ENVIRONMENTAL TAX REFORM AND ENDOGENOUS GROWTH

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Abstract

This paper explores the effects of an environmental tax reform on pollution, economic growth and welfare in an endogenous growth model with pre-existing tax distortions. We find that a shift in the tax mix away from output towards pollution may raise economic growth through two channels. The first channel is an environmental production externality, which determines the positive effect of lower aggregate pollution on the productivity of capital. The second channel is a shift in the tax burden away from the net return on investment towards profits. The paper also shows that, if taxshifting towards profits is large and environmental amenities are unimportant, the optimal tax on pollution may exceed its Pigovian level.

1 Introduction

The link between economic growth and environmental policy is a controversial issue. Environmentalists argue that a more ambitious environmental policy is required to ensure that growth is sustainable. Others, in contrast, maintain that society faces a trade-off between, on the one hand, a higher level of growth, and, on the other hand, a better quality of the environment. In this connection, environmental taxes seem a particularly attractive instrument to enhance environmental quality without seriously damaging growth prospects. In particular, by increasing taxes on dirty activities and using the revenues to cut distortionary taxes on income, governments may be able to reap a 'double dividend', namely, not only a cleaner environment but also a less distortionary tax system, thereby stimulating economic growth. The literature has explored the double dividend issue only in a static framework (see e.g. Bovenberg and Van der Ploeg (1994), Bovenberg and De Mooij (1994b)). We, in contrast, explore the link between environmental externalities and distortionary taxes in a dynamic model of endogenous growth.

Our paper is also closely related to the literature on pollution and long-term growth (see e.g. Bovenberg and Smulders (1993), Den Butter and Hofkes (1993), Gradus and Smulders (1993a,b), Ligthart and Van der Ploeg (1993), Hofkes (1993), Van Marrewijk, Van der Ploeg and Verbeek (1993), Verbeek (1993)). This literature focuses on optimal environmental policies and explores how the social optimum can be sustained in a decentralized economy. Our analysis departs from this first-best world in two major ways. First, public policy is not necessarily optimal. In particular, we explore the consequences of a reform of the tax system, starting from a sub-optimal initial equilibrium. Second, in addition to market failures associated with environmental externalities, we allow for tax distortions due to the absence of lump-sum taxation. Hence, the interaction between environmental externalities and distortionary taxes is explored in a second-best world. In this connection, we not only investigate the consequences of an environmental tax reform but also show how, in the presence of distortionary taxes, the optimal environmental tax deviates from the Pigovian tax.

We find that an environmental tax reform harms growth if two conditions are met. First, the positive externality of a better environmental quality on the productivity should be small compared to the production elasticity of pollution as a rival input into production. Second, substitution between pollution and other inputs should be rather easy. In these two conditions are met, the government faces a trade-off between environmental care and economic growth. If substitution between pollution and other inputs is difficult, pollution taxes are less powerful in cutting pollution. At the same time, however, they are a more effective device to tax the quasi-rents from pollution, thereby generating revenues to reduce the distortionary tax on output. Accordingly, an environmental tax reform may raise growth by improving the efficiency of the tax system as a revenue raising device. A double dividend may emerge also if the production externality is relatively large.

In a second-best world with distortionary taxes, the optimal environmental tax generally deviates from the Pigovian tax. We show that the optimal environmental tax lies below the Pigovian level if substitution between pollution and capital is relatively easy. However, if substitution is more difficult, the optimal pollution tax may exceed the Pigovian level, but only if environmental amenities are small.

We explore also the impact on growth of alternative instruments of environmental policy. In particular, we compare the effects of pollution taxes with environmental policy instruments that do not charge any price for pollution, such as freely issued pollution permits. We demonstrate that, compared to freely issued pollution permits, pollution taxes benefit growth if other inputs are poor substitutes for pollution. In that case, the base of pollution taxes is relatively inelastic so that pollution taxes are relatively efficient instruments for raising revenues. However, if pollution is easily replaced by other inputs, pollution taxes are worse for growth than freely issuing pollution permits. Intuitively, with easy substitution, pollution taxes are a relatively inefficient instrument to raise revenues because they generate a large negative impact on the after-tax return to investment.

The rest of this paper is organized as follows. Section 2 discusses the model. Section 3 explores the effects of various policy measures on growth, environmental quality, and welfare for the case of a Cobb-Douglas production function. The case with small substitution possibilities between capital and pollution is investigated in Section 4. Section 5 discusses the effects of environmental policy instruments that do not charge a price for pollution. Finally, section 6 concludes.

2 The model

This section presents the dynamic model for a closed economy. The model equations are contained in table I. Notation is explained at the end of the table. Subsection 2.1 elaborates the structure of the model, while subsection 2.2. discusses the linearized model.

2.1 Structure of the model

Production function

By using production technology (I.1), firms combine four factors of production to produce output (Y) (see figure 1). On the one hand, physical capital (K) and productive government spending (S), which can be thought of as infrastructure, are combined to produce an intermediate input, M. The function m(K,S) exhibits constant returns to scale with respect to the two inputs. On the other hand, by combining pollution (P)[1] and abatement (A), firms produce intermediate input, N. The function n(A,P) features constant returns with respect to abatement which, in contrast to pollution, is growing on a balanced growth path (see below). Combining the two intermediate inputs, N and M, by using a CRS production function, firms produce intermediate output. Together with the services from the environment (E), this intermediate output yields actual output (Y).[2]

For balanced growth to be feasible, the production function needs to meet a number of restrictions. First, the production function should exhibit constant returns with respect to the growing inputs:[3]

$$\delta + \beta + \gamma = 1 \tag{2.1}$$

Here δ, β and γ are the production elasticities of the growing inputs S, K and A, respectively.

The second condition for balanced growth is that the production elasticities of the various inputs at the left hand side (LHS) of (2.1) as well as the production elasticity of pollution ($\alpha\gamma$) remain constant over time. This imposes restrictions on the substitution possibilities between the various inputs. In particular, on a sustainable growth path the flow of pollution (P) is constant while abatement grows at the same rate as output. In the face of these diverging growth rates, the

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[1] Pollution can be interpreted as a polluting input with unlimited supply and zero extraction costs. Emissions are proportional to the use of this input.

[2] Pollution, P, can be viewed as the extractive, rival use of the environment. At the same time, the environment, E, yields non-extractive, non-rival services as an input into production.

[3] In the endogenous growth literature, this restriction is known as the 'core' property (see Rebelo (1991)).

production elasticities of pollution and abatement (i.e. $\alpha\gamma$ and γ) remain constant only if the elasticity of substitution between the non-growing input, P, and the growing input, A, equals unity. In particular, if substitution would be too easy (i.e. the substitution elasticity > 1), the production elasticity of pollution would approach zero as the growing input, abatement, would crowd out the non-growing input, pollution. If substitution would be too difficult (i.e. the substitution elasticity < 1), in contrast, the production elasticity of abatement would go to zero because the marginal product of abatement would fall substantially as the abatement-pollution ratio rises. For the same reasons, the elasticity of substitution between intermediate output, which grows on the balanced growth path, and the quality of the environment, which remains constant, should equal unity. Finally, the substitution elasticity between K and S, denoted by σ_m, as well as the substitution elasticity between M and N, denoted by σ_y, should be constant. However, these elasticities are allowed to differ from unity as the arguments in m and f (i.e. K, S and M, N, respectively) grow at the same constant rate. Consequently, the ratios K/S and M/N, which determine the production elasticities, are fixed on the balanced growth path.

Firm behavior

The model describes a decentralized economy. A representative firm maximizes its value (I.2) with respect to abatement, pollution and investment. It ignores the environmental production externality. The value of the firm amounts to the present value of all future dividends, represented by (I.3). T_y stands for a tax on output, T_a represents an abatement subsidy and T_p denotes a pollution tax. The interest rate, denoted by r, is constant on a balanced growth path. q in (I.4) is defined as the value of the firm per unit of capital. We abstract from depreciation of capital. Hence, the accumulation of capital ($\dot{K}$) equals gross investment (I) (see (I.5)). Profits in (I.6) are defined as dividends plus investment minus capital costs. Solving the maximization problem of the firm, we find the implicit demand equations for private inputs (I.7), (I.8) and (I.9).[4] These expressions reveal that firms equalize the marginal productivity of private inputs to their respective producer prices.[5]

[4] Productive government spending, S, is a public input and is determined by the government.

