ON MARKET DYNAMICS FOR A DURABLE GOOD, WITH AND WITHOUT A FINANCING CONSTRAINT

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It has become increasingly realised that Walrasian market adjustment mechanisms, at least in the form popularised by P.A. Samuelson (1941) in his Foundations, may in some circumstances yield an inadequate representation of price and quantity dynamics. In situations where nothing approximating in its effects the Walrasian auctionneer exists, there may be complicated internal dynamics incorporating the acquisition and processing of information by the agents concerned. These internal dynamics may imply a behaviour for manifest prices and quantities very different from the Walrasian scheme, where price is assumed to change only in response to a discrepancy between flow demand and flow supply. In the present paper we construct a model in which we attempt to allow for some of these internal dynamics. In the sense that it is cast in terms of a price distribution which is at least in principle easily observable, namely that at which transactions actually occur, the model represents an extension of Walrasian analysis. On the other hand, it stops short of the (possibly unattainable) task of representing in a simple way the full internal dynamics of the system, a burden to which the literature has so far not been equal.

A model of this kind should be able to tell us what happens when a variation in one or more of the underlying assumptions occurs. One of the most interesting and important of these, both from the theoretical and practical point of view, is the case where buyers may be subject to a financing constraint. The role of financing constraints in general has received widespread attention as a result of the so-called "Keynesian reinterpretation". To financial economists, it appears as the phenomenon of credit rationing, which may have either a temporary aspect, or else be of more permanent duration ("equilibrium rationing" in the terminology of D.M. Jaffee and F. Modigliani (1969)). Rationing of any kind has always been of interest in economic theory because of the implied contrast between two forms of stabilisation or control, the one based on a diktat of some kind, the other emerging out of the unencumbered operation of a pricing system. From the practical point of view this contrast is full of interest for those fortunate or unfortunate enough to live in a financial milieu of heavily regulated interest rates, particularly in respect of mortgage rates on real estate. Whether or not it is true that frustrated buyers succeed in obtaining finance from an uncontrolled fringe market, the question as to whether such direct controls would succeed of themselves in stabilising the housing market is obviously of interest. The model we establish appears to yield some useful insights into the effects of rationing of this kind.

The scheme of the paper is as follows. In section I we establish the basic model and its equilibrium properties. The dynamics are investigated in section II. We investigate here a question of some importance which does not seem to have been treated in the literature, namely whether a market is more or less stable the more imperfect is the information acquisition and processing on the part of its agents. Does "friction" increase or decrease a market's rate of convergence to equilibrium? In the third and final section we impose credit rationing constraints of two kinds and study equilibrium and disequilibrium properties of the resulting system.

I. The Model

One approach to the study of markets in disequilibrium starts from the presumption of stable flow demand and supply functions. The quantity transacted in any period is equal to the minimum of the quantities demanded or supplied; this and a rule for price adjustment based on the difference between demand and supply, determine the dynamics of clearing (R.C. Fair and D.M. Jaffee (1972), Fair and H.H. Kelejian (1973)). But while this may be adequate for fish, it will not do for houses. Where the commodity involved is durable, if the market is supposed to be in disequilibrium for a finite length of time, we have to consider what happens to the unsatisfied demand or supply. Does it simply go away? If it does not, that is, if at least some of it is backlogged, then if price change is to be related to a discrepancy between demand and supply, these schedules can no longer be regarded as stable, in the sense that they are invariant to the recent history of the process. It is this consideration that motivates our own model.

The determination of prices under conditions of imperfect information has received much recent attention (for topics related to the subject of this paper, some useful references are to be found in E. Phelps (ed) (1970), P. Diamond (1971) and the surveys by M. Rothschild (1973) and H.I. Grossman (1973)). A profusion of models analysing various aspects of search and price formation has emerged, bewildering in the diversity of their specifications and sometimes in their underlying logic. A formal distillation of these models where they attempt a complete price and quantity dynamics (not too often, admittedly) might be something like the following: Suppose for the sake of simplicity that only one good is purchased at a time; extension to the case where, given the purchase decision has been made, a demand function exists relating the number purchased to the price, poses no difficulties. At any point in time there is a certain number of sellers, and a corresponding distribution of asking prices for the product (which we shall call the sellers' price distribution). Now impose on this a selection rule, which selects a certain number of these sellers and a corresponding price distribution. This latter number is the quantity actually transacted, and the latter distribution we shall call the transactions price distribution. The burden of the literature has been to specify the details of the above process. The selection rule operates by the process of consumer search, itself a costly procedure. The task of specifying a fully adequate dynamics requires not only full specification of the number and search behaviour of buyers, but an account of how the sellers distribution is formed and changed, indeed as to why it should even persist as a proper distribution (in the mathematical sense). Particularly with respect to the latter aspect the literature has not so far been conspicuously successful. Presumbably, however, the characteristics of the sellers distribution can ultimately be related to the recent history of the transactions distribution. Likewise the numbers and perceptions of searching buyers must be related to the recent history of the transactions distribution. For in both cases, this distribution and in particular some measure of the central tendency thereof, represents the basic, observable data which help to motivate the entry, continuation, or exit of buyers or sellers from the pool of active agents. The other basic determinant of such entry or exits, is the search history of these agents, representing their hopeful prospects for the future or their accumulated frustrations from the past.

Thus in our own model, we have chosen not to attempt to model explicitly either the determination of the sellers price distribution or else the precise search characteristics of buyers. Rather, we have chosen to model the process in terms of the transactions price distribution, and in particular some single measure thereof such as the mean. Underlying our model, however, is some deeper structure such as that outlined above. By combining the idea of a ruling transactions price with a model based on underlying search behaviour, we are able to graft ideas derived from the probabilistic character of the search process on to the immensely fruitful stock of Marshallian demand and supply analysis. We shall now proceed to the details of the construction.

There is a flow of new entrants, both buyers and sellers, to the pool and for the sake of simplicity such flows will constitute our stable or invariant demand and supply schedules. There is a flow of agents out of the pool consisting (a) of those who have concluded agreements and (b) those who decided to exit for other reasons: either their costs of search (psychological or financial) may have mounted to the point where in the light of an unchanged subjective probability distribution of qualities or prices, they assess the probability of success in the coming period as not worthwhile. Or else their assessment of the distribution of prices may have changed so that they exit for the same kind of reason that they entered, namely their assessment of the ruling price. After each round of transactions, we imagine a new ruling price based on the discrepancy between the number wanting to buy and those wanting to sell at the previous price. Given this new price, unsatisfied buyers and sellers will consider whether or not they wish to remain in the pool. Their decisions will depend (i) on the new ruling price and (ii) on the length of time that they have already been in the pool, a quantity which we shall proxy by the expected "age" of buyers or sellers.

In defining the following functions and adjustment mechanisms we have chosen to remain quite general. (We have suppressed in particular a rate of interest variable, the effect of which will be given some attention in Section III. Depending on the market under consideration, many more variables might enter, and identical variables to those we do consider might enter in different ways - e.g. we could have a demand function depending also on $\frac{p}{p_x}$, where p_x was a perceived long-term or "permanent" price):

p_t = ruling purchase price, time t.

