IHS Economics Series
Working Paper 23
February 1996

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Impressum

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Title:
The Energy-Capital Complementarity Debate: An Example of a Bootstrapped Sensitivity Analysis

ISSN: Unspecified

1996 Institut für Höhere Studien - Institute for Advanced Studies (IHS)
Josefstädter Straße 39, A-1080 Wien
E-Mail: office@ihs.ac.at
Web: www.ihs.ac.at

Institut für Höhere Studien (IHS), Wien
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Reihe Ökonomie / Economics Series **No. 23**

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Abstract

The aggregate production function approach is one way to forecast future energy demand (a step in forecasting carbon dioxide emissions, for example) and to analyze the aggregate economic effects of measures such as the increase of taxes on energy use. The results of such an approach tend to hinge on whether energy and capital are substitutes, implying that increases in energy prices will increase the demand for capital stock or are complements, implying that increases in energy prices will reduce the demand for capital stock. In a famous but controversial paper, Berndt and Wood (1975) find energy and capital are complements using aggregate time series manufacturing data for the United States, 1947-1971. Ilmakunnas (1986) shows that much of this analysis is sensitive to the imposition of theoretical economic restrictions and provides a range of point estimates in a sensitivity analysis. The current paper discusses these issues further and taking the Berndt-Wood study as an empirical example, shows that the estimation sensitivity is due to one particular set of restrictions known as symmetry restrictions and provides a bootstrap analysis which suggests that estimation sensitivity is almost entirely in the means of the sampling distributions and not in their shapes or degrees of dispersion.

Keywords
energy-capital substitution, mixed estimation

JEL-Classifications
Q43, C15, E23

Comments

Baldev Raj wishes to acknowledge financial support in part from the Natural Sciences and Engineering Research Council of Canada; the Norwegian Research Council of Sciences and Humanities, and the Research Office at Wilfrid Laurier University. Michael Veall acknowledges the financial support of the Social Sciences and Humanities Research Council of Canada.

1. INTRODUCTION

Berndt and Wood (1975) is an early example of an aggregate production function approach to estimate substitution possibilities between inputs, where an empirical relationship is estimated between a measure of aggregate output (in this case U.S. total manufacturing output, 1947-1971) and a set of input aggregates (capital, labour, energy and materials). One goal of such research is the prediction of future energy demand as output increases (a step in forecasting the emissions of carbon dioxide or certain pollutants). Another goal is the analysis of effects of energy price increases or an energy tax on energy use, aggregate output, employment, investment and raw materials utilization.

The Berndt and Wood (1975) analysis yielded some noncontroversial findings, providing evidence for example that energy price increases would reduce energy demand and modestly increase the demand for labour. However a very controversial finding was that energy and capital were complements, that is that energy price increases would reduce the demand for capital and hence the rate of investment, and in addition, increases in the cost of capital would decrease the demand for energy. The Berndt-Wood finding was reanalyzed by Berndt and Wood (1979), Griffin and Gregory (1976), Griffin (1981a, 1981b), Kulatilaka (1984), Morrison and Berndt (1981), Pindyck (1979), Pindyck and Rotemberg (1983), Solow (1987) and Turnovsky, Folie and Ulph (1982), sometimes with different data sets and sometimes with more sophisticated dynamic models. Some of these other studies concluded that energy and capital were substitutes, that is, energy price increases would increase the demand for capital and the rate of investment.

As part of their analysis, Berndt and Wood impose theoretical economic restrictions known as homogeneity and symmetry restrictions on their estimated model. Homogeneity implies that the relative prices matter, not absolute prices, so that if the price of all inputs and outputs doubles, there is no impact on output or input demand. Symmetry implies that in setting the input levels that minimize cost, the percentage change in demand for input i due to a one per cent increase in the price of input j will be same as the percentage change in demand for input j due to a one percent increase in the price of input i. What is important to understand here is while these restrictions are mathematical consequences of cost minimization, it may well be that costs are not minimized in the production of aggregate output even if individual firms in the economy are well-modelled as cost-minimizers. Especially given that aggregate output is not homogeneous and consists of many components aggregated using varying prices, it seems that the imposition of homogeneity and symmetry in the Berndt-Wood estimation is dubious at best.

