	.370	.001	.210	.353	.000
slope	$\sigma_{\varsigma,i}$	.045	.110	.000	.000	.177
Tests						
$\sigma_i^2 = 0$	(W_1)	.04	5.15	.30	6.91	9.67
$\lambda_i = \lambda$	(LR_2)	—	.02	—	.24	1.54
$\theta_i = \theta_i^* = 0$	(W_3)	—	123.92	81.12	8.05	32.46
$\theta_i = 0$	(W_4)	—	86.11	78.49	.13	9.92
$\theta_i^* = 0$	(W_5)	—	49.14	0.51	8.00	28.30
Q(20)		25.56	23.42	31.60	24.68	28.81
JB		.85	1.09	.05	2.10	.63

Notes: The likelihood ratio (LR) and Wald (W) test statistics LR_2, W_4, W_5 follow a χ_1^2, the statistics W_3 a χ_2^2-distribution under the null. The distribution of test W_1 is discussed in section 4. Q(20) and JB denote the Ljung-Box and Jarque-Bera statistics for autocorrelation in and normality of standardized prediction errors, respectively (see Harvey, 1989: 259f). They are approximately distributed with χ_{20}^2 and χ_2^2. 5%-critical values of χ_1^2, χ_2^2 and χ_{20}^2 are 3.84, 5.99 and 31.41, respectively. The 5%-critical value for test W_1 is given by 2.71 (Kodde and Palm, 1986).

This result sheds some light on the conflicting conclusions of a number of studies that find the aggregate real wage to be either weakly pro- or counter-cyclical (Cushing, 1990; Mocan and Topian, 1993; Abraham and Haltiwanger, 1995) or do not find an association at all (Summer and Silver, 1989).[6] Some of these studies have argued that the cyclical behaviour of the real wage has been changing over time. The high factor loading yet suggests that the association at business cycle frequencies might be somewhat stronger and more stable than is usually concluded, which is revealed only if the high phase shift of the real wage is properly accounted for. Yet Fig. 3g indicates that the real wage might indeed have changed its phase to some extent. While the average lead with respect to output per hour is estimated with 2.5 quarters, the real wage moves roughly simultaneously with the latter during the seventies, though it exhibits a lead before and thereafter. Note further that the real wage is counter-cyclical with respect to employment lagging the latter by 2.8 quarters.

Prices are found to be strongly counter-cyclical with a pronounced lead of 3.4 quarters and a high factor loading of −0.83 which is quite in accordance with the majority of findings from the relevant literature (e.g., Backus and Kehoe, 1992; King and Watson, 1996). The nominal wage cycle exhibits a considerably smaller factor loading of −0.54 and lags the price cycle by 1.7 quarters, while the standard deviation is slightly smaller (see Table 2 and Fig. 3h). This pattern, along with the finding of a weak counter-cyclicality of the real wage with respect to employment seems quite consistent with a moderate degree of nominal wage stickiness.[7]

[6]These studies typically use static regressions where wages and employment are either detrended by the HP filter or taken in first differences.

[7]Approaches incorporating various non-Walrasian labour market features into RBC models (e.g., Burnside et. al., 1993; Danthine and Donaldson, 1990; Feve and Langot, 1996) typically predict a pro-cyclical pattern for real wages with respect to output *per hour,* as found in the data. Yet these approaches can hardly explain the findings that the real wage, on average, leads output per hour and the contemporaneous correlation with

Table 2: Cyclical components: parameters and implied cross correlations

		output	emp	hours	total hours	output p.hour	real wage	prices	nom. wage
std.dev		2.23	1.26	.48	1.64	1.08	.65	.91	.83
rel.to output	ϑ_i	1.00	.58	.22	.73	.48	.29	.40	.37
factor loading	α_i	1.00	.97	.98	.98	.95	-.67	-.83	-.54
phase shift	ξ_i	—	-2.06	0.29	-1.42	2.34	-4.88	3.44	1.70
Cross correlations with output									
t-9		-.65	-.64	-.62	-.68	-.23	.19	.09	.24
t-7		-.42	-.69	-.36	-.64	.09	.43	-.28	.03
t-5		.02	-.46	.09	-.33	.53	.55	-.60	-.23
t-3		.53	.00	.58	.17	.83	.46	-.73	-.44
t-2		.75	.27	.79	.44	.88	.33	-.69	-.50
t-1		.91	.54	.93	.69	.84	.16	-.58	-.51
t		1.00	.78	.98	.89	.71	-.04	-.40	-.47
t+1		.91	.88	.87	.93	.47	-.23	-.15	-.35
t+2		.75	.89	.69	.89	.20	-.39	.09	-.21
t+3		.53	.82	.46	.77	-.07	-.50	.30	-.05
t+5		.02	.49	-.06	.36	-.50	-.55	.59	.22
t+7		-.42	.03	-.47	-.12	-.70	-.40	.62	.37
t+9		-.65	-.37	-.66	-.48	-.62	-.14	.43	.37