$n^d(p_t)$ = flow of new entrants[2] to the buyers pool during period t.

$n^s(p_t)$ = flow of new entrants[2] to the sellers pool during period t.

N_t^d = stock of potential buyers in the pool at time t.

N_t^s = stock of potential sellers in the pool at time t.

n_t = transactions (agreements) during time t.

$\bar{n}_t$ = successfully concluded agreements, with finance available, during time t (used only in Section III).

(1) $$p_t - p_{t-1} = \theta(N_{t-1}^d - N_{t-1}^s), \qquad 0 < \theta < 1.$$

This expresses the change in ruling prices.

(2) $$N_t^d = \phi_{dt}(N_{t-1}^d - n_{t-1}) + n_t^d .$$

(3a) $$N_t^s = \phi_{st}(N_{t-1}^s - n_{t-1}) + n_t^s .$$

Equations (2) and (3a) need further explanation. Consider the demand version (2). The quantity $N^d_{t-1}(p_{t-1}) - n_{t-1}$ represents unsatisfied demand in t-1, i.e. the number of unsuccessful buyers in the pool. The new ruling price is p_t. A proportion ϕ_d, which (given p_{t-1}) will depend among other things on p_t, of the unsatisfied buyers then elect to stay in the pool. The number of searching buyers in period t is then given by (2). A similar interpretation holds for equation (3). We have not as yet specified in any detail the functions ϕ_d and ϕ_s; this is done below.

(4) $\quad n_t = h(|N^d_t - N^s_t|)\min.(N^d_t, N^s_t), \quad 0 \leqslant h \leqslant 1, \; h' \geqslant 0$.

The number of successful contacts during period t is determined by the short side of the market; but is closer to the minimum according to a measure of the discrepancy between the forces of demand and supply. This expresses some of the probabilistic character of search. The greater the discrepancy between the two different types of searcher, the higher the probability that a given agent of the type in short supply will be successful. Note that without the existence of credit rationing, n_t denotes successful contacts, which thereby exit from the pool. In Section III this will not be the case, and some redefinition will be necessary.

Let us now investigate the functions ϕ_d and ϕ_s, which reflect decisions by unsuccessful agents to try again. If his past search experiences were a matter of indifference to him, a decision to stay on would reflect only the agent's assessment of the new ruling price. In this respect, one could argue that he would then be on all fours with a new entrant, so that for example

$$\phi_d = \frac{n^d(p_t)}{n^d(p_{t-1})} \doteq 1 + (p_t - p_{t-1})\varepsilon_d,$$

where $\varepsilon_d = \frac{n^{d\prime}}{n^d}$ is the elasticity of the flow demand schedule, evaluated at p_{t-1}.

Such an assumption, however, is not palatable. As any prospective housebuyer knows, the process of search is rarely pleasant, and the longer it goes on, the more discouraging it becomes, in the light of point (b) at the start of this section. We therefore suppose that the functions ϕ_d and ϕ_s depend also upon the average age of the buyer (μ^d_t) and seller (μ^s_t), in the sense of the average time for which they have been in the pool. It is shown in Appendix I that

$$(5) \qquad \mu^d_t = 1 + \left(\frac{N^d_t - n^d_t}{N^d_t}\right)\mu^d_{t-1} \, ,$$

$$(6) \qquad \mu^s_t = 1 + \left(\frac{N^s_t - n^s_t}{N^s_t}\right)\mu^s_{t-1} \, .$$

The time convention applied in (5) and (6) refers to the end, rather than the beginning, of the period. Thus at the end of period 1, all unsatisfied searchers are 1 period old. We can now write

$$(7) \quad \phi_{dt} = \phi_d(p_t - p_{t-1}, \mu^d_{t-1}) \quad \phi_{d,1} < 0, \ \phi_{d,2} < 0.$$

$$(8) \quad \phi_{st} = \phi_s(p_t - p_{t-1}, \mu^s_{t-1}) \quad \phi_{s,1} > 0, \ \phi_{s,2} < 0.$$

The specification of our underlying model is now complete. Although we have used arguments which have a probabilistic flavour, the model is nevertheless deterministic. We achieve this by use of the "ruling price", which we might identify with the mean of the distribution of prices at which agreements are actually completed. The ruling price change (1) is taken to be the outcome of a probabilistic process in which ruling price and transacted quantities emerge jointly. The factors θ in (1) and h in (4) are therefore not independent, so that to assume θ constant is obviously only a first approximation. We have simply moved a little way from standard Marshallian supply and demand analysis towards a more realistic model of market clearing for a durable good, without wholly abandoning the great usefulness of the former.

Equations (1) to (8) constitute our model of market dynamics unhampered by the availability of finance. The system has a stationary point as follows:

(9) (a) $n^d(p^*) = n^s(p^*) = n^*,$

(b) $N^{d*} = N^{s*} = N^*,$

(c) $n^* = h(0)N^*,$

(d) $\mu_d^* = \dfrac{N^{d*}}{n^{d*}} = \dfrac{N^*}{n^*} = \dfrac{N^{s*}}{n^{s*}} = \mu_s^*,$

(e) $\phi_d(0, \mu^*) = \phi_s(0, \mu^*) = 1.$

Equation (a) tells us that this equilibrium is determined by equality of flow demand and flow supply, in classical fashion. However at any point in time there may be more searchers in the pool than the flow of completed transactions would indicate. The quantity h(0) can be taken to measure the equilibrium degree of "friction" in the market. Not every searcher can count on immediate success; the average age of searchers is $\frac{N^*}{n^*}$, which is greater than one if $h(0) < 1$, i.e. if friction exists. Condition (e) is not so much an independent equilibrium condition, as a condition on the functions ϕ to allow of a stationary solution. The situation is analogous to that of a hydro lake. In the stationary state, the flow into the lake is equal to the flow out; but this fact says nothing about the volume of the water in the lake at any point of time. This depends on the height of the dam - in our case, the distance 1 - h(0) as a measure of the imperfection of the market clearing process.

II. Dynamics of the Basic Model

In this section we shall study some of the dynamics of the model constructed in section I. Since backlogging of demand and supply is permitted, we should expect our model to possess at least some of the dynamic features of such models (see e.g. P.A. Samuelson (1941)). Thus a monotonic divergence of price away from equilibrium is ruled out, for this would imply a backlog of ever-increasing size which, under any reasonable price adjustment mechanism, would altimately reverse the price movement. We should expect also that the model would overshoot the equilibrium position, leading to cyclical behaviour, for at the point p^* where $n^d = n^s$, the existence of a backlog accumulated from past disequilibrium will nevertheless imply a non-zero price change. But apart from these intuitive impressions, the intrinsically non-linear nature of the system renders a more precise dynamics hard to give.