Ilmakunnas (1986) demonstrates this point using a sensitivity analysis based on Theil and Goldberger (1961). (In the following, we will briefly discuss a modification due to Mittelhammer and Conway (1988) that removes a technical difficulty associated with the Theil-Goldberger method commonly known as the admissibility problem.) This method allows such restrictions to be gradually relaxed to produce a spectrum of results, with restricted and unrestricted methods at either end and a variety of intermediate results corresponding to various degrees of belief in the restriction. Ilmakunnas finds that the capital-energy complementarity result is very sensitive to the imposed restrictions. We continue to use the Berndt-Wood study as an empirical example and show that the symmetry restrictions and not the homogeneity restrictions are the cause of the empirical sensitivity of the estimates. We also note that the Ilmakunnas approach is based on a series of point estimates with no standard errors or estimated sampling distributions. We remedy this by providing a bootstrap analysis; our findings suggest that the sensitivity is in the means of the relevant sampling distributions: studying changes in measures of dispersion or of shape does not suggest significantly more or less sensitivity.

Section 2 presents a brief discussion of the derived demand functions used by Berndt and Wood and discusses the restrictions of homogeneity and symmetry that microeconomic theory may place on such a system. Section 3 discusses the modified Theil-Goldberger mixed estimation system, shows how it can be used to provide a sensitivity analysis as in Ilmakunnas and discusses implementation for the Berndt and Wood example. Section 4 provides the empirical results of the sensitivity analysis. These include bootstrap estimates of the sampling distributions corresponding to estimates under different assumptions, so that these distributions may also be considered in the sensitivity analysis. A summary and the conclusions of the research are presented in the final section.

2. The Estimation of Substitution Elasticities

In the Berndt and Wood model, a translog unit cost function yields derived demand equations of the form:

(2.1) $$S_i = \alpha_i + \beta_{iL} \ln P_L + \beta_{iK} \ln P_K + \beta_{iE} \ln P_E + \beta_{iM} \ln P_M + \varepsilon_i$$

where i = L, K, E, M denote labour input, capital input, energy input and materials input respectively, S refers to the share of total costs spent on that input, P_i denotes the price of input i, ln denotes the natural logarithm and ε_i is a random error, which will be assumed throughout to be independently and identically distributed if data generated by this model are observed over time.

Most often the parameters of interest are the Allen partial elasticities of substitution which take a positive value if two inputs are substitutes and a negative value if two inputs are complements. For the translog system these are:

(2.2) $$\sigma_{ij} = (\beta_{ij} + S_iS_j + \delta_{ij}S_i)/S_iS_j$$

where $\delta_{ij} = 1$ if $i = j$ and zero otherwise. In this paper point estimates of the elasticities are evaluated using point estimates of the β_{ij} and fitted values for the S_i evaluated at the means of the exogenous variables. Standard errors of the elasticities are calculated by a bootstrap simulation which allows for the randomness in S_i.

The data for n observations on the factor demand equations (2.1) can be written in matrix form as:

$$y = Z\beta + u \tag{2.3}$$

where y is a 3n x 1 vector stacking first all the observations on S_L, then all the observations on S_K and finally all the observations on S_E. The matrix $Z = I_3 \otimes X$ is 3n x 15, where I_3 is the 3 x 3 identity matrix, X is the matrix consisting of a column of ones plus columns corresponding to $\ln P_L$, $\ln P_K$, $\ln P_E$ and $\ln P_M$ and $\otimes$ is the Kronecker Product. The vector β is 15 x 1, and consists of the unknown α_i and β_{ij} parameters in (2.1) for i = L, K and E and j = L, K, E and M and u is a 3n x 1 vector of ε_L, ε_K and ε_E errors, stacked to correspond to y. The covariance matrix of u is assumed to be $\Phi \otimes I_n$, where Φ is 3 x 3 and I_n is n x n. The materials share equation is dropped because of the adding-up problem. A property of the Maximum Likelihood /Iterative Zellner method of estimation (IZFP; Zellner, 1962) is that it makes no numerical difference as to which equation is dropped. Berndt and Wood (1975) estimate their system using Iterative Three Stage Least Squares (I3SLS), which allows for endogenous factor prices by employing a set of instrumental variables, assumed exogenous. However, Berndt and Christensen (1973) conduct an empirical comparison of I3SLS and IZFP and find no appreciable quantitative difference in estimates. We will therefore assume, for purposes of this paper, that factor prices are exogenously determined and hence that IZFP is appropriate.

The q linear restrictions may be imposed on (2.3):

(2.4) $r = R\beta$,

where r is q x 1 and R is q x 15. These restrictions could be the three homogeneity restrictions $\Sigma_j \beta_{ij} = 0$, i = K, L and E or the three symmetry restrictions $\beta_{ij} - \beta_{ji} = 0$ for i, j = K, L and E, $i \neq j$. Alternatively, all six restrictions could be imposed at once.