[5] The value of the firm per unit of capital, q, can be written as: $q = 1 + \frac{\delta - \alpha\gamma}{\beta}\frac{r}{r-\pi}$. q should be larger than unity to rule out negative firm profits (i.e. $s_w = (1 - T_y)(\delta - \alpha\gamma) > 0$). This requires that the production elasticity of pollution at the firm level ($\alpha\gamma$) should not exceed the production elasticity of public investment (δ). Since production features constant returns with respect to the growing inputs, S, A and K (see (2.1)), the non-negativity condition for profits guarantees that the production function does not exhibit increasing returns with respect to the 'private' inputs P, A and K. Merger across firms is thus not beneficial and a competitive equilibrium can be sustained. In equilibrium, positive profits can exist due to the presence of a fixed factor in production (e.g. land, know how, managerial talent).

Household Behavior

A representative household maximizes an intertemporal utility function (I.10) subject to a dynamic budget constraint (I.11). Three arguments enter instantaneous utility: private consumption (C), non-productive government services (G), and the quality of the environment (E) (see figure 2). To ensure that growth is balanced, sub-utility H(C,G) in (I.10) is homothetic. Moreover, the substitution elasticity between the growing variable, H, and the quality of the environment, E, must be unity to ensure that sustainable growth is optimal. Maximizing household utility subject to the household budget constraint, we find Keynes-Ramsey rule (I.12).[6]

Government

The government budget is balanced according to (I.13). The government does not issue public debt and raises revenues by adopting a tax on output (T_y) and a tax on pollution (T_p). The revenues are used to finance three types of government spending: public investment (S), public consumption (G) and abatement subsidies (T_a). The government balances its budget at each point in time by adjusting the tax rate on output.

Environmental Quality

Relation (I.14) formalizes the inverse relationship between the quality of the environment and the flow of pollution.[7] Environmental quality is assumed to be above a certain ecological threshold level in the initial equilibrium. Furthermore, we assume that $\lim_{P\to\infty} j[e(P)] = 0$, so that pollution should remain bounded for production to remain possible. Hence, on a balanced growth path, pollution cannot grow at a positive rate.

Walras Law: equilibrium conditions and budget constraints

The model describes a closed economy. The equilibrium condition on the goods market (I.15) can be found by combining (I.2), (I.3), (I.4), (I.5), (I.11) and (I.13) for the firm value, dividends, q,

[6] For the steady state to be meaningful, the consumption share of output should be non-negative. This condition ensures also that utility is bounded. Dividing (I.11) by Y and using the fact that $s_i = \pi s_k$, we derive: $s_c = (r - \pi) s_k q$. Hence, the growth rate, π, should not exceed the interest rate. This condition restricts the rate of time preference (θ) and the intertemporal elasticity of substitution (σ). In particular, by using (I.12), we find that a non-negative consumption share requires that: $\theta\sigma + (1-\sigma) r \geq 0$. Hence, the intertemporal substitution possibilities cannot be too large, especially if the rate of time preference is small.

[7] Alternatively, the quality of the environment could be modelled as a stock rather than as a flow (see e.g. Bovenberg and Smulders (1993)). However, this would complicate the model substantially as it would imply a second state variable, thereby introducing transitional dynamics. More importantly, the qualitative steady-state results do not change if a stock, rather than a flow, determines the external effects (see Gradus and Smulders (1993b)). Hence, if one is interested in long-term effects, the assumption of a flow of pollution involves no loss of generality.

investment, the household budget constraint and the government budget constraint, respectively.

Sustainable Balanced Growth

Most variables in the economy grow endogenously at the same constant rate. This situation is known as balanced economic growth. Growth is also sustainable because pollution and, therefore, environmental quality remain constant on the balanced growth path. In order to keep pollution from growing, the pollution tax grows at the same rate as output (Y) and the other inputs (K,A and S). Transitional dynamics in the model are absent. Hence, after a policy shock the economy moves immediately towards a new steady state in which ratios between the growing variables remain constant.

2.2 Linearized model

To solve the model, we log-linearize it around a steady state. Appendix A derives the log-linearized factor demand equations. Table II contains the log-linearized model. Notation is explained at the end of the table. A tilde (~) denotes a relative change, unless indicated otherwise. Relative changes in the growing variables are presented relative to the relative change in the capital stock and are presented as lower case variables. Hence, they represent relative changes of variables as a ratio of the capital stock (which is initially predetermined).

Inputs into production

Expression (II.2) for pollution reveals that a higher relative price of pollution induces substitution towards other inputs. The strength of the substitution effect towards either abatement or capital depends on the magnitude of σ_y. In particular, if $\sigma_y < 1$, substitution between the intermediate factors M and N is difficult relative to substitution between A and P. In that case, pollution and abatement are substitutes: a higher price for abatement increases pollution. However, if $\sigma_y > 1$, pollution is not a good substitute for abatement compared to other inputs. Therefore, a higher relative price for abatement increases pollution. Pollution can be affected also by public investment. In particular, lower public investment raises pollution if $\sigma_m < \sigma_y$. In that case, pollution is a better substitute for public investment than capital. The demand for abatement rises with the level of pollution (see (II.3)). Furthermore, an increase in the price of pollution in terms of abatement induces substitution away from pollution towards abatement.

Growth and Consumption

Relations (II.8) and (II.9) reveal that the economy grows faster if and only if the interest rate increases. This is because a higher interest rate is associated with a higher after-tax return to capital which stimulates savings and investment. According to (II.10), consumption rises if the value of the firm increases. Besides, consumption is affected by the income and intertemporal substitution effects associated with changes in the interest rate. In particular, if the intertemporal elasticity of substitution (σ) exceeds unity, the intertemporal substitution effect dominates and consumption declines on account of a higher interest rate.

Welfare

The welfare effects of small policy changes can be measured by the marginal excess burden. It amounts to the transfer that needs to be provided to the household to keep utility, after the policy shock, at its initial level. Hence, a negative value for the marginal excess burden corresponds to a rise in welfare. The marginal excess burden, derived in appendix B as the compensating variation divided by consumption (i.e. $\tilde{\lambda} = d\lambda / C$), amounts to:

$$\begin{aligned} \tilde{\lambda} = & - \left[\frac{\alpha\gamma - \epsilon\eta}{s_c} - \frac{\epsilon\phi}{1-\psi} \right] \tilde{P} - \frac{T_y - T_a}{1 - T_a} \frac{\gamma}{s_c} \left[\tilde{a} + \frac{\pi}{r-\pi} \tilde{\pi} \right] \\ & - \frac{\delta - s_s}{s_c} \left[\tilde{s} + \frac{\pi}{r-\pi} \tilde{\pi} \right] - \left[\frac{\psi}{1-\psi} - \frac{s_g}{s_c} \right] \left[\tilde{g} + \frac{\pi}{r-\pi} \tilde{\pi} \right] - \frac{T_y \beta}{s_c} \frac{\pi}{r-\pi} \tilde{\pi} \end{aligned} \qquad (2.2)$$

All coefficients at the RHS of (2.2) represent wedges between the marginal social costs and the marginal social benefits of an activity. If benefits exceed costs at the margin, an increase in such activity enhances welfare.[8] For the growing variables A, S, G and K we see that not only changes in current levels matter for welfare, but also the present value of future changes (indicated by the change in the growth rate).

The first term at the RHS of (2.2) stands for the welfare effect of a change in pollution, which is a non-growing variable. First, as a private input into production, pollution exerts a positive effect on output which is represented by $\alpha\gamma$. Second, by worsening environmental quality, pollution imposes an adverse production externality, captured by $\epsilon\eta$. The overall effect on productivity is positive only if $\alpha\gamma - \epsilon\eta > 0$, i.e. if the positive output effect of additional pollution as a rival input for the individual firm exceeds the negative external (non-rival) productivity effect of aggregate pollution. Finally, pollution yields an adverse consumption externality (see the second

[8] Note that these terms represent partial welfare effects as relation (2.2) is not a reduced form.

term between the first square brackets).

The second term at the RHS of (2.2) reveals that higher current or future abatement improves welfare if the output tax, which acts as an implicit tax on abatement, exceeds the abatement subsidy. Hence, more abatement benefits welfare only if it is taxed on a net basis.

The partial welfare effect of higher public investment is captured by the third term at the RHS of (2.2). A rise in public investment improves welfare if the productivity effect of public investment exceeds the share of public investment in output. This condition implies that the marginal product of public investment is larger than unity, i.e. $dY/dS > 1$ or, alternatively, that the marginal social benefits, dY, exceed the marginal social costs, dS.

The effect of higher current or future public consumption on welfare is represented by the fourth term at the RHS of (2.2). On the one hand, higher public consumption increases welfare on account of its effect on household utility. On the other hand, public spending crowds out private income as it is financed through taxation.

A higher growth rate affects welfare if a particular growing variable features a gap between its marginal social costs and -benefits. In particular, the last term at the RHS of (2.2) represents the welfare impact of capital accumulation. The tax on output drives a wedge between, on the one hand, the before-tax return to capital (which stands for the marginal social benefits of capital accumulation) and, on the other hand, the after-tax return to capital (which measures the marginal social costs of capital accumulation). Hence, if the output tax is positive a higher rate of capital accumulation improves welfare because the benefits in terms of higher future output exceed the costs in terms of current consumption foregone.