We can, however, obtain a degree of insight into its local dynamic properties by linearising the system about its equilibrium position, a procedure which will be of some interest in its own right. It is convenient to work in terms of the proportional deviations

$$\frac{N_t}{N^*} - 1, \ \frac{p_t}{p^*} - 1;$$

evidently we could simply redefine units so that $N^* = p^* = 1$. Define the elasticity

$$\varepsilon_d^* = \frac{1}{n^d} \cdot \frac{dn^d}{d(p/p^*)} \qquad \text{evaluated at } p = p^*.$$

In a similar fashion, define ε_s^* and the derivatives of the functions ϕ_d and ϕ_s, namely ϕ_{d1}^*, ϕ_{d2}^* and ϕ_{s1}^*, ϕ_{s2}^*. Reinterpret the adjustment parameter θ in equation (1) to refer to proportional deviations as defined above. A tilde over a variable will denote its proportional deviation from the equilibrium value. Thus $\tilde{N}_t = \frac{N_t}{N^*} - 1$. After a small amount of substitution, we obtain the following linearised (but see 10(d)) system:

$$\text{(10a)} \quad \tilde{N}_t^d = \alpha_d \tilde{p}_t + \tilde{N}_{t-1}^d - h\tilde{n}_{t-1} - \phi_{d1}^*(1-h)\tilde{p}_{t-1} + \phi_{d2}^*(1-h)\tilde{\mu}_{t-1}^d,$$

where $h = h(0)$ and $\alpha_d = \phi_{d1}^*(1-h) + h\varepsilon_d^*$,

$$\text{(10b)} \quad \tilde{N}_t^s = \alpha_s \tilde{p}_t + \tilde{N}_{t-1}^s - h\tilde{n}_{t-1} - \phi_{s1}^*(1-h)\tilde{p}_{t-1} + \phi_{s2}^*(1-h)\tilde{\mu}_{t-1}^s,$$

where $\alpha_s = \phi_{s1}^*(1-h) + h\,\varepsilon_s^*$.

$$\text{(10c)} \quad \tilde{p}_t = \tilde{p}_{t-1} + \theta(\tilde{N}_{t-1}^d - \tilde{N}_{t-1}^s).$$

(10d) (i) If $\tilde{N}_t^d \geqslant \tilde{N}_t^s$, $\tilde{n}_{t-1} = \beta\tilde{N}_{t-1}^d + (1-\beta)\tilde{N}_{t-1}^s$.

(ii) Otherwise, $\tilde{n}_{t-1} = (1-\beta)\tilde{N}_{t-1}^d + \beta\tilde{N}_{t-1}^s$,

where $\beta = N^* \dfrac{h'(0)}{h(0)}$

is defined on the same basis as previous elasticities. It is thus reasonable to specify $0 < \beta < 1$; the interpretation is that actual transactions are locally a fixed proper convex combination of N^d and N^s.

$$\text{(10e)} \quad \tilde{\mu}_t^d = (1-h)\tilde{\mu}_{t-1}^d - \varepsilon_d^*\tilde{p}_t + \tilde{N}_t^d \ .$$

$$\text{(10f)} \quad \tilde{\mu}_t^s = (1-h)\tilde{\mu}_{t-1}^s - \varepsilon_s^*\tilde{p}_t + \tilde{N}_t^s \ .$$

As it stands, this is a 6th order system. We can reduce it to 5th order by eliminating the transactions variable, recovering it later in terms of $\tilde{N}_t^d$ and $\tilde{N}_t^s$ from 10(d). The system can then be cast as a 1st-order matrix difference equation of the form

$$y_t = Ay_{t-1}, \text{ with } y_t' = [\tilde{N}_t^d, \tilde{N}_t^s, \tilde{p}_t, \tilde{\mu}_t^d, \tilde{\mu}_t^s].$$

We obtain:

(i) If $N_t^d \geqslant N_t^s$, so that 10(d) (i) holds,

$$A(i) = \begin{bmatrix} \gamma_1^d + \theta\varepsilon_d^*, & -(\gamma_2^d + \theta\varepsilon_d^*), & h\,\varepsilon_d^*, & (1-h)\,\phi_{d2}^*, & 0 \\ \gamma_1^s + \theta\varepsilon_s^*, & -(\gamma_2^s + \theta\varepsilon_s^*), & h\,\varepsilon_s^*, & 0, & (1-h)\,\phi_{s2}^* \\ \theta, & -\theta, & 1, & 0, & 0 \\ \gamma_1^d, & -\gamma_2^d, & -(1-h)\varepsilon_d^*, & (1-h)(1+\phi_{d2}^*), & 0 \\ \gamma_1^s, & -\gamma_2^s, & -(1-h)\varepsilon_s^*, & 0, & (1-h)(1+\phi_{s2}^*) \end{bmatrix}$$

where $\gamma_1^d = \gamma_2^d + (1-h)$,

$\gamma_2^d = (1-\beta)h + \theta(1-h)(\phi_{d1}^* - \varepsilon_d^*)$

$\gamma_1^s = \gamma_2^s + (1-h)$

$\gamma_2^s = -\beta h + \theta(1-h)(\phi_{s1}^* - \varepsilon_s^*)$.

ii) If $N_t^d < N_t^s$ so that 10(d) (ii) holds, we need only replace ß by 1-ß in the above matrix to obtain A(ii).

Now in most tatonnement adjustment mechanisms, price change can be expressed solely in terms of the excess-demand differences $N^d - N^s$, rather than in either N^d or N^s by itself. That is, price movements do not depend directly upon the past history of transacted quantities. Now inspection of the above coefficients reveals that $\gamma_1^d - \gamma_2^d = 1-h = \gamma_1^s - \gamma_2^s$, so that $\gamma_1^d - \gamma_1^s = \gamma_2^d - \gamma_2^s$. If we look at the equation for price adjustment we can see that only the differences $N_{t-1}^d - N_{t-1}^s$ will appear so far as these stock variables are concerned. Unfortunately, this does not nevertheless imply that price movements are independent of transactions. For the effect of these transactions shows up in the search history of participants as reflected in the age variables μ_{t-1}^d and μ_{t-1}^s. Thus we should not be surprised to find that the dynamic behaviour of the system is a great deal more complicated as a result. Note also that our model is not completely linearised, in that we retain the switching from one regime to another incorporated in equation 10(d) (i) and (ii). This fact makes stability analysis difficult; all we could assert is that if the characteristic roots of both A(i) and A(ii) were less than one in absolute value, the model would be locally stable. Thus the very weak sufficiency criterion together with the fact that in either regime the system is of the 5th order, make stability analysis very difficult.