Table 3: Implied contemporaneous correlations

	output	emp	hours	total hours	output p.hour	real wage	prices
emp	.78						
hours	.99	.71					
tot.hours	.89	.93	.84				
output p.hour	.71	.19	.76	.37			
real wage	-.04	-.42	.02	-.32	.40		
prices	-.40	.11	-.46	-.05	-.75	-.47	
nom. wage	-.47	-.21	-.49	-.30	-.51	-.17	.39

the nominal wage is close to zero (Table 3).

The results appear to be robust with respect to alternative specifications of the trends. The benchmark model includes an innovation dummy in the employment level equation in order to account for a sharp outlier in 1975:2. While this improves the diagnostics set out in Table 1, it does not affect any other result. The same holds for relaxing the orthogonality restrictions on the trends, allowing for stochastic slopes in hours per worker and hourly compensation, or eliminating the stochastic slope term in productivity. The only notable difference to the benchmark model occurs if the employment trend is restricted to follow a random walk with drift, which yields an estimate of the cycle length of 29.5 quarters. However, relative variances, factor loadings, and phase shifts relative to the cycle length again remain essentially unchanged. The main results for this case are shown in Table 4.

It is finally of interest to compare the implied cross correlations with those obtained from univariate HP filtering (Table 5). For output, employment, and hours the similarity of the findings is remarkably high. The results deviate somewhat more as regards prices and wages. The high correspondence of the findings for output (Fig. 2) might reflect the fact that the smoothing parameter of the HP filter has been tailored in order to conform with prior views about the size of the cyclical component in this series. However, as argued by Harvey and Jäger (1993), the implied signal-noise ratio is not necessarily adequate for other data under consideration.

Table 4: Cyclical components
(stochastic slope in employment eliminated)

		output	emp	hours	total hours	output p.hour	real wage	prices	nom. wage
std.dev		2.30	1.44	.41	1.77	1.20	.93	1.00	.92
(rel. to output)	ϑ_i	1.00	.62	.18	.77	.52	.40	.43	.40
factor loading	α_i	1.00	.92	1.00	.95	.88	.62	-.93	-.48
phase shift	ξ_i	—	-2.79	0.32	-2.08	3.47	6.70	4.99	2.68

Table 5: HP filter: cross correlations

	output	emp	hours	total hours	output p.hour	real wage	prices	nom. wage
std. dev.	2.19	1.46	.45	1.73	1.16	.98	1.42	.95
(rel. to output)	1.00	.66	.20	.78	.52	.45	.64	.43
Cross correlations with output								
t-9	-.33	-.54	-.23	-.52	.10	.12	.02	.16
t-7	-.26	-.48	-.08	-.42	.15	.28	-.25	-.07
t-5	-.02	-.28	.22	-.17	.26	.38	-.53	-.40
t-3	.42	.08	.49	.20	.50	.43	-.72	-.64
t-2	.64	.32	.61	.43	.57	.43	-.75	-.67
t-1	.84	.57	.71	.67	.59	.41	-.71	-.64
t	1.00	.79	.72	.85	.62	.33	-.60	-.54
t+1	.84	.86	.53	.87	.30	.21	-.45	-.45
t+2	.64	.84	.30	.79	.03	.11	-.28	-.30
t+3	.42	.74	.08	.65	-.18	-.01	-.10	-.16
t+5	-.02	.39	-.29	.25	-.39	-.21	.20	.07
t+7	-.26	.04	-.50	-.29	-.35	-.28	.42	.34
t+9	-.33	-.21	-.46	-.09	-.22	-.19	.48	.52

Notes: The smoothing parameter has been set to 1,600.

6 Summary and conclusions

The present study proposed a multivariate unobserved components (UOC) approach for modelling cyclical dynamics the basic building block of which is the stochastic cycle (SC). It was shown that, in a multivariate latent factor setting, the specific symmetry properties of the SC allow for a testable parametrization in terms of the parameters of interest. Long-run neutrality restrictions on the trend components might be imposed.