3 Effects on growth, pollution and welfare

This section explores the economic, ecological and welfare effects of various policies. In particular, we examine the effects of higher pollution taxes and abatement subsidies. We explore also how changes in productive- and non-productive government spending affect the steady state. Appendix C solves the model from table II for the special case that $\sigma_m = \sigma_y$. This section discusses the results for a Cobb-Douglas production function (i.e. $\sigma_m = \sigma_y = 1$). Subsection 3.1

explores the effects on pollution and growth. The welfare effects are investigated in subsection 3.2. Section 4 examines the more general case where σ_y and σ_m are allowed to differ from unity.

3.1 Effects on growth and environment

The reduced-form equation for the growth rate is given by:

$$\tilde{\pi} = \frac{1}{\Delta}\frac{r}{r-\theta}(\tilde{X} - \tilde{H}) \tag{3.1}$$

where $\Delta = \beta + (\delta - s_s) + (\epsilon\eta - \alpha\gamma) - s_g > 0$.[9] The first term between round brackets at the RHS of (3.1) is called the 'productivity effect' and is defined as:

$$\tilde{X} = -(\alpha\gamma - \epsilon\eta)\tilde{t}_p + \gamma\tilde{T}_a + \delta\tilde{s} \tag{3.2}$$

A positive productivity effect corresponds to a rise in the before-tax (or gross) marginal product of capital and, therefore, boosts the growth rate. The second term at the RHS of (3.1) represents the so-called 'public burden effect' and is defined as:

$$\tilde{H} = s_a\tilde{T}_a + s_s\tilde{s} + s_g\tilde{g} \tag{3.3}$$

By raising the tax on output, a larger public burden reduces the after-tax marginal product of capital, thereby harming growth.

The solutions for pollution and abatement look as follows:

$$\tilde{P} = -\tilde{t}_p + \frac{1}{\Delta}(\tilde{X} - \tilde{H}) \tag{3.4}$$

[9] The determinant, Δ, can be interpreted as net household income. It consists of four elements. The first term indicates income from the privately owned capital stock (β). The second term (i.e. δ-s_s) represents net benefits from public investment. The third term (i.e. $\epsilon\eta$-$\alpha\gamma$) amounts to the net benefits from a lower level of pollution. The final term (i.e. s_g) stands for the public claim on private income aimed at financing public consumption. Alternatively, the determinant can be interpreted as the tax base of the public sector, i.e. the before-tax household income minus the tax burden.

$$\tilde{a} = \tilde{T}_a + \frac{1}{\Delta}(\tilde{X} - \tilde{H}) \tag{3.5}$$

The reduced-form for the output tax is given by:

$$\tilde{T}_y = - [s_s + s_g] \frac{1}{\Delta} \tilde{X} + [\beta + \delta + (\epsilon\eta - \alpha\gamma)] \frac{1}{\Delta} \tilde{H} \tag{3.6}$$

The first term at the RHS of (3.6) reveals that, by expanding the tax base (i.e. $s_s + s_g$), productivity gains allow for a reduction in the output tax. The second term indicates that a larger public sector, associated with a larger public burden, requires a rise in T_y.

We now explore how each of the four exogenous policy variables affects the two dividends, namely, the growth rate and the quality of the environment.

Pollution taxes

Expression (3.4) reveals that a pollution tax exerts two effects on pollution. First, pollution decreases proportionally due to input substitution (i.e. the first term at the RHS of (3.4)). Hence, the *composition* of economic activity becomes cleaner. Second, changes in the growth rate affect pollution (see (3.1) and (3.2)). On account of this growth effect, the *level* of economic activity changes. Whether a higher pollution tax raises or reduces the growth rate depends on the difference between, on the one hand, the production elasticity of pollution, $\alpha\gamma$, and, on the other hand, the environmental externality in production, ϵ, multiplied by the elasticity that measures the effect of pollution on environmental quality, η (see (3.2)). In particular, if $\alpha\gamma - \epsilon\eta = 0$, a higher pollution tax yields no aggregate productivity effect because the negative effect of lower pollution on the before-tax return to capital ($\alpha\gamma$) is exactly offset by the positive effect on that return of a better environmental quality ($\epsilon\eta$). In this case, the endogenous tax rate on output, T_y, remains unaffected (see (3.6)). The reason is that the higher pollution tax does not yield any additional public revenues: the slope of the Laffer curve is zero. Since both the before-tax return and the output tax are unaffected, the after-tax return to capital, and hence growth, also remain unchanged. If $\alpha\gamma - \epsilon\eta > 0$, higher pollution taxes reduce the growth rate because the adverse input effect of lower pollution dominates the favourable externality effect, thereby reducing the before-tax return to capital. In this case, the Laffer curve for pollution taxes is downward sloping. Consequently, the government requires a higher tax rate on output in order to meet its revenue constraint. If $\alpha\gamma - \epsilon\eta < 0$, higher pollution taxes raise the before- and after-tax returns to capital and thus stimulate economic growth, thereby allowing for a reduction in the tax rate on output. This result suggests that employing pollution taxes may yield a double dividend if the environmental externality is large. In that case, higher pollution taxes do not only improve the quality of the environment, but also raise tax revenues and boost economic growth. Previous analysis of the double dividend issue (see e.g. Bovenberg and De Mooij (1994b) and Bovenberg and Van der Ploeg (1994)) have abstracted from production externalities (i.e. $\epsilon\eta = 0$). In that case, a higher pollution tax always implies a net burden on the production sector and thus does not yield a double dividend (as $\alpha\gamma > 0$).

Abatement subsidies

In practice, governments face serious political and administrative obstacles in implementing pollution taxes. Therefore, they often employ alternative instruments, such as subsidies. In our model, subsidies on abatement have two effects on the demand for abatement. On the one hand, they increase the demand for abatement by a proportional amount (see (3.5)). On the other hand, they raise abatement through a higher growth rate. The growth effect depends on the difference between, on the one hand, the productivity effect of abatement subsidies in (3.2) and, on the other hand, the public burden effect in (3.3). This difference can be rewritten as:

$$(\gamma - s_a)\tilde{T}_a = \frac{T_y - T_a}{1 - T_a}\gamma\tilde{T}_a \tag{3.7}$$

A marginal increase in the abatement subsidy boosts economic growth only if the initial abatement subsidy is smaller than the output tax, which acts as an implicit tax on abatement. Accordingly, abatement raises the growth rate only if it is taxed rather than subsidized on a net basis. The effect of abatement subsidies on pollution is determined solely by their consequences for growth (see (3.4) and (3.1)). Hence, abatement subsidies improve the environment only if they harm growth so that they never yield a double dividend.

Public spending

Public investment, on the one hand, stimulates growth as it increases the before-tax return to capital (see (3.2)). On the other hand, it raises the public burden on the private sector, thereby reducing the after-tax return to capital and hence growth (see (3.3)). Public investment reduces pollution only if it harms growth. Hence, it never produces a double dividend. More public consumption always reduces both the growth rate and pollution; it does not raise gross productivity but reduces the after-tax marginal product of capital through a higher output tax.

3.2 Welfare effects

Armed with the economic and ecological effects, we now turn to the effects on welfare, defined by the marginal excess burden in (2.2). By substituting the reduced-form equations for growth, pollution and abatement from (3.1), (3.4) and (3.5) into (2.2), we derive the following solution for the marginal excess burden:

$$\tilde{\lambda} = - \frac{\psi}{1-\psi}\tilde{g} - \frac{\epsilon\phi}{1-\psi}\tilde{t}_p - [\frac{s_c + s_i}{s_c} - \frac{\epsilon\phi}{1-\psi}]\frac{1}{\Delta}(\tilde{X} - \tilde{H})$$
$$- \frac{\pi}{r-\pi}\frac{r}{r-\theta}[\frac{T_y - T_a}{1-T_a}\frac{\gamma}{s_c} + \frac{\delta - s_s}{s_c} + (\frac{\psi}{1-\psi} - \frac{s_g}{s_c}) + \frac{T_y\beta}{s_c}]\frac{1}{\Delta}(\tilde{X} - \tilde{H}) \quad (3.8)$$

The first two terms at the RHS of (3.8) capture the direct welfare effects of changes in public consumption and changes in environmental amenities due to pollution taxes. Apart from these direct effects, policies that raise the after-tax return to capital (i.e. $\tilde{X} - \tilde{H} > 0$) affect social welfare through their impact on household income, environmental amenities and growth. In particular, the third term at the RHS of (3.8) reveals that, on the one hand, a higher after-tax return to capital raises househould income available for consumption and savings (first term between square brackets). On the other hand, a higher after-tax return implies more production and, therefore, more pollution. This harms welfare through the environmental amenity effect (second term between square brackets). The fourth term at the RHS of (3.8) reveals that a higher after-tax return to capital impacts welfare by raising the growth rate (see (3.1)). This effect depends on the wedges between the marginal social costs and -benefits of the growing variables (i.e. A, S, G, and K respectively). The overall effect of a higher after-tax return to capital on welfare is assumed to be positive (i.e. the environmental amenity effect is dominated by the income effect and the growth effect).