3. Theil-Goldberger Mixed Estimation

We now change (2.4) so that the q linear restrictions only hold with some random error:

(3.1) $r = R\beta + v$

where v is a q x 1 random vector distributed independently of u with known mean γ and known covariance matrix Ψ. The distribution of v represents the degree of belief of the researcher that various behavioural restrictions will hold exactly. Just as model imperfections may lead to the random error in the demand system (2.3), the error in (3.1) may be justified by arguing that the restrictions hold imperfectly due to aggregation, optimization errors etc.

This approach corresponds to Theil-Goldberger mixed estimation which is straightforward to apply. It is a simple way to incorporate prior information as in a Bayesian approach, yet it may be appealing to researchers trained in the classical statistics tradition. Restricted or unrestricted estimates are special cases of this framework and as we now show, it is straightforward to develop a sensitivity analysis within this framework. The Theil-Goldberger mixed estimator is:

(3.2) $\beta^{TG} = \Omega^{-1}((\Phi^{-1} \otimes X')y + R'\Psi^{-1}r)$

where $\Omega = \Phi^{-1} \otimes X'X + R'\Psi^{-1}R$. β^{TG} has covariance matrix Ω^{-1}.

Mittelhammer and Conway argue that this formulation has led to some confusion with respect to the estimator, in that r is random in (3.1) but seems to be more sensibly thought of as a fixed, known constant in (3.2). They argue that researchers who have used this method have implicitly assumed that r was distributed with mean ω and variance Ψ so that the actual estimator used is:

(3.3) $$\beta^{TGM} = \Omega^{-1}((\Phi^{-1} \otimes X')y + R'\Psi^{-1}\omega)$$

which has variance covariance matrix $\Omega^{-1} (\Phi^{-1} \otimes X'X) \Omega^{-1}$. Under this formulation, Mittelhammer and Conway show that the covariance matrix of β^{TG} exceeds that of β^{TGM} by a positive definite matrix. Related to this result, β^{TG} is inadmissible because it has higher expected quadratic loss than β^{TGM}. β^{TGM} has a lower mean square error and is superior using the minimax criterion for average risk.

In the following we shall assume $\omega = 0$. In this case, the restricted IFZP estimator corresponds to $\Psi = 0$ and the unrestricted estimator corresponds to $\Psi^{-1} = 0$. Ilmakunnas (1986) shows that a sensitivity analysis can be based on choosing alternative values of Ψ. For simplicity, he assumes that Ψ is a diagonal matrix, $\Psi = \sigma_v^2\Psi^*$. In our implementation (which we interpret in the spirit of Mittelhammer and Conway), we follow Ilmakunnas and make the simplifying assumption that $\Psi^* = 10^{-k}I_q$, where σ_v^2, which could be any positive number, is set equal to 1.13 for reasons given below. We then choose a value of k algebraically small enough to give values very close to the unrestricted estimator. This turns out to be k = 1 in this case. We also choose a value of k large enough to give values very close to the restricted estimator; this turns out to be k = 9. The sensitivity analysis just involves varying k from 1 to 9, where we choose k = 1, 2, 3,...,9. The reason we chose $\sigma_v^2 = 1.13$ is that the OLS estimate of Φ, which is the starting estimate for the IFZP estimator, has diagonal elements which average to 1.13×10^{-5}. Therefore k = 5 might arguably be of special interest, because in some weak sense it assigns almost equal weight to the parameter restrictions and the data (as it equalizes the variance postulated for the restrictions, the diagonal elements of Ψ, to an estimate of the variance of the disturbances, the average of the diagonal elements of an estimate of Φ).

Our sensitivity analysis is in fact slightly different than this (and that of Ilmakunnas). We wish to consider homogeneity and symmetry restrictions separately. When we wish to consider a sensitivity analysis for the first three homogeneity restrictions with the latter three restrictions of symmetry imposed, we choose $\Psi^* = \text{diag}\ (10^{-k},10^{-k},10^{-k},10,^{-10},10^{-10},10^{-10})$, effectively assuming that the symmetry restrictions hold exactly while testing for sensitivity to the homogeneity restriction. When we wish to impose homogeneity and do the sensitivity for the symmetry restrictions we choose $\Psi^* = \text{diag}\ (10^{-10},10^{-10},10^{-10},10^{-k},10^{-k},10^{-k})$.