Consequently, the multivariate UOC approach has the potential to yield more consistent and substantially sharper results compared to univariate filtering techniques. As the most striking outcome, there appears to be a significant association of the hourly real wage with the business cycle which is subject to a phase shift relative to output of close to one quarter of the cycle length. It would yet be grossly misleading to interpret the resulting contemporaneous zero correlation as the absence of any association. With regard to measuring the business cycle, accounting for phase shifts therefore might also yield additional insights in comparison with other dynamic factor models as used, for instance, by Quah and Sargent (1993) or Norrbin and Schlagenhauf (1996) that focus on contemporaneous relationships only.

In sum, the results once more confirm the need to endow business cycle models with various features that account for pronounced phase shifts of employment, labour productivity, and the real wage. First, they make a strong case for mechanisms that have been proposed in order to explain the negative phase shift of employment with respect to output, i.e., indivisible labour (Hansen, 1985), labour adjustment costs (Burnside et al., 1993), or search costs (Feve and Langot, 1996). Yet, in turn, such mechanisms seem also entirely sufficient for explaining the lead of output per hour, as the cyclical dynamics of output, employment, and hours per worker is found to be represented by one common cycle. Second, while the real wage is strongly pro-cyclical with respect to output per hour, the findings on the cyclical behaviour of prices and nominal wages seem to indicate some role of nominal wage stickiness as, e.g., investigated by Cho and Cooley (1995) in an intertemporal equilibrium framework.