Pollution taxes

The effect of higher pollution taxes on welfare depends on the difference between, on the one hand, the direct amenity effect of lower pollution as indicated by the second term at the RHS of (3.8) and, on the other hand, the social value of the change in the after-tax return to capital (i.e. the third and fourth terms at the RHS of (3.8)). If a lower pollution level raises the after-tax return to capital (i.e. $\alpha\gamma - \epsilon\eta < 0$), higher taxes on pollution always improve welfare as they yield a double dividend: they enhance both environmental amenities and productivity and, therefore, growth. If, however, the production externality ($\epsilon\eta$) is smaller than the production elasticity ($\alpha\gamma$), higher pollution taxes reduce the after-tax return to capital. In that case, a trade-off exists between, on the one hand, lower economic growth and, on the other hand, higher environmental amenities.

Public Consumption

Higher public consumption exerts two distinct effects on welfare. On the one hand, it increases welfare because it is an argument of the utility function. On the other hand, it raises the public burden, which negatively affects the after-tax return to capital and hence growth. However, in the presence of environmental amenity values, this adverse effect on growth is less detrimental to social welfare as compared to the case without environmental amenities. This result resembles that of Ligthart and Van der Ploeg (1994), who show that an increased weight of environmental amenity in utility reduces the marginal costs of public funds.

Optimal pollution tax

In a first-best world, the optimal pollution tax is the Pigovian tax which fully internalizes the environmental externalities in production and consumption (see (2.2)):

$$\frac{\alpha\gamma - \epsilon\eta}{s_c} - \frac{\epsilon\phi}{1-\psi} = 0 \qquad (3.9)$$

Our model is not first-best. In particular, the government does not have access to lump-sum taxation to finance its public goods. We investigate whether, in this second-best world, the government still finds it optimal to set the environmental tax at its Pigovian level. To that end, we explore the welfare effects of a marginal change in the pollution tax away from its Pigovian level. Hence, (3.9) holds in the initial equilibrium. We abstract from an abatement distortion, i.e. $T_y - T_a = 0$, and assume that $\tilde{T}_a = \tilde{s} = \tilde{g} = 0$. Substituting (3.9) into (2.2), we arrive at:

$$\tilde{\lambda} = \frac{\pi}{r-\pi}\left[(\delta - s_s) + \left(\frac{\psi s_c}{1-\psi} - s_g\right) + T_y\beta\right]\tilde{\pi} \qquad (3.10)$$

Hence, raising the pollution tax above its Pigovian level affects welfare by impacting the growth rate. At the Pigovian tax, the production elasticity of pollution exceeds the environmental externality in production if environmental amenities are positive (i.e. $\frac{\alpha\gamma - \epsilon\eta}{s_c} = \frac{\epsilon\phi}{1-\psi} > 0$, see (3.9)). Hence, a higher pollution tax reduces the growth rate (see (3.1)). This affects welfare if either public investment, public consumption or private capital features a gap between its marginal social costs and social benefits. In particular, if public consumption or public investment is below its social optimum (i.e. $\delta - s_s < 0$ or $\frac{\psi s_c}{1-\psi} - s_g < 0$), a lower growth rate harms welfare on account of this distortion. Furthermore, the government needs to adopt distortionary taxes on output to finance public spending. This implies a wedge between the marginal social costs and -benefits of capital accumulation. On balance, a lower growth rate reduces welfare if profits and public consumption are positive.[10]

These results demonstrate that pollution taxes aimed at internalizing environmental externalities may, as a side effect, either alleviate or exacerbate existing distortions in the economy. Indeed, in a second-best world, the optimal tax on pollution differs from its first-best level. In particular, in the absence of an abatement distortion, the optimal pollution tax lies below the Pigovian level. Bovenberg and De Mooij (1994a) reach similar conclusions when exploring the interaction between environmental and labor-market distortions. They argue that the optimal pollution tax lies below its Pigovian level because pollution taxes exacerbate the distortion in the labor market.

4 Tax Shifting

Section 3 assumed a Cobb-Douglas production function. In that particular case, substitution from pollution towards physical capital is relatively easy. Hence, a pollution tax is a rather effective instrument to cut pollution through input substitution. However, the large drop in pollution implies that the base of the pollution tax erodes substantially. Hence, the pollution tax is not successful in raising revenues so that the distortionary tax on output can hardly be reduced. Indeed, if the growth rate declines, the tax rate on output even needs to be increased (see (3.6)). This section

[10] More specifically, by using the relations between the shares at the end of table II, we can rewrite the term between square brackets in (3.10) as: $(\delta\text{-}s_s) + (\frac{\psi}{1-\psi}s_c\text{-}s_g) + T_y\beta = \frac{q-1}{q} + \frac{\psi}{1-\psi} > 0$. Hence, lower growth harms welfare if profits are strictly positive (i.e. $q > 1$) or if the government provides public consumption services (i.e. $\psi > 0$).

allows the substitution elasticities σ_y and σ_m to be smaller than unity. Accordingly, pollution taxes imply less factor substitution and thus are more effective as a revenue-raising device. Moreover, compared to capital, abatement is a better substitute for pollution. This does better justice to the term 'abatement'. We restrict our analysis to the case that $\sigma_m = \sigma_y$ and concentrate on the effects of pollution taxes. To simplify, we abstract from an initial abatement distortion, i.e. $T_a = T_y$.

Pollution

Assuming that $\tilde{s} = \tilde{g} = \tilde{T}_a = 0$, the reduced form equation for pollution looks as follows (see appendix C):

$$\tilde{P} = -[1 - \frac{(1-\sigma_y)(1-T_y)(1-\gamma)\alpha}{\Delta^*}]\tilde{t}_p - \frac{\sigma_y(\alpha\gamma - \epsilon\eta)}{\Delta^*}\tilde{t}_p \tag{4.1}$$

where $$\Delta^* = \Delta + (1-\sigma_y)[\alpha\gamma - \epsilon\eta + (1-T_y)\alpha] > 0$$

If $\sigma_y = 1$, (4.1) reduces to (3.4). If substitution between the intermediate factors M and N becomes more difficult, pollution taxes are less effective in cutting pollution as they induce less substitution from pollution towards capital. Nevertheless, (4.1) reveals that pollution taxes always reduce pollution, even if σ_y approaches zero.

Growth

The growth rate is given by (see appendix C):

$$\frac{1}{\sigma} s_i \tilde{\pi} = (\alpha\gamma - \epsilon\eta)\tilde{P} - s_w \tilde{w} \tag{4.2}$$

This expression reveals that the growth performance is affected by two factors, namely, first, the net costs of environmental policy borne by the production sector (i.e. the first term at the RHS of (4.2)), and, second, the part of the costs borne by profits (i.e. the second term at the right-hand side of (4.2)). A tighter environmental policy, which is associated with a decline in pollution, generates an adverse impact on growth if a lower pollution level implies a net burden on the production sector. This is the case if $\alpha\gamma - \epsilon\eta > 0$, i.e. if the negative output effect of a reduction in pollution as a private rival input exceeds the positive effects on productivity of less adverse production externalities (see also section 3). The second term at the RHS of (4.2) indicates the effect of shifting the tax burden away from the after-tax return towards profits. In particular, lower profits produce a higher growth rate. Intuitively, lower profits create room to cut the distortionary tax rate on output, thereby raising the return on capital accumulation.