There is one important case, however, where this is not so. Suppose that $\phi_{d2}^* = \phi_{s2}^* = \phi_2^*$, so that buyers and sellers possess the same discouragement factor with regard to unfulfilled search. In this case we can express our price movements in terms of the differences $\tilde{\mu}_t^d - \tilde{\mu}_t^s$ in the ages of buyers and sellers, rather then each individually. Subtracting the 2nd row of A(i) from the 1st, and the 5th row from the 4th, we can form the matrix A^R corresponding to the reduced state-vector $[\tilde{N}_t^d-\tilde{N}_t^s, \tilde{p}_t, \tilde{\mu}_t^d-\tilde{\mu}_t^s]$:

$$A^R = \begin{bmatrix} \gamma_1^d - \gamma_1^s + \theta(\varepsilon_d^* - \varepsilon_s^*), & h(\varepsilon_d^* - \varepsilon_s^*) & , & (1-h)\phi_2^* \\ \theta & , \quad 1 & , & 0 \\ \gamma_1^d - \gamma_1^s & , -(1-h)(\varepsilon_d^* - \varepsilon_s^*), & & (1-h)(1+\phi_2^*) \end{bmatrix} .$$

Now if we do the same with A(ii) and bear in mind the value of the coefficients $\gamma_1^{d,s}$ and $\gamma_2^{d,s}$ we arrive at exactly the same reduced matrix. Thus the same linear system applies regardless of whether we are in the regime defined by (i) or that defined by (ii), so that not only is the system of only 3rd order, but stability conditions derived from a consideration of A^R will be necessary as well as sufficient.

We can write the characteristic equation of A^R as:

$$(11) \qquad F(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_o = 0,$$

where: $a_2 = -(1-h)(1+\phi_2^*)+1+\gamma_1^d-\gamma_1^s+\theta(\varepsilon_d^*-\varepsilon_s^*)$

$$a_1 = (1-h)(1+\phi_2^*)+(2-h)(\gamma_1^d-\gamma_1^s)+\theta(1-h)(2+\phi_2^*)(\varepsilon_d-\varepsilon_s)$$

$$a_o = -(1-h)(1+\theta(1-h)(\phi_{d1}^*-\phi_{s1}^*)).$$

The following 4 conditions are necessary and sufficient for all the roots of (11) to lie inside the unit circle (E.I. Jury (1964), p.93):

(i) $F(1) > 0$

(ii) $F(-1) < 0$

(iii) $|a_o| < 1$

(iv) $a_o^2 - 1 < a_o a_2 - a_1$.

In terms of our elementary parameters, these conditions emerge from a little manipulation as follows:

(i) $F(1) = \theta(\varepsilon_d^* - \varepsilon_s^*)(-h^2 + (1-h)\phi_2^*)$.

Since $\varepsilon_d^* < 0$, $\varepsilon_s^* > 0$ and $-1 < \phi_2^* < 0$, this is positive as required, for any admissible parameter values.

(ii) $$-F(-1) = 4(2-h) + \phi_2^* \theta(1-h)(\varepsilon_d^* - \varepsilon_s^*) - \theta h(2-h)(\phi_{d1}^* - \phi_{s1}^* - (\varepsilon_d^* - \varepsilon_s^*)) + 2\phi_2^*(1-h) + \theta(2-h)^2(\phi_{d1}^* - \phi_{s1}^*).$$

This condition is evidently fairly complicated, and the above represents the most informative nould in which we have been able to cast it. We require the R.H.S. > 0. The first two terms are positive, the first large. The third may be of either sign, but we might expect its absolute value to be small. Considering the demand parameters, for instance, ϕ_{d1}^* is an elasticity representing the reactions to a price change of those already in the market; ε_d^* is the elasticity of the flow demand schedule. If we suppose that any interaction effects with age are small - which our linearisation does indeed presuppose - then these two factors should be of similar magnitude. Similarly for the supply parameters, so that the difference defined by the third term above will be small. The last two terms are both negative. It is evident that condition (ii) is by no means stringent.

Only if the elasticity difference $\phi^{*}_{d1} - \phi^{*}_{s1}$ is very large indeed in absolute value will it be in danger of violation. If the adjustment parameter $\theta < 1$, it is certainly sufficient for stability that the elasticities ϕ^{*}_{d1} and ϕ^{*}_{s1} are both less than one in absolute value, but much weaker sufficient conditions than these can be derived from (ii).

(iii) The condition $|a_o| < 1$ can be recast as

$$-\frac{1}{\theta(1-h)} \cdot \left(1 + \frac{1}{1-h}\right) < \phi^{*}_{d1} - \phi^{*}_{s1} < \frac{h}{\theta(1-h)^2} \; .$$

Only the left-hand inequality is ever binding, and once again the magnitude of the elasticity difference $\phi^{*}_{d1} - \phi^{*}_{s1}$ is involved. The condition is not stringent. Note that the larger the value of h, the less stringent it becomes. We investigate in more detail below the relationship of this friction coefficient to stability.

(iv) This is more complicated. It can be cast as:

$$0 < -h^2(1-h)^2(1+\phi^{*}_{2})^2 + \theta\phi^{*}_{2}(1-h)(\phi^{*}_{d1} - \phi^{*}_{s1} - (\varepsilon^{*}_{d} - \varepsilon^{*}_{s})) -$$

$$-\theta h(1-h)(1+\phi^{*}_{2})(\phi^{*}_{d1} - \phi^{*}_{s1}) + \left[\chi - \tfrac{1}{2}h(1-h)(1+\phi^{*}_{2})\right]^2 ,$$

where $\chi = 1 + \theta(1-h)(\phi^{*}_{d1} - \phi^{*}_{s1})$.

Under the argument of (iii) regarding the 2nd term, the leading negative term is the first. All others are semi-positive. Evidently the first term, consisting of a squared product of three fractions, will be very small. Thus condition (iv) is once again far from stringent,indeed we have been unable to find any plausible parameter combination that does not satisfy it.

The overall impression is therefore quite unambiguous. Unless the elasticities involved are quite unrealistically large in absolute value, the model will be stable for any combination of parameter values. Before proceeding further with the topic of stability it is convenient to look briefly at the question of cyclical properties. A full examination of this question relying upon Rozanov's & Fuller's determinants (Jury p 112) is too complicated. Note however that a negative real root will exist if $a_o > 0$, i.e. if $1+ \theta(1-h)(\phi^{x}_{d1}-\phi^{x}_{s1}) < 0$. Thus the effect of numerically very large price elasticities is to introduce the posibility of saw-tooth fluctuations associated with one or more negative real roots.

If we discount this possibility, this leaves us either with three semipositive real roots, indicating a monotonic convergence to equilibrium, or else two complex conjugate roots and one semipositive real root. For reasons earlier outlined, we might expect the latter case to be applicable, so that the model will exhibit damped cycles about the equilibrium position. Let us now return, for this case, to the question of stability. The quantity 1-h we shall call the coefficient of friction for this system, after the analogy with physical systems. Thus if 1-h=0, we have $n^{x} = N^{x}$ and markets are fully cleared in every period. As an inverse measure of the stability of the system we can take $/\det A^R/ = /a_o/$. (Viewed as such a measure, it is decidedly imperfect; the only perfect measure is the modulus of the root with greatest absolute value, which is not available). Now if the elasticity parameters are not too large numerically, so that no negative real roots exist, we have $a_o < 0$. Then

$$(12) \quad \frac{\partial}{\partial(1-h)} /\det A^R/ = -\frac{\partial}{\partial h}(-a_o) = -(1+\frac{2a_o}{1-h}).$$

Thus if $/a_o/$ is very small, so that the model exhibits a very high degree of stability to begin with, the effect of increasing the friction in the system is to decrease $/\det A^R/$, in other words, to increase the stability. But if $/a_o/$ is not so small, the reverse conclusion holds: if the model does not exhibit a high degree of stability to begin with, increasing friction will, as a matter of comparative dynamics, decrease its stability. These are interesting conclusions even if not as rigorous as we should prefer. In a recent study of the housing market, Bowden (1975) suggested in a rather nebulous way that imperfections in information on the part of agents may increase the stability of a market. The present study shows that this is only true if the market is pretty stable to start with. In the more useful context where the market is not so stable, precisely the reverse is true. A final point concerns negative roots. If these are present, then condition (12) has its sign reversed, since a_o is now positive. The conclusion is then that increasing friction unambiguously decreases the stability of the system.