4. Empirical Results

The sensitivity analyses for the homogeneity restrictions and for the symmetry restrictions are given in Tables 1 and 2 respectively. Table 1 shows that for the homogeneity restriction, the estimates of the Allen partial elasticities of substitution tend to be insensitive to the choice of k, for the energy-capital substitution elasticity estimate as well as the other estimates. Table 2 shows that some of the cross-elasticities are very sensitive to the degree of belief in the symmetry restriction. In particular, the estimated Allen partial elasticity of substitution of capital for energy (σ_{KE}, based on the capital share equation estimates and not σ_{EK}, which is based on the energy share equation estimates) varies from -3.28 to 10.84 i.e. from indicating strong complementarity to indicating strong substitutability.

There are also similar sign changes for the materials-capital elasticity, the capital-materials elasticity and for the materials-energy elasticity. It appears as if the conclusions about the cross-elasticity estimates in Berndt and Wood are sensitive to the imposition of the symmetry restriction, and that it is difficult to sign these estimates without either strongly believing or strongly disbelieving in the appropriateness of the application of the symmetry restriction from microeconomic theory. Hence we support the finding of Ilmakunnas (1986) but show further that the results are sensitive to the degree of belief in the symmetry restriction and not that in the homogeneity restriction.

In Table 3 we look at bootstrap estimates of the entire sampling distribution of the estimates of σ_{KM}, σ_{KE} and σ_{KL}. (These are representative of other detailed results not presented here.) The bootstrap is used because the Allen partial elasticities of substitution are nonlinear functions and we desire to guard against inferential problems due to potential nonnormality or the relatively small sample (e.g. see Efron (1982), Fiebig and Theil (1983) and Freedman and Peters (1984) Vinod and Raj (1988) and Raj (1992, 1992a)). In any case, it is clear from the table that while the mean of the estimates varies with k, for each choice of k the sampling variance of the estimates is very small. (While not strictly comparable, Krinsky and Robb (1991) calculate similarly-sized standard errors for ordinary demand elasticities and cross-elasticities for the restricted version of this model with these same data). Moreover, the shapes and degrees of dispersion of the distribution change very little with k and tend to be very close to the normal distribution as illustrated with Figures 1a, 1b and 1c. In all cases, the null hypothesis of normality is not rejected at the 5 per cent level, using the test of Jarque and Bera (1980). It is perhaps interesting to note that for the intermediate value k=5, both the estimated 90% and 95% confidence bounds for the σ_{KE} only include positive values (which would imply energy and capital are substitutes). The bottom line is that the bootstrap experiment has revealed that for this model and data, the size of k matters because of its effects on the point estimates of the coefficients, not on the degree of variation or the distribution of the sampling error. The Ilmakunnas result that the degree of belief in the theoretical restrictions is of paramount importance for inference is sustained.

5. Concluding Remarks

This paper discusses the Ilmakunnas (1986) framework for undertaking a sensitivity analysis on linear restrictions using modified Theil-Goldberger mixed estimation. The example comes from the well known Berndt and Wood (1975) study which concluded from estimates based on U.S. manufacturing aggregate time series data that energy and capital were complements. Our sensitivity analysis shows that this result depends very much on the researcher's degree of belief in the symmetry restriction that Berndt and Wood imposed in their estimation of the factor demand equations. However a bootstrap technique shows that this sensitivity is almost entirely in the mean of the sampling distribution of the estimates of the Allen partial elasticities of substitution and that the shape and dispersion of the distribution are not sensitive.

Acknowledgements.

Baldev Raj wishes to acknowledge financial support in part from the Natural Sciences and Engineering Research Council of Canada; the Norwegian Research Council of Sciences and Humanities, and the Research Office at Wilfrid Laurier University. Michael Veall acknowledges the financial support of the Social Sciences and Humanities Research Council of Canada.

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TABLE 1

Point Estimates of Allen Cross Elasticity of Substitution:

A Sensitivity Analysis on the Homogeneity Postulate

Elasticity \ k		1	2	3	4	5	6	7	8	9
1.	σ_{KL}	0.96	0.96	0.94	0.91	0.85	0.93	0.97	0.97	0.97
2.	σ_{LK}	same as above								
3.	σ_{EK}	-3.94	-3.94	-3.92	-4.01	-3.93	-3.46	-3.30	-3.28	-3.28
4.	σ_{KE}	same as above								
5.	σ_{KM}	0.45	0.45	0.47	0.53	0.62	0.49	0.44	0.43	0.43
6.	σ_{MK}	same as above								
7.	σ_{EL}	1.25	1.25	1.24	1.18	0.99	0.73	0.65	0.64	0.64
8.	σ_{LE}	same as above								
9.	σ_{LM}	0.33	0.33	0.35	0.43	0.55	0.58	0.59	0.59	0.59
10.	σ_{ML}	same as above								
11.	σ_{EM}	0.45	0.45	0.47	0.54	0.70	0.82	0.85	0.85	0.85
12.	σ_{ME}	same as above								
13.	σ_{LL}	-1.41	-1.41	-1.42	-1.49	-1.59	-1.63	-1.63	-1.63	-1.63
14.	σ_{KK}	-7.08	-7.08	-7.06	-7.22	-7.36	-7.33	-7.31	-7.31	-7.31
15.	σ_{EE}	-15.62	-15.62	-15.62	-15.54	-14.77	-12.79	-12.12	-12.03	-12.02
16.	σ_{MM}	-0.42	-0.42	-0.41	-0.37	-0.35	-0.34	-0.36	-0.36	-0.36

TABLE 2

Point Estimates of Allen Cross Elasticity of Substitution:

A Sensitivity Analysis on the Symmetry Postulate