Fig. 2: Output cycle

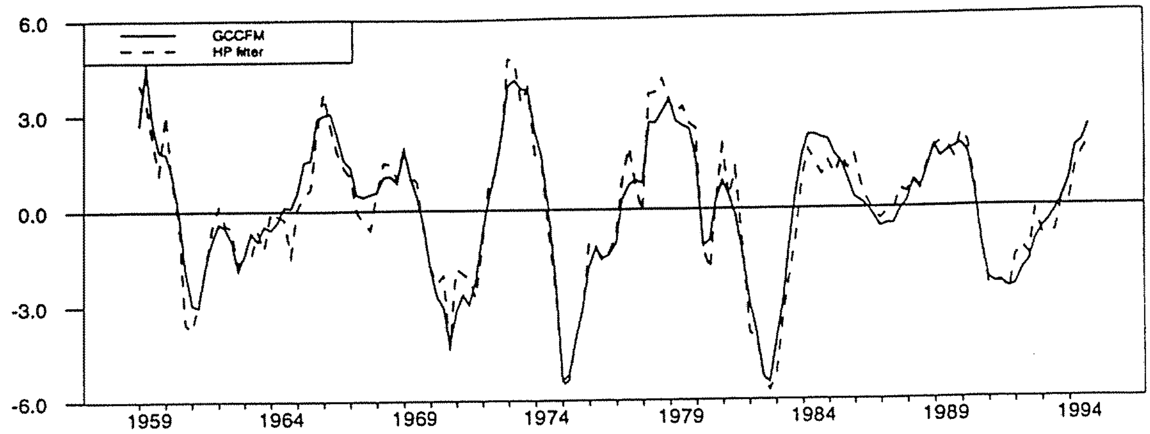


Fig. 3a: Output and employment

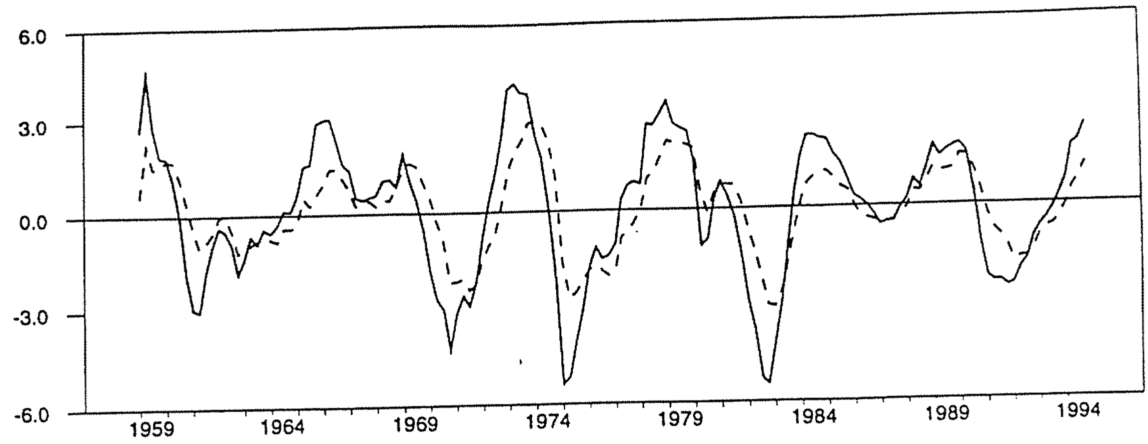


Fig. 3b: Output and hours per worker

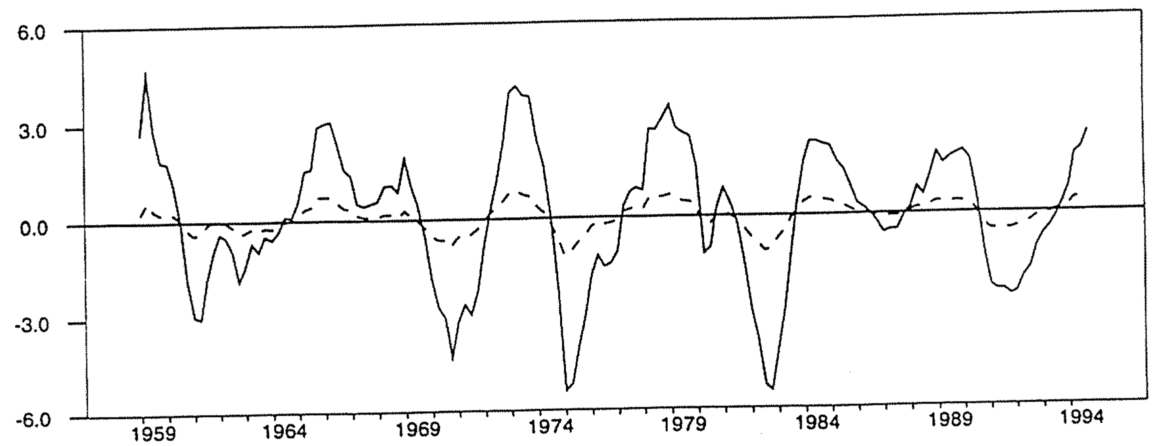


Fig. 3c: Output and hourly real wages

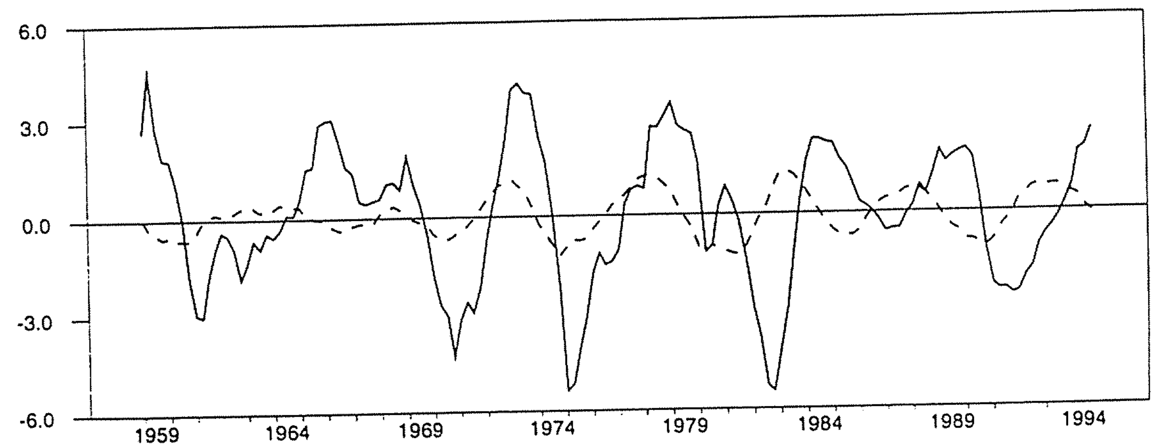


Fig. 3d: Output and productivity

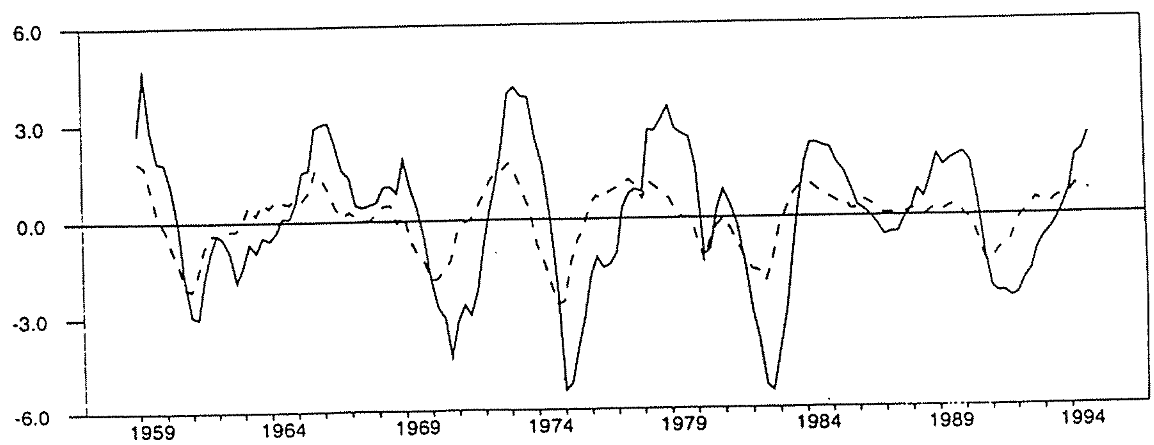


Fig. 3e: Output and prices