Preliminary to introducing the next section on credit rationing, we report here a simulation on the same basis, and with similar numbers as will be described therein. This revealed an oscillatory approach to an equilibrium solution, which was fairly well damped. Starting with the equilibrium price perturbed 50%, the system settled down to within 1% of the equilibrium price within about 35 to 40 periods, depending upon the particular combination of parameters. This simulation, done with a full nonlinear model corresponding to equations (1)-(8), can thus be taken to support the view of the local dynamics presented in this section.

III. Credit Rationing

Implicit in the foregoing treatment has been the idea that if finance is required to complete transaction, it is always available, so that if necessary interest rates rise in the derived market for funds. Our flow demand and supply schedules are assumed to reflect possible interest effects. But suppose now that there is some imperfection in the market for funds, so that the rate of interest cannot rise or fall to maintain equality of the derived demand and supply for funds. The number of transactions that can be successfully completed will then depend on the flow of finance during the period. We shall suppose[3] two cases

(a) A constant flow of funds (M) is available for financing each period. Or

(b) Funds may be backlogged, so that if not all the available funds are required in a period, the excess may be transferred forward to finance transactions in the following period.

Since successful contacts will turn into completed contracts only if finance is available, we have now to distinguish n_t, the number of contacts as defined in Section I, from $\bar{n}_t$, the number of consummated agreements. Thus in case (a) above,

$$(13) \qquad \bar{n}_t = \min(n_t, \frac{M}{ap_t}),$$

expressing the requirement that $a\bar{n}_t p_t \leqslant M$. Here a is a constant reflecting the financing requirement for each transaction. Henceforth we simply absorb it in the quantity M. In case (b) above,

$$\text{(13b)} \qquad \bar{n}_t = \min(n_t, \frac{M_t}{p_t}), \text{ where}$$

$$\text{(14)} \qquad M_t = M + (M_{t-1} - \bar{n}_{t-1}p_{t-1})$$

expresses the backlogging of funds. We consider first case (a).

<u>(a) No backlogging of funds</u>

Suppose that the system is (as described in Section I) initially in equilibrium at the point E in Figure 1, the intersection of the flow demand and supply schedules. Imagine that there is now a sudden drop in the flow of available financing, so that new transactions are contained to lie below or on the hyperbola np = M, as indicated. The immediate temptation is to imagine that this merely creates a new effective flow demand function CDE'FG, as distinct from the notional schedule n^d. The intersection E' of this effective demand schedule with the supply schedule n^s would then indicate the new constrained equilibrium. We shall shortly show that this is a temptation which must be resisted; E' is <u>not</u> a stationary point of the system.

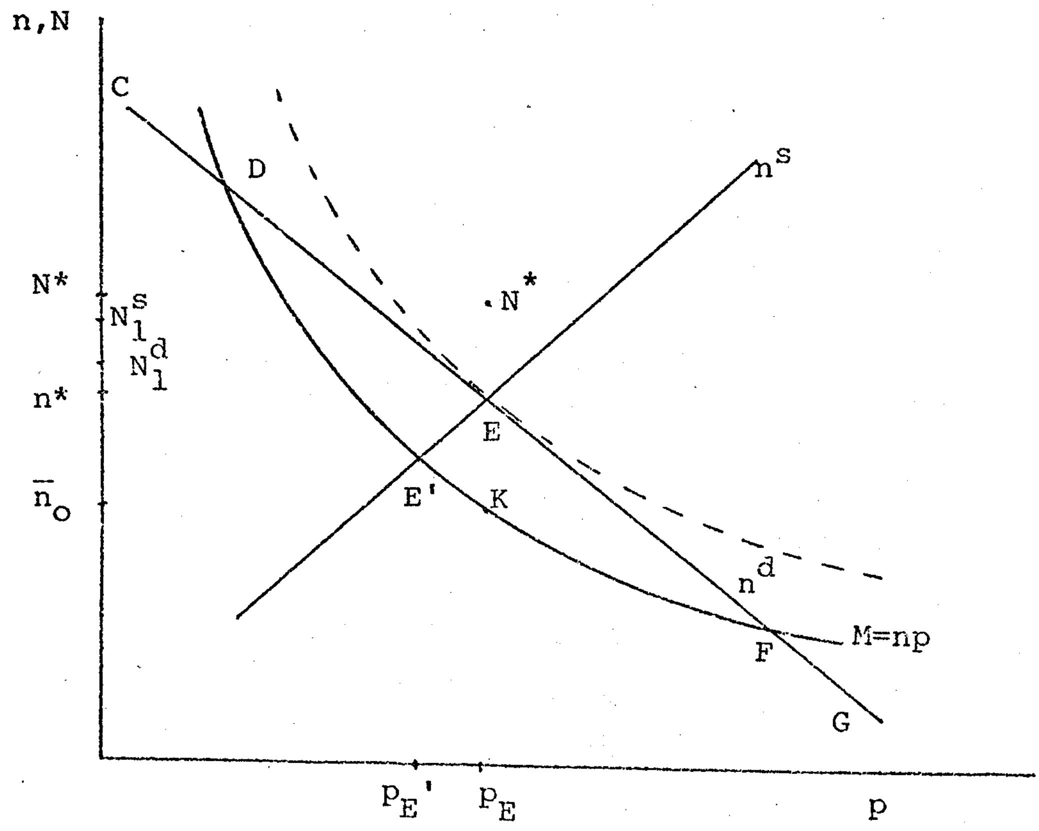

Figure 1

Following the imposition of the finance constraint in period 0, $n^*-\bar{n}_0$ transactions cannot now be consummated for lack of finance. Let us consider the effect of this on decisions to remain in the pool. These will not be symmetric as between buyers and sellers. If a buyer is refused finance, this is likely to have a rather shattering effect on his decision to remain in the pool. He may judge it better to withdraw from the pool until such time as his prospects for obtaining finance appear to have improved. As a matter of contrast, we shall thus imagine that if a buyer is refused finance, he immediately exits from the pool of searchers. So far as buyers are concerned, equation (2) of Section I remains unchanged as a description of total demand pressure. The exit of buyers from the pool comprises all those who have made contacts, n_t. Under the above discouragement assumption, it is immaterial whether or not all of the n_t pencilled agreements have been consummated with finance.