The reduced form equation for profits is given by:

$$s_w \tilde{w} = - \frac{s_w}{\Delta^*}(\alpha\gamma - \epsilon\eta)\tilde{t}_p - \frac{(1-\sigma_y)(1-T_y)[\alpha\gamma - \epsilon\eta + rs_k\alpha]}{\Delta^*}\alpha\gamma\tilde{t}_p \tag{4.3}$$

The first term at the RHS of (4.3) represents the effect of input substitution. It reveals that, through this channel, profits share in the burden of environmental policy on the production sector. In particular, pollution taxes harm profits if the adverse pollution externalities on production are small compared to the input share of pollution as a rival input (i.e. $\alpha\gamma - \epsilon\eta > 0$). The second term at the RHS of (4.3) represents the shifting of the tax burden away from the after-tax return on investment towards profits.[11] If $\sigma_y < 1$, this tax shifting effect causes the replacement of the output taxes by pollution taxes to raise the return on investment by reducing profits, thereby boosting growth. The overall effect on growth is positive if the aggregate burden of environmental policy on the production sector is small (i.e. $(\alpha\gamma - \epsilon\eta)\tilde{P}$ is small) and a large share of this burden can be shifted unto profits (i.e. $\tilde{w}$ is large and negative). By substituting (4.1) and (4.3) into (4.2), we derive the following condition for pollution taxes to boost growth:

$$\frac{\alpha\gamma - \epsilon\eta}{\alpha} < (1-\sigma_y)(1-T_y)\alpha\gamma \tag{4.4}$$

The LHS of (4.4) represents the net burden of less pollution on the production sector. In particular, if $\alpha\gamma - \epsilon\eta > 0$, a decline in pollution hurts the production sector. The parameter α determines how effective pollution taxes are in cutting pollution. In particular, if α is large, pollution is an important factor of production relative to abatement (i.e. $\alpha\gamma$ is large relative to γ). In that case, substantially lowering pollution is not attractive for firms as the marginal product of

[11] Profits can be maintained in equilibrium due to the presence of a fixed factor in production which constitutes a barrier for new firms to enter the market.

pollution is very sensitive to the flow of pollution.[12]

The RHS of (4.4) shows the benefits of pollution taxes in terms of shifting the tax burden away from the return on investment towards profits. If $\sigma_y = 1$, the pollution tax and output tax imply the same relative burden on profits and the return on capital accumulation. Hence, an ecological tax reform does not imply a shift toward non-distortionary lump sum taxation of profits. Accordingly, the tax shifting effect exerts no positive effect on growth. If $\sigma_y < 1$, however, compared to the output tax, the pollution tax is more effective in raising revenues without depressing the return to investment. The reason is that the tax on pollution is a more effective instrument to tax away profits (and the quasi-rents from pollution, in particular). Thus, compared to an output tax, a pollution tax incorporates a more important lump sum element. Accordingly, by using the revenues from the pollution tax to cut the distortionary tax on output, the government shifts the tax burden away from the return on investment towards profits, thereby boosting growth.

On balance, an environmental tax reform produces a double dividend by improving the environment and stimulating growth if the positive effect of tax shifting (which allows for a reduction in the wedge between the before- and after-tax return to capital) dominates the negative effect of input substitution on gross productivity associated with more public consumption in the form of more environmental amenities. Also Bovenberg and Van der Ploeg (1994) find that, if an environmental tax reform shifts the burden of taxation from labor towards the rents from a fixed factor in production, it may boost employment. Similarly, Bovenberg and De Mooij (1994b) demonstrated that environmental taxes can yield a double dividend, by expanding employment and improving the environment, if the burden of the public sector is shifted from workers towards transfer recipients.

Optimal pollution tax

In section 3 we found that, in the Cobb-Douglas case, the optimal pollution tax is always below the Pigovian tax in the presence of environmental amenities. The reason is that lowering the pollution tax from the Pigovian level always boosts growth and, therefore, enhances welfare (see (3.10)). If substitution is more difficult, the welfare effect of raising the pollution tax from its Pigovian level continues to be represented by (3.10). Hence, pollution taxes impact welfare if they

[12] In the extreme case that α goes to infinity, pollution is not affected by a rise in the pollution tax. Hence, the costs of pollution taxes, represented by the LHS of (4.4), are zero.

affect the growth rate. Contrary to the Cobb-Douglas case, pollution taxes may raise the growth rate if $\sigma_y < 1$, even if environmental amenities cause the production elasticity to exceed the production externality (i.e. $\alpha\gamma - \epsilon\eta = \frac{\epsilon\phi}{1-\psi} s_c > 0$). This is because raising pollution taxes may improve the efficiency of taxation by shifting the tax burden from the return on investment towards profits. In particular, if environmental amenities are unimportant (i.e. ϕ is small), a rise in pollution taxes from the Pigovian level is most likely to stimulate growth and thus to improve welfare. Hence, the optimal tax on pollution exceeds the Pigovian level. Intuitively, in that case additional environmental amenities are small (i.e. pollution does not fall much if σ_y is small) and cheap (i.e. ϕ small). At the same time, the pollution tax acts as an implicit lump sum tax on profits.

5 Pollution Permits

So far, we have focussed on financial instruments aimed at cutting pollution, such as pollution taxes and abatement subsidies. These instruments charge a price for (the remaining) pollution. Hence, as shown in section 4, pollution taxes may, as a side effect from reducing pollution, yield public revenues if substitution between pollution and other inputs is difficult enough (i.e. if $\sigma_y <$ 1). In that case, growth can be stimulated if the government uses these revenues to cut the distortionary tax on output. This section explores the consequences for growth if the government adopts environmental policy instruments that do not charge a price for pollution. In particular, we assume that the government exogenously determines the economy wide level of pollution, P, by freely providing pollution permits to existing firms.

Appendix D solves the model if the level of pollution is exogenous. We assume that $T_a = T_y$ and $\tilde{s} = \tilde{g} = \tilde{T}_a = 0$. Hence, we concentrate on the effects of a change in the number of pollution permits. Appendix D reveals that expression (4.2) for the growth rate continues to hold. Hence, a tighter environmental policy (i.e. $\tilde{P} < 0$) reduces the growth rate by depressing the after-tax return to investment if two conditions are met. First, environmental policy should impose a net burden on the production sector (i.e. $\alpha\gamma - \epsilon\eta > 0$). Second, this burden can not be shifted to profits. With freely issued pollution permits, the reduced form for profits is given by:

$$s_w \tilde{w} = \frac{\delta}{\beta + \delta}(\alpha\gamma - \epsilon\eta)\tilde{P} \tag{5.1}$$

Expression (5.1) reveals that a reduction of the number of pollution permits reduces profits if and only if environmental policy imposes a net burden on the production sector (i.e. $\alpha\gamma$ - $\epsilon\eta$ > 0). This condition differs from the corresponding condition for the case of pollution taxes (i.e. expression (4.3)) if the substitution elasticity, σ_y, differs from unity. In particular, pollution taxes are more likely to reduce profits than freely issued pollution permits if $\sigma_y < 1$. Intuitively, if firms can not easily replace pollution by other inputs, a large share of the burden of pollution taxes is borne by profits. Hence, pollution taxes succeed in shifting the tax burden away from the after-tax return to investment towards profits.

The reduced form equation for the growth rate is found by substituting (5.1) into (4.2):

$$\tilde{\pi} = \frac{r}{r - \theta}\frac{1}{\Delta'}(\alpha\gamma - \epsilon\eta)\tilde{P} \tag{5.2}$$

where $\Delta' = \beta + (\delta - s_s) - s_g = \Delta + (\alpha\gamma - \epsilon\eta)$. Expression (5.2) shows that a double dividend (i. e. not only a lower level of pollution but also a higher rate of growth) occurs only if a tighter environmental policy confers a net benefit to the production sector, i.e. if the fall in gross productivity on account of less pollution as a rival input into production ($\alpha\gamma$) is dominated by the boost in the gross productivity of capital on account of less adverse production externalities ($\epsilon\eta$). This condition generally differs from the corresponding condition (4.4) for the case in which the government adopts pollution taxes rather than freely issued pollution permits to cut pollution. The reason is that, in contrast to freely issued pollution permits, pollution taxes may shift the tax burden between profits and the after-tax return.

This difference between the growth performance of alternative instruments of environmental policy reveals that the distribution of property rights of the environment may affect not only the distribution of income but also growth. In particular, if the government owns the property rights of the environment, it can adopt instruments that raise public revenues from the sale of the publicly owned environment, e.g. environmental taxes or auctioned pollution permits. However, if the property rights are assigned to pollutors, environmental policy instruments, such as pollution standards or freely distributed pollution permits, should be adopted that do not raise public revenues.

The ease with which other inputs can substitute for pollution determines whether environmental policy instruments that change the price for pollution are more likely to enhance growth than instruments that do not. In particular, comparision of (4.4) and (5.2) reveals that, compared to freely issued pollution permits, pollution taxes are more likely to enhance growth if $\sigma_y < 1$. Intuitively, if substitution possibilities are limited, pollution constitutes a relatively inelastic tax base. Hence, pollution taxes are an attractive instrument to raise revenues because a large part of the burden of these taxes is borne by profits rather than the after-tax return to capital accumulation.

If pollution can be easily replaced by other inputs (i.e. $\sigma_y > 1$), freely issued pollution permits are more likely to improve growth than pollution taxes. The reason is that, with easy substitution, the base of the pollution tax is relatively elastic. Accordingly, the pollution tax is a relatively inefficient device to raise revenues. In particular, the burden of the pollution tax is reflected mainly in a lower after-tax return to investment rather than in a lower profit level. Thus, the revenues generated by the pollution tax are relatively expensive in terms of a lower after-tax return on investment. Indeed, for any given amount of revenues and given the environmental objective, the pollution tax hurts the after-tax return to capital more than the output tax because the pollution tax is less effective in taxing away profits.