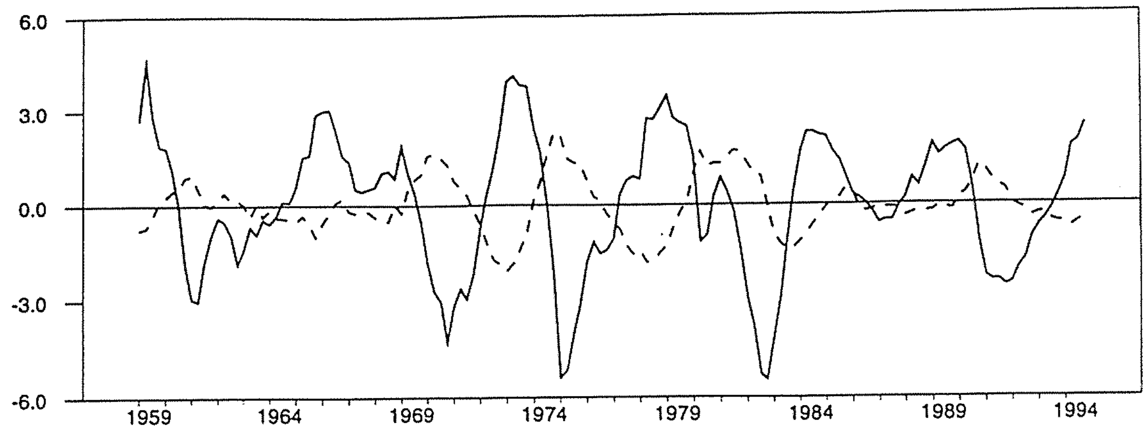


Fig. 3f: Output and hourly nominal wages

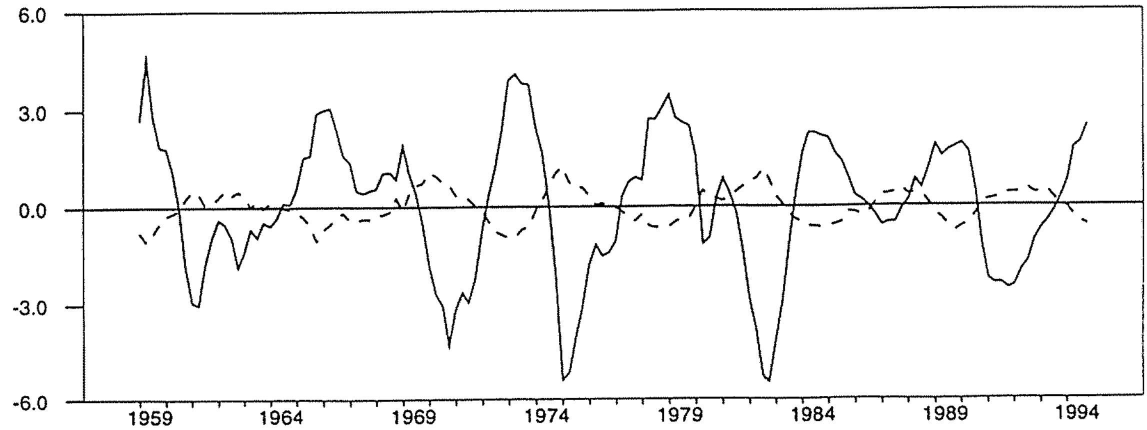


Fig. 3g: Real wage and productivity

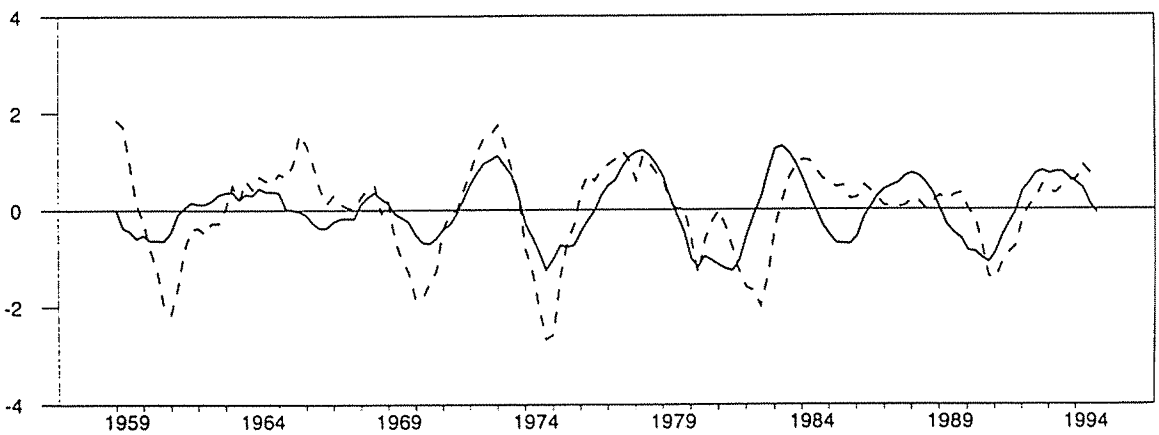
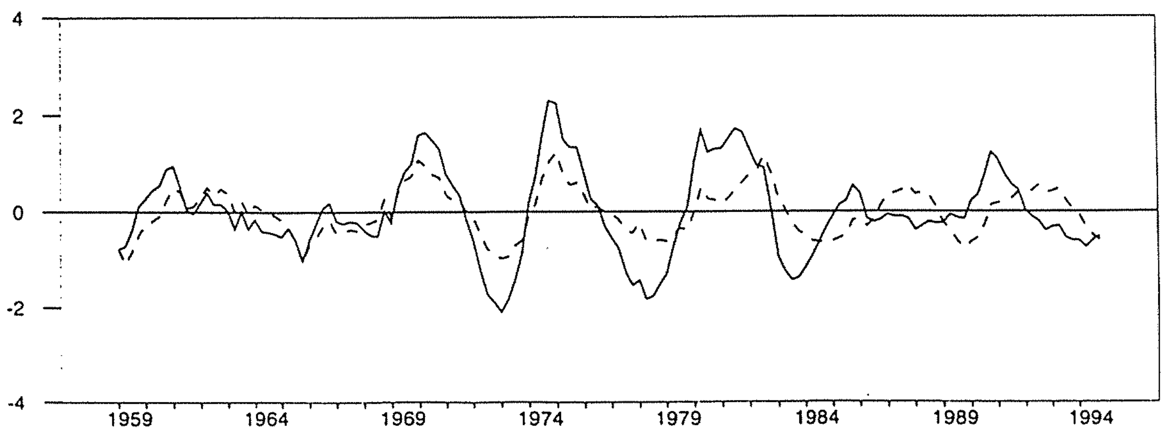


Fig. 3h: Prices and hourly nominal wages



A.1. Auto covariance function of the GCCFM

The ACF $\Gamma(s)$ of the SC as from equation (4) is found from the set of equations

$$\begin{aligned}\Gamma(s) &= [\rho C(\lambda)]^s \Gamma(0) \\ \Gamma(0) &= \rho C(\lambda)\Gamma(0)C(\lambda)' + \sigma_i^2 I_2\end{aligned}$$

It is easily verified that $\Gamma(0) = \sigma_i^2(1-\rho^2)^{-1}I_2$. Moreover, it holds $[C(\lambda)]^s = C(s\lambda)$. Hence, the ACF for $\widetilde{\varphi}_{i,t}$ is given as in equation (4) in the text.