However, the effect of a buyer's failing to obtain finance on the seller, i.e. on the other half of such a pencilled agreement, is not as clear-cut. Failure to obtain finance is more often a reflection on the creditworthiness of the buyer, rather than on the quality of the good offered by the seller. The discouragement effect of the falling through of an agreement on the buyer's decisions is consequently not likely to be quite as dramatic. He may elect to remain in the pool of active sellers. Under these circumstances, equation (3a) of Section I must be replaced with

(3b) $$N_t = \phi_{st}(N^s_{t-1} - \bar{n}_{t-1}) + n^s_t \ .$$

Strictly speaking, the function ϕ_{st} should now be altered, to reflect the fact that the collective decision on the part of sellers to remain in the pool should now depend on the history of unconsummated contacts $(n_t-\bar{n}_t)$. That is, an additional argument, as well as the (unchanged) arguments p_t-p_{t-1}, and μ^s_t, should appear in ϕ_{st}. We will assume, however, to highlight the contrast with buyers, that sellers are indifferent to such failures in their decision to stay on the market.

Let us return then to Period 0, with the initial shock provided by the decrease in available finance. As of Period 0, there are no directly consequential pressures on price. However it follows from the arguments given above, that in the following period, there will be more sellers than buyers at the unchanged price p_E. That is $N_1^s > N_1^d$, as indicated on the vertical axis of Figure 1. In this sense, the initial gap EK between n^s and the "effective" demand curve DCE'KM provides a clue as to the depressant effect on price of our initial financial shock.

However, the interpretation of the latter curve as a new effective flow demand does not extend to consideration of new equilibrium positions. Note first that, if $*$ denotes a presumed stationary solution, we cannot have $\bar{n}^* < \frac{M}{p^*}$. For this would correspond to an unconstrained system as in Section I. Given that the equilibrium is unique in the latter case, this rules out the above contingency. We must therefore have $\bar{n}^* = \frac{M}{p^*}$. Consider, then, the following equations, which will define any new stationary solution:

(15) (i) $$N^{d*} = N^{s*},$$

(ii) $$N^{d*} = \phi_d(0, \mu_d^*)(N^{d*} - n^*) + n^{d*},$$

(iii) $$N^{s*} = \phi_s(0, \mu_s^*)(N^{s*} - \bar{n}^*) + n^{s*},$$

(iv) $$\bar{n}^* p^* = M,$$

(v) $$n^* = h(0) N^*,$$

(vi) $$\mu_d^* = 1 + \left(\frac{N^{d*} - n^*}{N^{d*}}\right)\mu_d^*,$$

(vii) $$\mu_s^* = 1 + \left(\frac{N^{s*} - \bar{n}^*}{N^{s*}}\right)\mu_s^*.$$

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(vii) $$\mu_s^* = 1 + \left(\frac{N^{s*} - \bar{n}^*}{N^{s*}}\right)\mu_s^*.$$

Combining (ii) and (vi), it follows that $\mu_d^* = \frac{N^{d*}}{n^{d*}}$, and similarly $\mu_s^* = \frac{N^{s*}}{n^{s*}}$. We can now reduce 15(i) to (vii) to the two equations:

$$N^* = \phi_d(0, \frac{N^*}{n^{d*}})(1-h(0))N^* + n^{d*} \tag{16}$$

$$N^* = \phi_s(0, \frac{N^*}{n^{s*}})(N^* - \frac{M}{p^*}) + n^{s*}. \tag{17}$$

Bearing in mind that $n^{d*} = n^{d*}(p^*)$ and $n^{s*} = n^{s*}(p^*)$, these two equations define any equilibrium in terms of N^* and p^*. By considering the slope of (16) and (17) in the N-p plane, one can show that along (16), $\frac{dN^*}{dp^*} < 0$. And provided that $N^* \geqslant \frac{M}{p^*}$, which is certainly a sensible requirement for a stationary solution, $\frac{dN^*}{dp^*} > 0$ along (17) in such a region. It follows that if a stationary solution exists, it must be unique.

Particular interest attaches to the question of whether E' of Figure 1 can constitute a new equilibrium. This point is characterised by $n^s(p) = \frac{M}{p}$, defining $p_{E'}$. It accordingly satisfies equation (17) only if either (i) $\phi_s(0, \frac{N_{E'}}{n^s_{E'}}) = 1$ or (ii) $N_{E'} = n^s_{E'}$ and $\phi_s(0, \frac{N_{E'}}{n^s_{E'}}) = 0$. Only case (i) is sensible. The equation

$$\phi_s(0, \frac{N_{E'}}{n^s_{E'}}) = 1, \text{ with the condition } n^s_{E'} = \frac{M}{p_{E'}},$$

defines $N_{E'} = N_{E'}(M)$, $p_{E'} = p_{E'}(M)$.

Substituting these values into equation (16) yields an <u>independent</u> condition that E' must satisfy if it is to constitute a stationary solution. Evidently there will in general be at most one value of M for which this condition is satisfied. Thus in general, i.e. for an arbitrary M, the point E' cannot be a stationary solution.

Let us now turn to the dynamics of disequilibrium adjustment. Even if $\phi_{d2} = \phi_{s2}$, as in section II, it is no longer possible to cast our price dynamics in terms of the difference $N_t^d - N_t^s$, since the transactions quantity relevant for N_t^d, namely n_t, is different from that apposite to N_t^s, namely $\bar{n}_t$. We therefore resorted to a simulation study in the hope that this would yield some insight into possible dynamics. The numerical values and specifications for equations of the simulations are given in Appendix II to this paper, which reproduces a representative set of results (Table 1). In the present case (a constant flow of funds) a well-defined limit cycle emerged after quite a short time. This is sketched in Figure 2, where the continuous line represents transactions (n) and the broken line the number of goods on the market (N^s).