At the optimal level of pollution permits, the environmental benefits of a marginal reduction in pollution exceed private abatement costs if the environment yields positive environmental amenities (substitute (5.2) into (2.2) and use $\tilde{\lambda} = 0$):

$$\frac{\alpha\gamma - \epsilon\eta}{s_c} - \frac{\epsilon\phi}{1-\psi} < 0 \tag{5.3}$$

Intuitively, in this second best world, increasing the environmental distortion (by raising pollution above the level where environmental benefits equal abatement costs) reduces the distortion due to distortionary taxation. In particular, distortionary taxation implies that growth is 'too low:' an increase in growth yields a first-order improvement in welfare. If environmental damages of increased pollution equal private costs of pollution abatement, the change in environmental quality yields only second-order welfare effects. However, an increase in pollution boosts growth (if environmental amenities cause the private costs of pollution abatement to exceed the positive productivity effects of less pollution). Accordingly, a marginal increase in pollution above the level where marginal environmental costs equal the marginal reduction in abatement costs boosts welfare: it yields a first-order gain as a result of the growth wedge but only a second-order loss as a result of the environmental wedge.

6 Conclusions

This paper has explored the consequences of an environmental tax reform for economic growth, pollution and welfare in a second-best framework in which the government has to employ distortionary taxes to finance public spending. Our analysis uncovered two channels through which an environmental tax reform may yield a double dividend, i.e. not only improve environmental quality but also stimulate economic growth. The first channel, discussed in section 3, is a positive environmental externality on production associated with the role of the environment as a public production factor. Previous analyses of the double dividend issue modelled the environment as a public consumption good rather than as a public production factor. In that case, environmental policy always implied a net burden on the production sector of the economy as the benefits accrued only to the household sector. Accordingly, these analyses have ignored this channel through which a double dividend can occur (see, e.g., Bovenberg and De Mooij (1994b) and Bovenberg and van der Ploeg (1994)). However, the role of the environment as a public production factor does not overturn the result derived by Bovenberg and De Mooij (1994a) that the optimal pollution tax is lower than the Pigovian tax. As long as the environment produces some environmental amenities as a public consumption good, the Pigovian tax exceeds the optimal pollution tax, which internalizes the adverse pollution externalities on consumption. Intuitively, at the optimal tax level, the pollution tax reduces growth, thereby worsening distortions due to the absence of lump-sum taxation.

The second channel through which an environmental tax reform may boost growth is a shift in the tax burden away from the return on capital accumulation towards profits. In particular, section 4 showed that a pollution tax may improve the efficiency of the tax system as a revenue raising device by taxing away the quasi-rents from pollution. This second channel operates only if substitution between pollution and other inputs is difficult so that the base of the pollution tax is inelastic. Moreover, the government should not have access to more direct ways to tax profits and should not have to compensate the owners of the firm for the decline in the value of the firm.

The role of the environmental tax as a lump-sum tax on rents suggests the importance of how environmental policy assigns property rights to the environment. If the property rights are assigned to pollutors, the quasi-rents from pollution accrue to the owners of the firms rather than to the government. Accordingly, a double dividend cannot occur if the environmental externality in production is relatively small so that a decline in pollution imposes a net cost on the production sector. If the government owns the environment, in contrast, it can use the revenues from the sale of this public resource (i.e. the revenues from pollution taxes or the auctioning off of pollution permits) to cut distortionary taxes. In this case, a tighter environmental policy implies a capital loss for profit earners if substitution away from pollution is difficult. Hence, part of the burden of environmental policy on the production sector is borne by profits rather than the after-tax return to investment. If the tax shifting away from the after-tax return towards profits is large enough, an ecological tax reform may enhance growth, even if environmental policy implies a net burden on the production sector. Similar results were derived by Bovenberg and De Mooij (1994b) and Bovenberg and van der Ploeg (1994) in a static framework. Bovenberg and de Mooij showed that an environmental tax reform may boost employment if environmental taxes are allowed to erode the real value of income transfers. Similarly, Bovenberg and van der Ploeg (1994) demonstrated that employment may expand if environmental taxes act as an implicit way to tax away profits due to the presence of a fixed factor in production.

This paper shows, however, that, compared to other environmental policy instruments that do not charge a price for pollution, a pollution tax is not always a more efficient instrument to cut pollution. In particular, if substitution between pollution and other inputs is easy, freely issued pollution permits rather than pollution taxes protect the after-tax return to investment the best. Intuitively, with easy substitution, the pollution tax is a relatively inefficient device to raise revenues. In particular, the revenues generated by environmental policy instruments damage the after-tax return more and profits less than revenues generated by the output tax.

In a second-best world with both environmental distortions and tax distortions, the optimal environmental tax typically differs from the Pigovian level. In particular, we demonstrated that the optimal environmental tax may exceed its Pigovian level if the government does not have access to more direct ways to tax rents. Intuitively, in the absence of an explicit lump-sum tax on profits, the pollution tax acts an implicit way to tax rents. The conditions for the optimal environmental tax to exceed the Pigovian level are twofold. First, environmental amenities are unimportant so that a cut in pollution is relatively cheap in terms of losses in the productivity of capital. Second, the pollution tax should be rather ineffective in cutting pollution. Under these two conditions, the shifting of the burden of public spending and environmental amenities to the owners of the firm offsets the costs of additional environmental amenities, thereby alleviating the net burden on investors. Hence, the after-tax return on investment and hence growth rise.

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Figure 1: The production structure

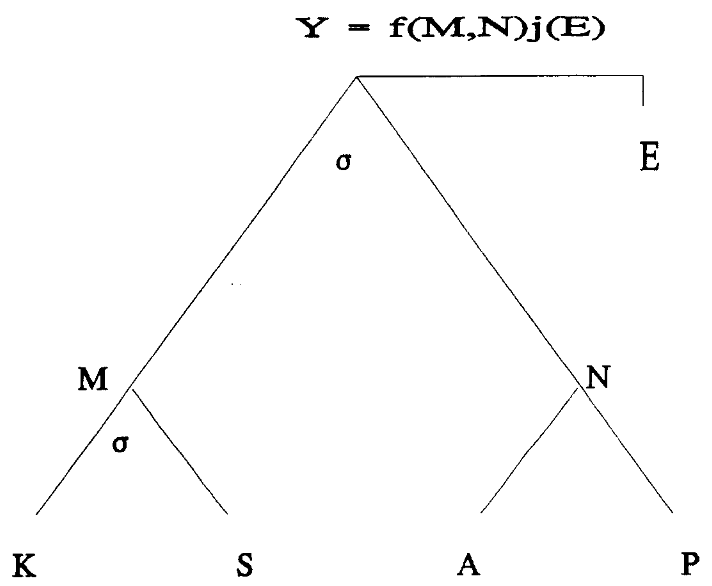

Figure 2: Household Instantaneous Utility

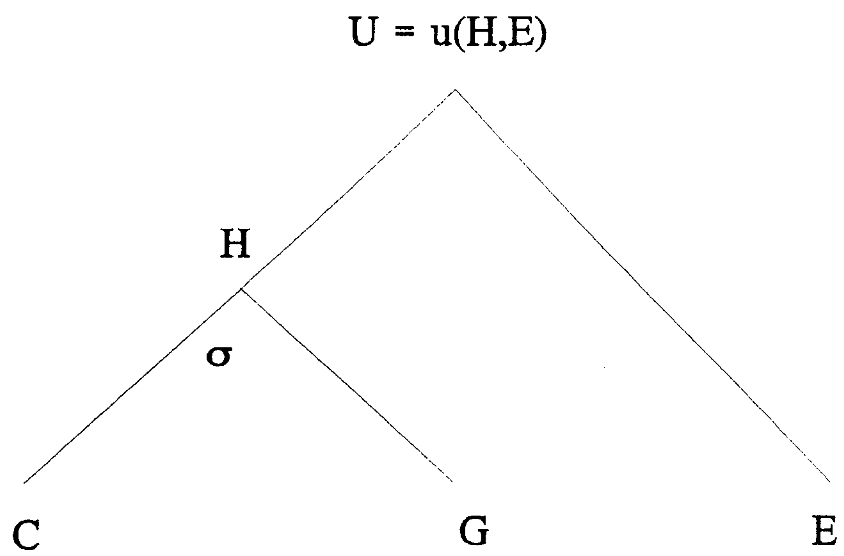

Table I: The model in levels

Firms

Production function

$$Y = f[m(K,S), n(A,P)]\,j(E) \qquad \text{(I.1)}$$

$$where \quad n(A,P) = A\,P^{\alpha}$$

Firm Value

$$V = \int_0^{\infty} D\,e^{-rt}\,dt \qquad \text{(I.2)}$$

Dividends

$$D = (1 - T_y)\,Y - (1 - T_a)A - T_p P - I \qquad \text{(I.3)}$$

Firm Value per unit of Capital

$$q \equiv \frac{V}{K} \qquad \text{(I.4)}$$

Investment

$$I \equiv \dot{K} \qquad \text{(I.5)}$$

Profits

$$W = D + I - rK \qquad \text{(I.6)}$$

First Order Conditions

$$(1 - T_y)\,\frac{\partial Y}{\partial A} = (1 - T_a) \qquad \text{(I.7)}$$

$$(1 - T_y)\,\frac{\partial Y}{\partial P} = T_p \qquad \text{(I.8)}$$

$$(1 - T_y)\,\frac{\partial Y}{\partial K} = r \qquad \text{(I.9)}$$

Households

Utility

$$U = \int_0^{\infty} \frac{[H(C,G)\,E^{\phi}]^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}\,e^{-\theta t}\,dt \qquad \text{(I.10)}$$