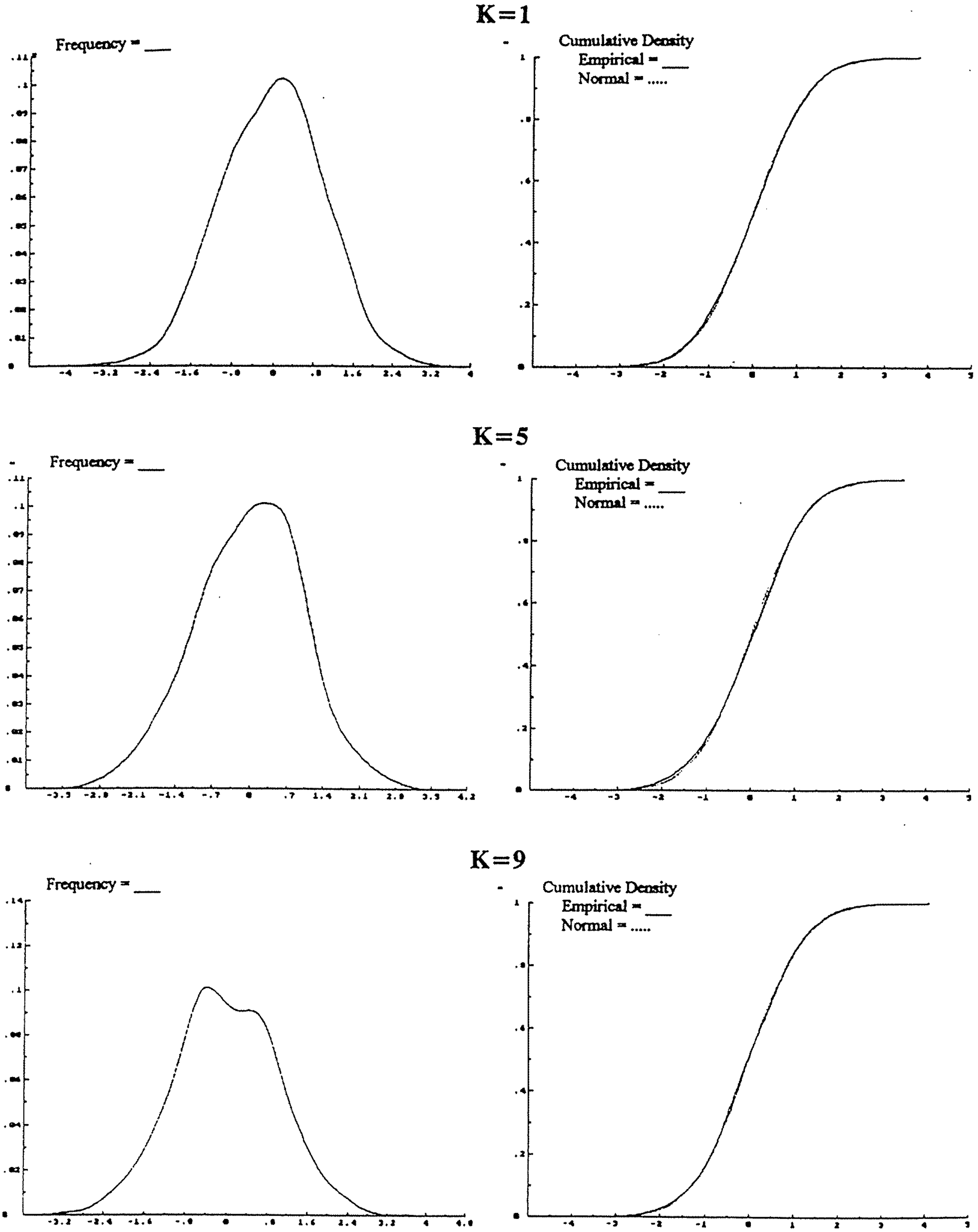
Elasticity	k	1	2	3	4	5	6	7	8	9
1.	σ_{KL}	1.28	1.28	1.27	1.19	1.03	0.98	0.98	0.98	0.98
2.	σ_{LK}	1.99	1.98	1.95	1.73	1.27	1.02	0.98	0.98	0.98
3.	σ_{EK}	-2.22	-2.22	-2.25	-2.44	-2.92	-3.22	-3.28	-3.28	-3.28
4.	σ_{KE}	10.84	10.80	10.36	7.29	0.19	-2.81	-3.23	-3.27	-3.28
5.	σ_{KM}	-0.87	-0.87	-0.83	-0.55	0.10	0.39	0.43	0.43	0.43
6.	σ_{MK}	-0.25	-0.21	0.00	0.21	0.38	0.42	0.43	0.43	0.43
7.	σ_{EL}	0.82	0.82	0.82	0.77	0.67	0.64	0.64	0.64	0.64
8.	σ_{LE}	5.57	5.54	5.28	3.59	1.12	0.68	0.65	0.64	0.64
9.	σ_{LM}	0.10	0.11	0.13	0.29	0.53	0.59	0.59	0.59	0.59
10.	σ_{ML}	0.68	0.68	0.66	0.61	0.57	0.57	0.58	0.59	0.59
11.	σ_{EM}	0.08	0.68	0.11	0.29	0.68	0.83	0.85	0.86	0.86
12.	σ_{ME}	-3.15	-3.12	-2.94	-1.76	0.20	0.76	0.85	0.85	0.86
13.	σ_{LL}	-1.52	-1.52	-1.52	-1.53	-1.57	-1.63	-1.63	-1.63	-1.63
14.	σ_{KK}	-5.46	-5.46	-5.51	-5.82	-6.66	-7.21	-7.30	-7.31	-7.31
15.	σ_{EE}	-3.52	-3.55	-3.84	-5.57	-10.14	-11.78	-12.00	-12.02	-12.02
16.	σ_{MM}	-0.02	-0.02	0.00	-0.11	-0.29	-0.35	-0.36	-0.36	-0.36