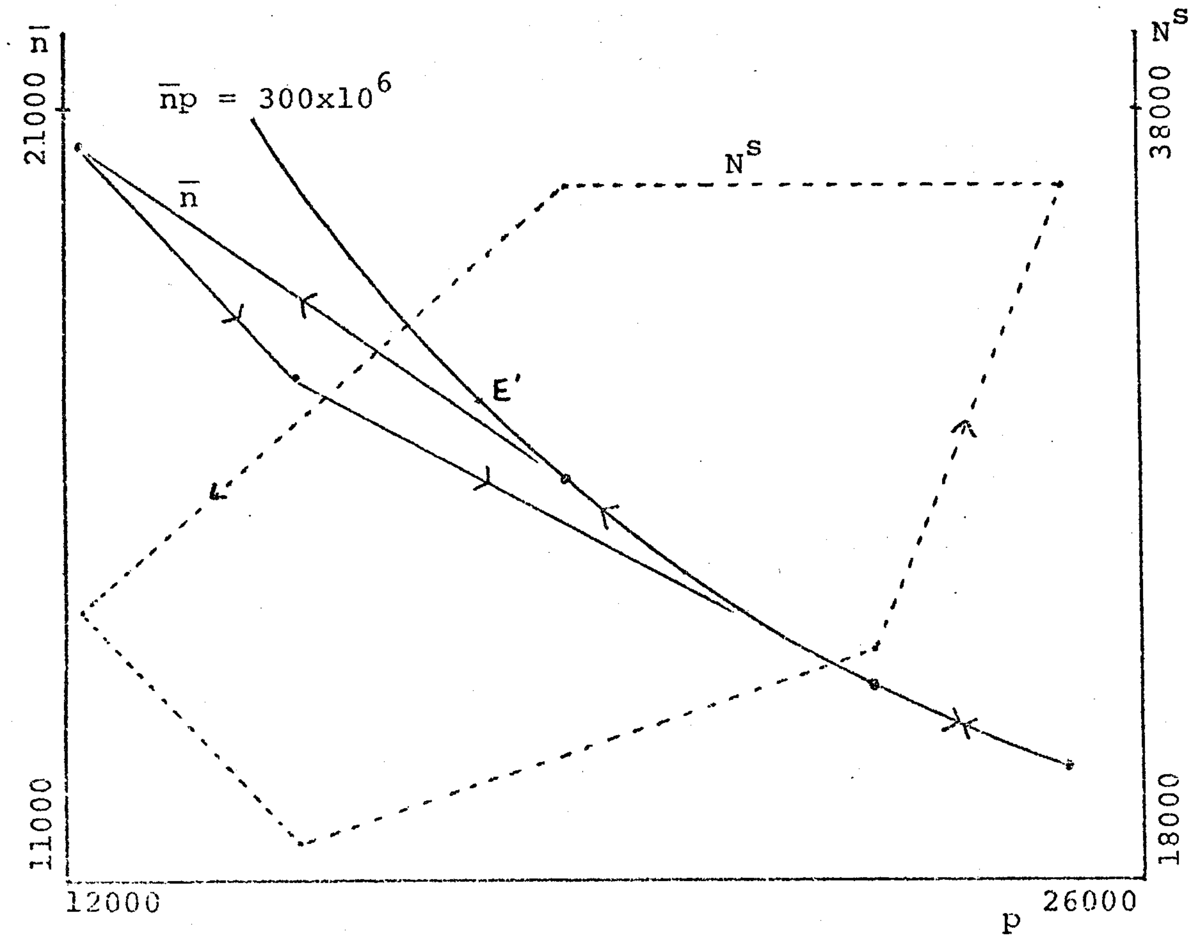

Figure 2: Limit Cycle of Table 1 (sketched).

It is apparent that the closed transactions - price cycle does not include the point E'. One of the most interesting features of this cycle is that the price fluctuations set up by credit rationing may, at the lower end of the price scale, depress supply to the point where all the available credit is not needed. The system does not, therefore, automatically move along the financing constraint or frontier. So far as the number of goods on the market is concerned, this too shows quite a wide degree of fluctuation.

In short, the imposition of credit rationing, perhaps to control a situation where the equilibrium price is regarded as too high, can hardly be regarded as a stabilising device.

(b) Backlogging of Funds

Let us now substitute equation (13b) with (14) for equation (13a), to allow for the possibility that funds not used in any period may be backlogged. The comparative statics of stationary states are the same as those of case (a) above, where no backlogging occurs. For if a stationary state were to occur, it would follow from equation (14) above that $M^{*} = M$, the constant of the previous analysis.

A dynamic simulation on a similar basis for that in case (a) is also presented in Appendix II. Once again, price and transacted quantities oscillate, and the amplitude of oscillation in price, at least, is very little different from that where no backlogging is permitted. However, no limit cycle, in the exact sense, appears to exist. Instead the dynamics consist of a series of oscillations, each of which folds back on itself in the n-p plane, of roughly similar shape. It may be that a true limit cycle does exist, but if so would take a very long time indeed to emerge.

We have tried in this section to build a model of market dynamics under a finance constraint which while undoubtedly polar in nature and abstract in the sense that it might be applicable to a wide range of particular markets, is hopefully reasonably realistic in its description of the behaviour of participants. Naturally, there have been implicit comparisons and contrasts with existing models of market clearing. We single out two:

(a) The first is between our model of credit rationing and the case where interest rates rise quickly to preserve equality between the demand and supply for funds. The polar case is where interest rate adjustment is instantaneous, by a tatonnement process within one of our single periods. In this case we can write our flow demand function as $n^d = n^d(p,r)$, to reflect the fact that higher interest rates will affect decisions to enter the pool. Moreover ϕ_s and ϕ_d will also depend on r, in the sense that the interest-rate tatonnement is supposed to take place jointly with the formation of the ruling price. Assume now a lowering of the available funds for transactions. By replacing the rationing device by the rate of interest as a pricing device, the result is to create a stable equilibrium, in the sense that no limit cycle will in general emerge. This stable equilibrium will now coincide with the point E' of Figure 1, at which point $n^d(p,r)$, evaluated at the new equilibrium interest rate, will intersect the flow supply curve, $n^s(p)$, assumed unchanged. In this sense, interest rate flexibility, as a pricing device, can do what credit rationing cannot.

(b) Our second comparison is with the case where market behaviour is assumed to follow more standard concepts of demand and supply. Thus suppose $h(0) \simeq 1$ and $\phi_s \equiv \phi_d \equiv 0$, so that pure flow concepts are applicable. The system (without backlogging of funds) is then

$$n_t = \min(n_t^d, n_t^s),$$

$$\bar{n}_t = \min(n_t, \frac{M}{p_t}).$$

> Suppose we start from the point E of full equilibrium in Figure 1. M is now lowered as in the previous experiment. The new quasi-equilibrium is now at point K. For in the absence of backlogging of any kind, there are no pressures on price. Moreover, if the system starts at any other point, price adjustment will follow classical rules and come to rest at p_E. Equilibrium transactions are solely determined by the rationing constraint and price gets stuck at p_E. Intuitively, such a separation of price and quantity adjustments seems a little implausible.

If we regard the matter as one of judging the relative efficacy of a price mechanism or a rationing mechanism for the use of funds, the conclusions of this paper are fairly clear-cut. In the model we have used, rationing is responsible for providing a wrong signal to sellers as to the state of effective demand. If the cost of funds rises, the price which buyers are willing to bid falls, and sellers will be faced with longer periods without a contact, inducing them to lower their reservation prices directly. But if the price of funds is held constant, so that rationing becomes necessary, there is no such direct signal. The failure of a buyer to find the necessary finance may be seen by the seller only as a reflection on the buyer's creditworthiness. So far as the seller is concerned, there is no direct incentive to lower his price, for the existing price has been seen to attract attention. Only when frustrated buyers exit, does the more indirect force of the growing relative number of sellers to buyers start to make itself felt.

Footnotes

1. I should like to thank Esther Ching who programmed very efficiently the simulation of the model. I am grateful also to an unknown referee, whose criticisms stimulated a material improvement in the paper.

2. Note that the assumption that this is stable may under many circumstances be questionable. For example, the world at large will include those who have just emerged from the pool of searchers, and their recent experience will affect their decision to re-enter. One would certainly require a large population of holders ("reservation stock demand") relative to the size of the pool for the presumption of stability.