Household Budget Constraint

$$\dot{K} = rK - \frac{C}{q} \qquad \text{(I.11)}$$

First Order Condition

$$\frac{\dot{C}}{C} = \sigma(r - \theta) \qquad \text{(I.12)}$$

Government

Government Budget Constraint

$$T_y Y + T_p P = T_a A + S + G \qquad \text{(I.13)}$$

Environmental Quality

$$E = e(P) \qquad \text{(I.14)}$$

Walras Law

Goods market equilibrium

$$I = Y - S - G - A - C \qquad \text{(I.15)}$$

Exogenous: T_a, T_p, S, G, K_0

Endogenous: Y, P, A, V, D, I, W, C, T_y, E, q, r, π

Growth rate

$$\pi = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{S}}{S} = \frac{\dot{A}}{A} = \frac{\dot{C}}{C} = \frac{\dot{I}}{I} = \frac{\dot{G}}{G} = \frac{\dot{T}_p}{T_p} = \frac{\dot{V}}{V} = \frac{\dot{D}}{D} = \frac{\dot{W}}{W}$$

Notation

Y = output

S = public investment

P = pollution

V = value of the firm

C = private consumption

E = quality of the environment

T_a = abatement subsidy

r = interest rate

q = value of the firm per unit of capital

K = capital stock

A = abatement spending

I = investment in private capital

D = Dividends

G = public consumption

T_y = tax on output

T_p = pollution tax

π = growth rate

W = profits

Parameters

ϕ = environmental externality parameter in utility

σ = intertemporal substitution elasticity

σ_h = substitution elasticity between public- and private consumption

θ = pure rate of time preference

$\psi = \frac{\partial H}{\partial G}\frac{G}{H}$ household preference parameter for public consumption goods

σ_m = substitution elasticity between private capital and public investment

σ_y = substitution elasticity between intermediates N = n(A,P) and M = m(K,S)

Table II: The model in relative changes

Output

$$\tilde{y} = \delta \tilde{s} + \gamma \tilde{a} + \alpha \gamma \tilde{P} + \eta \tilde{E} \tag{II.1}$$

Pollution

$$[1 + \alpha(1 - \sigma_y)]\tilde{P} = \frac{\delta(\sigma_m - \sigma_y)}{\sigma_m(\delta + \beta)} \tilde{s} - (1 - \sigma_y)(\tilde{t}_p + \tilde{T}_a) + \sigma_y(\tilde{r} - \tilde{t}_p) \tag{II.2}$$

Abatement

$$\tilde{a} = \tilde{P} + \tilde{T}_a + \tilde{t}_p \tag{II.3}$$

Rate of return to capital

$$\tilde{y} = \frac{\delta(\sigma_m - \sigma_y)}{\sigma_m(\delta + \beta)} \tilde{s} + (1 - \sigma_y) \eta \tilde{E} + \sigma_y [\tilde{r} + \tilde{T}_y] \tag{II.4}$$

Dividends

$$s_d \tilde{d} = - s_i \tilde{i} + (1 - T_y) [\delta \tilde{s} + \gamma \tilde{T}_a - \tilde{T}_y - \alpha \gamma \tilde{t}_p - \epsilon \eta \tilde{P}] \tag{II.5}$$

Value of the firm

$$\frac{q}{q-1} \tilde{q} = \tilde{w} - \frac{r}{r - \pi}(1 - \sigma) \tilde{r} \tag{II.6}$$

Profits

$$s_w \tilde{w} = s_d \tilde{d} + s_i \tilde{i} - r s_k \tilde{r} \tag{II.7}$$

Growth rate

$$\tilde{\pi} \equiv \tilde{i} \tag{II.8}$$

Household savings

$$\tilde{i} = \frac{r}{r - \theta} \tilde{r} \tag{II.9}$$

Household budget constraint

$$\tilde{c} = \tilde{q} + \frac{r}{r - \pi}(1 - \sigma) \tilde{r} \tag{II.10}$$

Government Budget constraint

$$(1 - T_y) \tilde{T}_y + T_y \tilde{y} + T_p^* \tilde{t}_p + T_p^* \tilde{P} = (1 - T_a) s_a \tilde{T}_a + T_a s_a \tilde{a} + s_g \tilde{g} + s_s \tilde{s} \tag{II.11}$$

Environmental Quality

$$\tilde{E} = - \epsilon \tilde{P} \tag{II.12}$$

Goods market equilibrium (Walras Law)

$$s_i \tilde{i} = \tilde{y} - s_c \tilde{c} - s_a \tilde{a} - s_g \tilde{g} - s_s \tilde{s} \qquad \text{(II.13)}$$

Capital letters (X) denote levels of a variable while lower case variables (x) denote levels as a ratio of the capital stock (K), i.e. x = X/K for x = y, s, a, t_p, d, v, w, i, c, g, and $\tilde{x} = \tilde{X} - \tilde{K}$.

Elasticities

$$\beta = \frac{\frac{\partial Y}{\partial K} K}{Y}, \quad \delta = \frac{\frac{\partial Y}{\partial S} S}{Y}, \quad \gamma = \frac{\frac{\partial Y}{\partial A} A}{Y}, \quad \alpha\gamma = \frac{\frac{\partial Y}{\partial P} P}{Y}$$

$$\epsilon = -\frac{\partial E}{\partial P}\frac{P}{E} > 0 \qquad \eta = \frac{\partial j}{\partial E}\frac{E}{j} > 0$$

Taxes

$$\tilde{T}_a = \frac{dT_a}{1-T_a}, \quad \tilde{T}_y = \frac{dT_y}{1-T_y}, \quad \tilde{T}_p = \frac{dT_p}{T_p}$$

Shares

s_a = A/Y	s_d = D/Y	T_p^* = T_pP/Y	s_g = G/Y
s_s = S/Y	s_c = C/Y	s_i = I/Y	s_k = K/Y
s_v = V/Y	s_w = W/Y		

Relations between the shares:

Dividends	$s_d = (1-T_y) - (1-T_a)s_a - T_p^* - s_i = (r-\pi)s_k q$
Firm value per unit of capital	$q = s_v/s_k = 1 + \frac{\delta - \alpha\gamma}{\beta}\frac{r}{r-\pi}$
Profits	$s_w = s_d + s_i - rs_k = (1-T_y)(\delta-\alpha\gamma)$
Household Budget Constraint	$s_i = rs_k - s_c/q$; $s_c = s_d$; $(r-\pi)q = s_c/s_k$
Definition of investment	$s_i = \pi s_k$
Government Budget Constraint	$s_s + s_g + T_a s_a = T_y + T_p^*$
Goods Market Equilibrium	$s_i = 1 - s_s - s_g - s_a - s_c$

Production elasticities

$$(1-T_y)\gamma = (1-T_a)s_a$$

$$(1-T_y)\alpha\gamma = T_p^*$$

$$(1-T_y)\beta = rs_k$$

Determinant

$$\Delta^* = \sigma_y[\beta + (\delta - s_s) + (\epsilon\eta - \alpha\gamma) - s_g] + (1-\sigma_y)(1-T_y)(1-\gamma)(1+\alpha)$$

Appendix A: Linearizing the factor demand equations

We log-linearize the model around an initial equilibrium. A tilde (~) above a variable denotes a relative change unless indicated otherwise.