The ACF $\Gamma_x(s)$ for the model $x_t^C = \widetilde{\Theta}\widetilde{f}_t^C$ as from equations (9) yet with $\varphi_{i,t} \equiv 0$ follows from $\Gamma_x(s) = \widetilde{\Theta}\Gamma(s)\widetilde{\Theta}'$. The elements of $B(s) = \widetilde{\Theta}C(s\lambda)\widetilde{\Theta}'$ are found as

$$\begin{aligned}B_{ii}(s) &= (\theta_i^2 + \theta_i^{*2})\cos(s\lambda) \\ B_{ij}(s) &= (\theta_i\theta_j + \theta_i^*\theta_j^*)\cos(s\lambda) + (\theta_i\theta_j^* - \theta_i^*\theta_j)\sin(s\lambda) \\ &= \theta_{ij}\cos(s\lambda) + \theta_{ij}^*\sin(s\lambda)\end{aligned}$$

The cross terms can be simplified from the identity (Harvey, 1993: 227)

$$\theta_{ij}\cos(s\lambda) + \theta_{ij}^*\sin(s\lambda) = \text{sign}(\theta_{ij})r_{ij}\cos(s\lambda - \gamma_{ij})$$

where

$$\begin{aligned}r_{ij} &= \sqrt{\theta_{ij}^2 + \theta_{ij}^{*2}} \\ \gamma_{ij} &= \tan^{-1}(\theta_{ij}^*/\theta_{ij})\end{aligned}$$

and $\gamma_{ij} \in [-\pi/2; \pi/2]$ denotes the phase shift between $x_{i,t}^C$ and $x_{j,t}^C$. Note that $(\theta_{ij}, \theta_{ij}^*) = (\theta_{ji}, -\theta_{ji}^*)$ and, hence, $\gamma_{ij} = -\gamma_{ji}$. As a special case emerge equations (7) for $\varphi_{i,t} \equiv 0$. In order to establish equation (10) it remains to show that the phase shifts are additive. Denoting

$$\begin{aligned}r_k &= \sqrt{\theta_k^2 + \theta_k^{*2}} \\ u_k &= \theta_k^*/\theta_k \\ \gamma_k &= \tan^{-1}(u_k)\end{aligned}$$

it is, first, easily shown that $r_{ij} = r_i r_j$. Additivity follows from

$$\begin{aligned}\gamma_{ij} &= \tan^{-1}(u_j - u_i)/(1 + u_i u_j) \\ &= \tan^{-1} u_j - \tan^{-1} u_i = \gamma_j - \gamma_i\end{aligned}$$

The above identity holds if $|\gamma_j - \gamma_i| \leq \pi/2$ but needs some adjustment of signs otherwise from the requirement $|\gamma_{ij}| \leq \pi/2$. Yet $|\gamma_j - \gamma_i| > \pi/2$ is just equivalent to

$$1 + u_i u_j = \theta_{ij}/(\theta_i \theta_j) < 0 \Leftrightarrow \operatorname{sign}(\theta_{ij}) = -\operatorname{sign}(\theta_i)\operatorname{sign}(\theta_j)$$

in which case the sign of the tangens switches. It finally follows that

$$\begin{aligned}\Gamma_{x,ii}(s) &= \frac{\rho^{|s|}\sigma_0^2}{1-\rho^2} r_i^2 \cos(s\lambda) \\ \Gamma_{x,ij}(s) &= \frac{\rho^{|s|}\sigma_0^2}{1-\rho^2} \operatorname{sign}(\theta_i)\operatorname{sign}(\theta_j) r_i r_j \cos(\lambda(s - \xi_j + \xi_i))\end{aligned}$$

where $\xi_i = \lambda^{-1}\gamma_i$. Adding uncorrelated idiosyncratic cycles $\varphi_{i,t}$ leaves the covariances unaffected but changes the expressions for $\Gamma_{x,ii}(s)$ by replacing r_i^2 with $\vartheta_i^2 = r_i^2 + (\sigma_i/\sigma_0)^2$ according to equation (7). Finally, the ACF for any linear combination $z_t^C = a' x_t^C$ is again of the same form as in (10).

A.2. Spectral generating function

Consider equations (9) and let the SGF of $\widetilde{\varphi}_{i,t}$ be given by

$$G_i(\omega) = (2\pi)^{-1}\sigma_i^2 \begin{bmatrix} g_1(\omega) & g_{12}(\omega) \\ \overline{g_{12}(\omega)} & g_2(\omega) \end{bmatrix}$$

where $\overline{g(.)}$ denotes the complex conjugate to $g(.)$. Under the symmetry conditions $g_1(\omega) = g_2(\omega)$ and $\operatorname{Re}(g_{12}(\omega)) = 0$ the SGF $G_x(\omega) = \widetilde{\Theta} G_0(\omega) \widetilde{\Theta}'$ for $x_t^C = \widetilde{\Theta}\widetilde{\varphi}_{0,t}$ is found as

$$\begin{aligned}g_{x,ii}(\omega) &= \sigma_0^2 r_i^2 g_1(\omega) \\ g_{x,ij}(\omega) &= \sigma_0^2(\theta_{ij} g_1(\omega) + \theta_{ij}^* g_{12}(\omega))\end{aligned}$$