3. In assuming credit rationing to take this form, we are supposing that the probability of any particular buyer obtaining finance is independent of any of his other characteristics such as his search history. More generally the representation of credit rationing must depend on a specific theory of such, and we refer the reader on this question to Jaffee and Modigliani (1969) and the references cited therein. The approach of this paper does not appear to be incompatible with such previous work, although this is really a matter for substantive research.

APPENDIX I

We prove equations (5) and (6) of Section I. We consider only buyers; the case of sellers is identical.

Leftovers from Period t-1 trading are $N^d_{t-1} - n_{t-1}$. Let $P^d_{t-1}(\tau)$ be the age distribution of these leftovers; $= 1,2 \ldots$. The number of t-1 leftovers who decide to remain for Period t trading is $\phi_{dt}(N^d_{t-1} - n_{t-1})$. The number of new entrants is n^d_t. The total is N^d_t. Assume that the probability that a buyer is successful is independent of whether he is one of the new entrants or one of the old hands. Then

$$P^d_t(\tau) = \frac{n^d_t}{N^d_t} \qquad \tau = 1,$$

$$= \left(\frac{N^d_t - n^d_t}{N^d_t}\right) \times P^d_{t-1}(\tau - 1) \qquad \tau \geqslant 2.$$

So

$$\mu_t = \sum_{\tau=1}^{\infty} \tau P^d_t(\tau) = \frac{n^d_t}{N^d_t} + \sum_{\tau=1}^{\infty} \left(\frac{N^d_t - n^d_t}{N^d_t}\right) \tau P^d_{t-1}(\tau-1).$$

Since $\mu_{t-1} = \sum_{\tau=1}^{\infty} \tau P^d_{t-1}(\tau)$, by writing $s = \tau - 1$ we obtain

$$\mu_t = \frac{n^d_t}{N^d_t} + \sum_{s=1}^{\infty} \left(\frac{N^d_t - n^d_t}{N^d_t}\right) (1-s) P^d_{t-1}(s),$$

which reduces to equation (5) of the text.

APPENDIX II

The Simulation of Section III

The following is the complete system specification:

(1) $$p_t - p_{t-1} = \theta(N^d_{t-1} - N^s_{t-1})$$

We tried $\theta = 0.5, 0.75$.

(2) $$N^d_t = \phi_{dt}(N^d_{t-1} - n_{t-1}) + n^d_t \ .$$

We specified $n^d_t = 40{,}000 - p_t$.

(3a) $$N^s_t = \phi_{st}(N^s_{t-1} - n_{t-1}) + n^s_t.$$

We specified $n^s_t = p_t$.

Or

(3b) $$N^s_t = \phi_{st}(N^s_{t-1} - \bar{n}_{t-1}) + n^s_t.$$

(4) $$n_t = h(|N^d_t - N^s_t|)\min(N^d_t, N^s_t).$$

We chose $h_t = 1 - 0.25 \exp - \frac{1.6094}{20{,}000} |N^d_t - N^s_t|$. So $h(0) = 0.75$.

(5) $$\mu^d_t = 1 + \left(\frac{N^d_t - n^d_t}{N^d_t}\right)\mu^d_{t-1}$$

(6) $$\mu^s_t = 1 + \left(\frac{N^s_t - n^s_t}{N^s_t}\right)\mu^s_{t-1} \ .$$

(7) $$\phi_{dt} = \max(0,\min(1,1 - (p_t - p_{t-1})\cdot \frac{1}{n^d_{t-1}} - (\mu^d_{t-1} - 1.33)\beta_d)).$$

We tried $\beta_d = 0.25, 0.25$. The above specification is intended to represent a linearisation about the point $p_t - p_{t-1} = 0$, $\mu^* = \frac{4}{3}$, the latter corresponding to the unconstrained equilibrium expected age.

(8) $$\phi_{st} = \max(0,\min(1,1 - (p_t - p_{t-1})\cdot \frac{1}{n^s_{t-1}} - (\mu^s_{t-1} - 1.33)\beta_s)).$$

We tried $\beta_s = \beta_d = 0.15, 0.25$.

(10a) $$\bar{n}_t = \min(n_t, \frac{M}{p_t}).$$

$$M = 300 \times 10^6.$$

or

(10b) $$M_t = M + (M_{t-1} - \bar{n}_{t-1}p_{t-1}).$$

We reproduce below a selection of figures for two sets of results, (a) with no backlogging of funds (b) with backlogging. In both cases the simulation is that of the conceptual experiment discussed in Section III; i.e. initial equilibrium price p_0 is set at 20,000, the unconstrained equilibrium. The precise value of the parameters as given above turned out to make little difference to the substantive character of the results. We accordingly present only values for $\theta = 0.5$, $\beta_s = \beta_d = 0.15$.

TABLE 1

(No backlogging)

(to nearest dollar or unit)

t	p_t	N_t^s	μ_t^d	μ_t^s	n_t	$\bar{n}_t$
1	20,000	26,667	1.33	1.33	20,000	15,000
2	20,000	31,667	1.33	1.49	33,208	15,000
3	17,500	31,689	1.22	1.67	22,352	17,143
4	15,135	26,985	1.19	1.73	21,462	19,822
5	16,378	23,541	1.30	1.53	20,415	18,317
6	20,308	25,532	1.41	1.31	20,670	14,772
7	21,999	32,759	1.41	1.43	21,910	13,637
8	18,339	33,999	1.20	1.66	22,090	16,359
t=491	22,488	23,901	1.51	1.08	19,976	13,341
492	25,098	35,659	1.51	1.32	20,504	11,953
493	18,489	35,999	1.12	1.64	21,313	16,226
494	12,213	24,359	1.08	1.82	20,467	20,467
495	14,995	18,887	1.28	1.37	17,473	17,473
496	22,488	23,901	1.51	1.08	19,976	13,341
497	25,098	35,659	1.51	1.32	20,504	11,953
498	18,489	35,999	1.12	1.64	21,313	16,226
499	12,213	24,359	1.08	1.82	20,467	20,467
500	14,995	18,887	1.28	1.37	17,473	17,473

TABLE 2
(Backlogging)
(to nearest dollar or unit)

t	p_t	N_t^s	μ_t^d	μ_t^s	n_t	$\bar{n}_t$
t=491	12,470	23,672	1.10	1.79	20,225	20,225
492	15,828	19,275	1.31	1.32	17,734	17,734
493	22,913	24,454	1.52	1.08	19,952	16,021
494	24,815	33,248	1.49	1.27	20,017	12,090
495	19,360	36,053	1.15	1.59	20,960	15,496
496	12,814	25,628	1.08	1.80	20,817	20,817
497	14,594	19,405	1.26	1.45	17,851	17,851
498	21,665	23,220	1.49	1.10	19,810	17,204
499	24,971	30,987	1.52	1.21	19,985	12,014
500	20,985	37,271	1.21	1.53	20,426	14,296

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