TABLE 3

The Bootstrap Distribution of Some Estimates of the Allen Cross Elasticity of Substitution: Alternative Summary Statistics Based on 1000 Samples (A Sensitivity Analysis on the Symmetry Postulate)

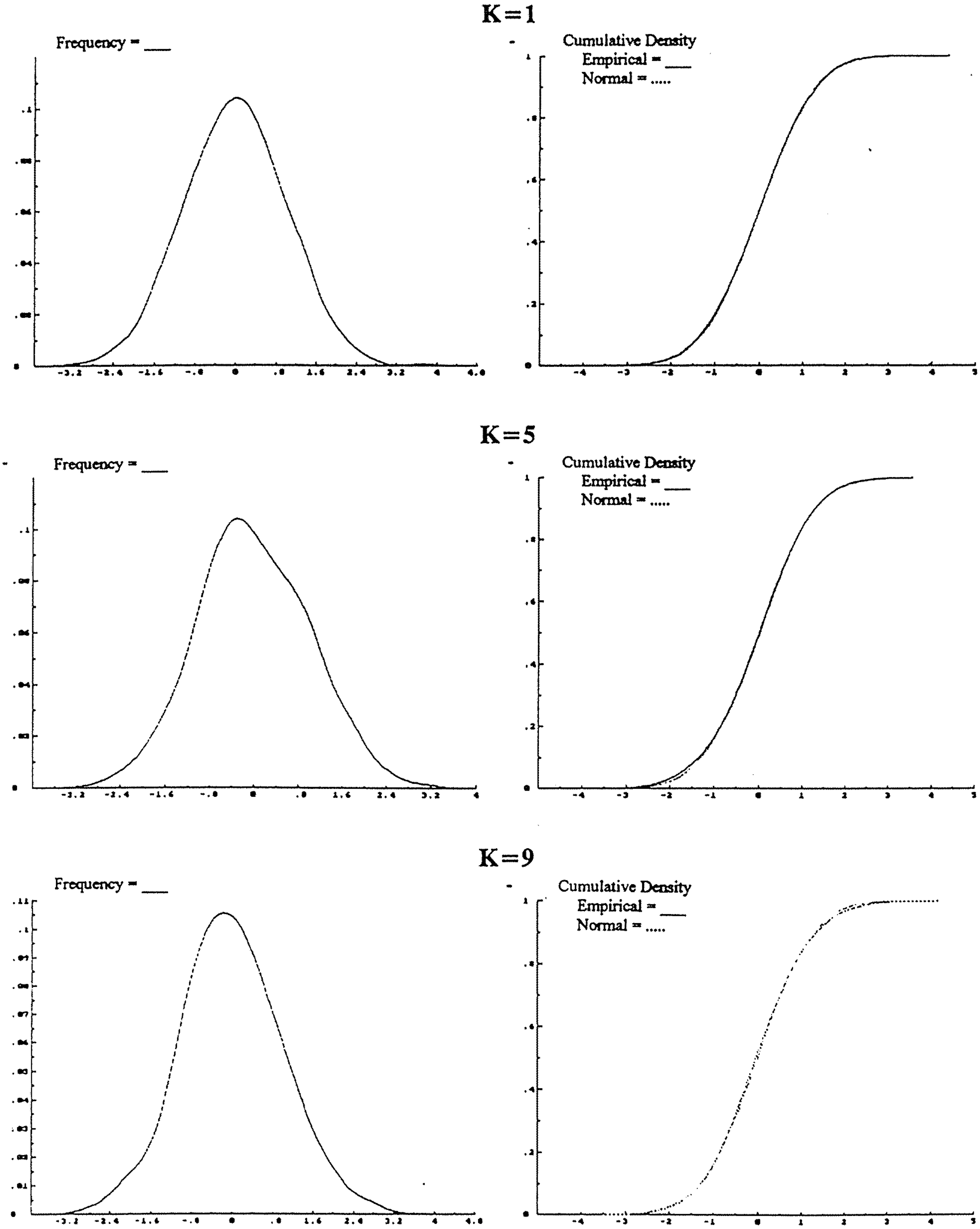
Statistic k	Mean (1)	Standard Deviation (2)x10^{-2}	Lower Tail 2.5% (3)	Lower Tail 5% (4)	Median (5)	Upper Tail 5% (6)	Upper Tail 2.5% (7)
				σ_{KM}			
1	-0.8701	0.2155	-0.8742	-0.8737	-0.8701	-0.8667	-0.8661
4	-0.5487	0.2337	-0.5535	-0.5526	-0.5487	-0.5450	-0.5443
5	0.1036	0.2907	0.0976	0.0986	0.1038	0.1082	0.1093
6	0.3858	0.3134	0.3792	0.3804	0.3859	0.3908	0.3915
9	0.4304	0.1925	0.4266	0.4271	0.4303	0.4335	0.4341
				σ_{KE}			
1	10.8421	2.2300	10.7991	10.8063	10.8418	10.8782	10.8854
4	7.2944	2.2330	7.2511	7.2576	7.2936	7.3344	7.3416
5	0.1842	2.2935	0.1293	0.1373	0.1824	0.2339	0.2416
6	-2.8094	3.3210	-2.8687	-2.8600	-2.8101	-2.7553	2.7395
9	-3.2754	1.6731	-3.3088	-3.3023	-3.2760	-3.2471	-3.2411
				σ_{KL}			
1	1.2775	0.1197	1.2751	1.2754	1.2774	1.2793	1.2798
4	1.1945	0.1273	1.1920	1.1924	1.1945	1.1965	1.1969
5	1.0299	0.1522	1.0267	1.0273	1.0299	1.0323	1.0327
6	0.9806	0.1778	0.9771	0.9776	0.9806	0.9835	0.9841
9	0.9749	0.1471	0.9719	0.9724	0.9750	0.9774	0.9777

Figure 1a. The empirical density and the cumulative (with the standard normal) distribution functions of $\hat{\sigma}_{KM}$