Hence, all $x_{i,t}^C$ share the same spectral density and this consequently holds for any linear combination. The expressions for coherence and the phase spectrum are then given by

$$\begin{aligned} \underset{ij}{\mathrm{Coh}}(\omega) &= (r_i r_j)^{-2}\left[\theta_{ij}^2 + \theta_{ij}^{*2}\left(\frac{\mathrm{Im}(g_{12}(\omega))}{g_1(\omega)}\right)^2\right] \\ \underset{ij}{\mathrm{Ph}}(\omega) &= \tan^{-1}\left[-\frac{\theta_{ij}^*}{\theta_{ij}}\frac{\mathrm{Im}(g_{12}(\omega))}{g_1(\omega)}\right] \end{aligned}$$

$\mathrm{Coh}_{ij}(\omega)$ is independent of $\mathrm{sign}(\theta_{ij}^*/\theta_{ij})$ while $\mathrm{Ph}_{ij}(\omega)$ changes its sign with the latter. This shows that the above conditions are sufficient for establishing the relevant symmetry properties as discussed in the text. Again, adding idiosyncratic cycles leaves the expressions unchanges apart from scaling. The necessity of the conditions can be established from an inspection of the respective expressions for a more general SGF $G_i(\omega)$.

The power spectrum of the SC is calculated from the moving average representation $\widetilde{\varphi}_{i,t} = M(L)\widetilde{\kappa}_{i,t}$ (e.g, Priestley, 1981: 689) as

$$G_i(\omega) = (2\pi)^{-1}\sigma_i^2 M(e^{-i\omega})M'(e^{+i\omega})$$

where $M'(e^{+i\omega})$ denotes the transposed conjugated matrix to $M(e^{-i\omega})$. From Harvey (1993: 183) the SGF follows as

$$\begin{aligned} 2g_1(\omega) &= \sigma_i^2\frac{1+\rho^2-2\rho\cos\lambda\cos\omega}{D} \\ \mathrm{Im}(g_{12}(\omega)) &= -2\sigma_i^2\frac{\rho\sin\lambda\sin\omega}{D} \end{aligned}$$

and

$$D = \left[1+\rho^4+2\rho^2-4\rho(1+\rho^2)\cos\lambda\cos\omega+2\rho^2(\cos 2\lambda+\cos 2\omega)\right]$$

while the real part of the cross spectrum is identical to zero.

A.3. State-space form

The state-space form is outlined for a model with $n = 2$ and one idiosyncratic cycle $\psi_{2,t}$ as from equations (6). It is given by

$$\begin{aligned} x_t &= Z a_t + \nu_t \\ a_t &= T a_{t-1} + \varepsilon_t \end{aligned}$$

with the vector $x_t = (x_{1,t}, x_{2,t})'$ of the observations at time t, the state vector a_t comprising the unobserved state variables, and ν_t and ε_t representing the irregular components and innovations respectively. More specifically, with

$$\begin{aligned} a_t &= (x_{1,t}^{tr}, \mu_{1,t}, x_{2,t}^{tr}, \mu_{2,t}, \varphi_{1,t}, \varphi_{1,t}^*, \varphi_{2,t}, \varphi_{2,t}^*)', \text{ and} \\ \varepsilon_t &= (\eta_{1,t}, \zeta_{1,t}, \eta_{2,t}, \zeta_{2,t}, \kappa_{1,t}, \kappa_{1,t}^*, \kappa_{2,t}, \kappa_{2,t}^*)'. \end{aligned}$$

the matrices Z and T are given by

$$Z' = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & \theta_2 \\ 0 & \theta_2^* \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho \mathrm{c}_\lambda & \rho \mathrm{s}_\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho \mathrm{s}_\lambda & \rho \mathrm{c}_\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho \mathrm{c}_\lambda & \rho \mathrm{s}_\lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho \mathrm{s}_\lambda & \rho \mathrm{c}_\lambda \end{bmatrix}$$

where c_λ and s_λ denote $\cos(\lambda)$ and $\sin(\lambda)$, respectively.

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Institut für Höhere Studien
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna
Austria

Phone: +43-1-599 91-145
Fax: +43-1-599 91-163