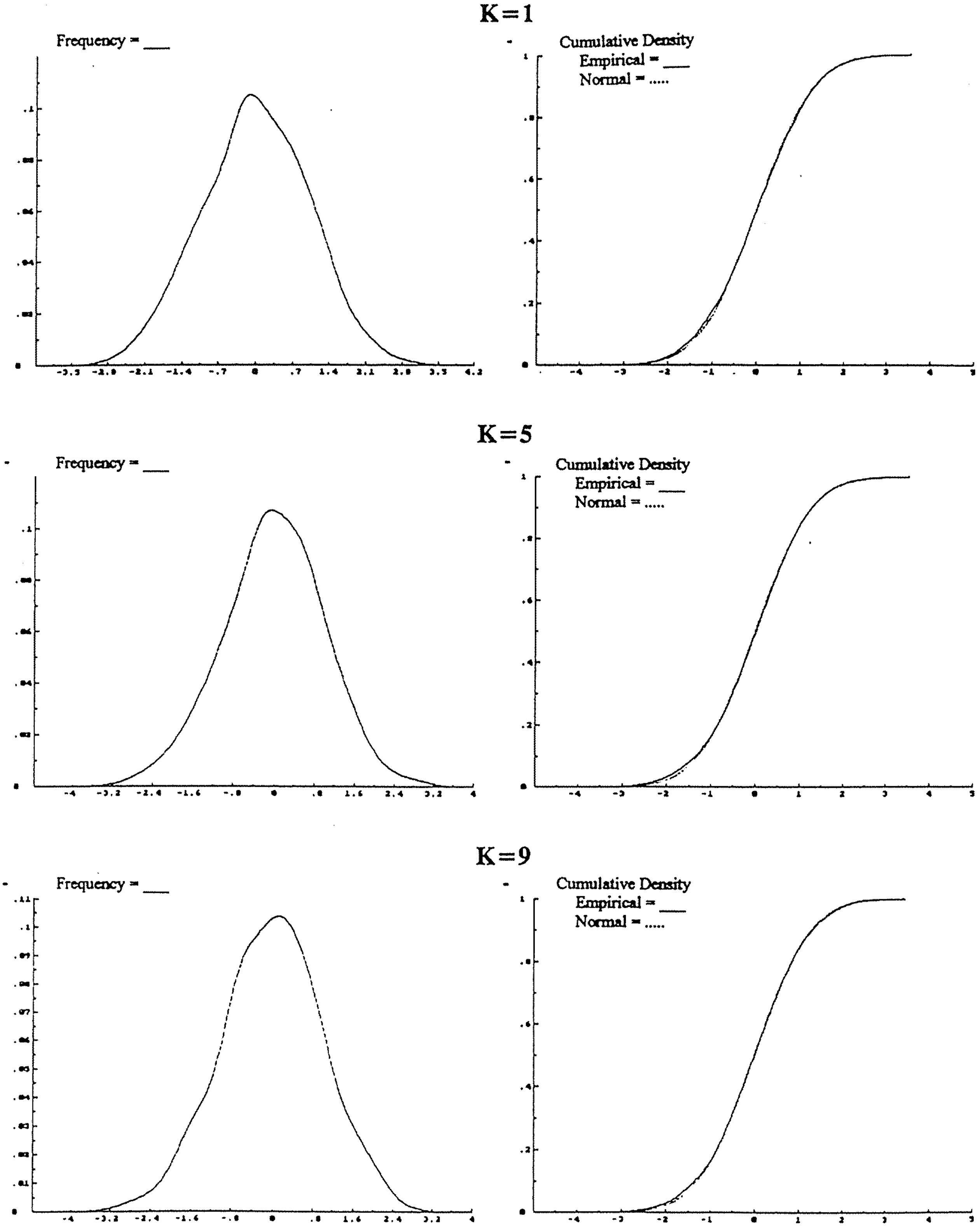
The panels on the left are empirical densities of the bootstrap estimates, smoothed using the technique of Silverman (1986). The panels on the right are empirical cumulative distribution functions of the bootstrap estimates, plotted along with the virtually indistinguishable normal cumulative distribution function.

Figure 1b. The empirical density and the cumulative (with the standard normal) distribution functions of $\hat{\sigma}_{KE}$



The panels on the left are empirical densities of the bootstrap estimates, smoothed using the technique of Silverman (1986). The panels on the right are empirical cumulative distribution functions of the bootstrap estimates, plotted along with the virtually indistinguishable normal cumulative distribution function.

Figure 1c. The empirical density and the cumulative (with the standard normal) distribution functions of $\hat{\sigma}_{KL}$



The panels on the left are empirical densities of the bootstrap estimates, smoothed using the technique of Silverman (1986). The panels on the right are empirical cumulative distribution functions of the bootstrap estimates, plotted along with the virtually indistinguishable normal cumulative distribution function.

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