Specification of F

F = f(M,N) is homogenous of degree 1 in M and N. Hence, F/M = f(1,N/M) $\rightarrow \phi = \phi(z)$ where ϕ = F/M and z = N/M. The first order derivatives of F are:

$$\frac{\partial F}{\partial N} = \phi' \tag{A1}$$

$$\frac{\partial F}{\partial M} = \phi - z\phi' \tag{A2}$$

The substitution elasticity between M and N is defined as:

$$\frac{1}{\sigma_y} = -\frac{\partial\left(\frac{\partial F/\partial N}{\partial F/\partial M}\right)}{\partial\frac{N}{M}}\frac{\frac{N}{M}}{\frac{\partial F/\partial N}{\partial F/\partial M}} = \frac{-z\phi''}{\phi'}\frac{\phi}{\phi - z\phi'} \tag{A3}$$

Homogeneity of F implies:

$$\tilde{F} = \gamma\tilde{N} + (\delta+\beta)\tilde{M} = \tilde{N} + (\delta+\beta)[\tilde{M} - \tilde{N}] \tag{A4}$$

where we have used (2.1).

Specification of M

M = m(K,S) is homogenous of degree 1 in K and S. Hence, M/K = m(1, S/K) $\rightarrow \mu = \mu(s)$ where μ = M/K and s = S/K. The first order derivatives of M are:

$$\frac{\partial M}{\partial S} = \mu' \tag{A5}$$

$$\frac{\partial M}{\partial K} = \mu - s\mu' \tag{A6}$$

The substitution elasticity between K and S is defined as:

$$\frac{1}{\sigma_m} = -\frac{\partial\left(\frac{\partial M/\partial S}{\partial M/\partial K}\right)}{\partial\frac{S}{K}}\frac{\frac{S}{K}}{\frac{\partial M/\partial S}{\partial M/\partial K}} = \frac{-s\mu''}{\mu'}\frac{\mu}{\mu - s\mu'} \tag{A7}$$

Homogeneity of M implies:

$$\tilde{M} = \frac{\beta}{\beta + \delta}\tilde{K} + \frac{\delta}{\beta + \delta}\tilde{S} \tag{A8}$$

Specification of N and Y

The Cobb Douglas specification for N implies for the relative change in N:

$$\tilde{N} = \tilde{A} + \alpha\tilde{P} \tag{A9}$$

Moreover, the externality of pollution specified in (I.1) implies for the relative change in Y:

$$\tilde{Y} = \tilde{F} + \eta\tilde{E} \tag{A10}$$

Linearizing marginal factor productivity

Armed with this set of definitions and relations, we are able to log-linearize the factor demand equations (I.7), (I.8) and (I.9) for abatement, pollution and capital, respectively. The relative change in the marginal product of abatement is derived by using (A1), (A3) and the Cobb Douglas specification of N in the production function (I.1):

$$\begin{aligned} \frac{\Delta\partial Y/\partial A}{\partial Y/\partial A} &= \frac{\Delta\partial F/\partial N}{\partial F/\partial N} + \eta\tilde{E} + \frac{\Delta\partial N/\partial A}{\partial N/\partial A} = \\ &- \frac{1}{\sigma_y}(\beta+\delta)[\tilde{N}-\tilde{M}] + \alpha\tilde{P} + \eta\tilde{E} \end{aligned} \tag{A11}$$

The linearized marginal product of pollution is found by using the same relations as for abatement:

$$\begin{aligned} \frac{\Delta[\partial F/\partial P]j(E)}{\partial Y/\partial P} &= \frac{\Delta\partial F/\partial N}{\partial F/\partial N} + \eta\tilde{E} + \frac{\Delta\partial N/\partial P}{\partial N/\partial P} = \\ &- \frac{1}{\sigma_y}(\beta+\delta)[\tilde{N}-\tilde{M}] + \tilde{A} - (1-\alpha)\tilde{P} + \eta\tilde{E} \end{aligned} \tag{A12}$$

To derive an expression for the relative change in the marginal product of capital, we use (A2), (A3), (A6), (A7) and the specification of the production function (I.1):

$$\begin{aligned} \frac{\Delta\partial Y/\partial K}{\partial Y/\partial K} &= \frac{\Delta\partial F/\partial M}{\partial F/\partial M} + \eta\tilde{E} + \frac{\Delta\partial M/\partial K}{\partial M/\partial K} = \\ &\frac{1}{\sigma_y}\gamma[\tilde{N}-\tilde{M}] + \frac{1}{\sigma_m}\frac{\delta}{\beta+\delta}[\tilde{S}-\tilde{K}] + \eta\tilde{E} \end{aligned} \tag{A13}$$

Linearizing factor demand equations

Using (A11), we linearize the demand for abatement (I.7) as:

$$\frac{1}{\sigma_y}(\beta + \delta)[\tilde{N} - \tilde{M}] - \alpha\tilde{P} - \eta\tilde{E} = \tilde{T}_a - \tilde{T}_y \tag{A14}$$

From (A12), we arrive at the following linearized demand for pollution (I.8):

$$-\frac{1}{\sigma_y}(\beta + \delta)[\tilde{N} - \tilde{M}] + \tilde{A} - (1 - \alpha)\tilde{P} + \eta\tilde{E} = \tilde{T}_p + \tilde{T}_y \tag{A15}$$

By adding (A14) and (A15), we find the following relation between abatement and pollution:

$$\tilde{A} = \tilde{P} + \tilde{T}_p + \tilde{T}_a \tag{A16}$$

Combining (A9) and (A16), we derive for abatement and pollution:

$$\tilde{A} = \frac{1}{1+\alpha}\tilde{N} + \frac{\alpha}{1+\alpha}[\tilde{T}_p + \tilde{T}_a] \tag{A17}$$

$$\tilde{P} = \frac{1}{1+\alpha}\tilde{N} - \frac{1}{1+\alpha}[\tilde{T}_p + \tilde{T}_a] \tag{A18}$$

Finally, (A13) and (I.9) yield for the desired demand for capital:

$$\frac{1}{\sigma_y}\gamma[\tilde{N} - \tilde{M}] + \frac{1}{\sigma_m}\frac{\delta}{\beta + \delta}[\tilde{S} - \tilde{K}] + \eta\tilde{E} = \tilde{r} + \tilde{T}_y \tag{A20}$$

Now we are able to derive the factor demand equations as presented in table II. First, the capital market equilibrium (II.4) is found by substituting (A4), (A8) and (A10) into (A20) to eliminate $\tilde{N}$, $\tilde{M}$ and $\tilde{F}$, respectively. Second, relation (II.2) for pollution is found by substituting (A8) and (A9) into (A14) to eliminate $\tilde{N}$ and $\tilde{M}$, and then using (II.4) to eliminate $\tilde{Y}$. Finally, (II.3) is equivalent with (A16).

Appendix B: The marginal excess burden

This appendix derives the marginal excess burden as a measure for changes in social welfare. It represents a monetary equivalent of welfare changes due to various policy measures.

Utility is defined in (I.10), where the function H(C,G) is homogenous of degree one. On the sustainable balanced growth path private- and public consumption grow at an equal rate, π, while pollution remains constant. Hence, we can rewrite utility as:

$$U = \frac{[C_0 h(v) E^{\phi}]^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \int_0^{\infty} e^{[\pi(1-\frac{1}{\sigma})-\theta]t} dt \tag{B1}$$

where $v = G/C$, $h = H/C$ and C_0 is the initial consumption level. To solve the integral at the RHS of (B1), we require that $\pi(1 - 1/\sigma) - \theta < 0$ so that that utility is bounded. Using (I.12) from table I, we rewrite this condition as follows:

$$\theta - \pi(1-\frac{1}{\sigma}) = r - \pi > 0 \tag{B2}$$

Solving the integral in (B1) yields the following expression for utility:

$$U = \frac{[C_0 h(v) E^{\phi}]^{1-\frac{1}{\sigma}}}{(1 - \frac{1}{\sigma})(\theta - \pi(1-\frac{1}{\sigma}))} \tag{B3}$$

For utility to remain constant after a policy shock we need:

$$0 = dU = \frac{\partial U}{\partial C}dC + \frac{\partial U}{\partial G}dG + \frac{\partial U}{\partial E}dE + \frac{\partial U}{\partial \pi}d\pi \tag{B4}$$

The derivatives at the right hand side of (B4) can be derived by differentiating (B3) with respect to the respective arguments. This yields:

$$\frac{\partial U}{\partial C}dC = \frac{[C_0 h(v) E^{\phi}]^{1-\frac{1}{\sigma}}}{r - \pi}(1 - \frac{h'v}{h})\tilde{C} \tag{B5}$$

$$\frac{\partial U}{\partial G}dG = \frac{[C_0 h(v) E^{\phi}]^{1-\frac{1}{\sigma}}}{r - \pi}\frac{h'v}{h}\tilde{G} \tag{B6}$$

$$\frac{\partial U}{\partial E}dE = \frac{[C_0 h(v) E^{\phi}]^{1-\frac{1}{\sigma}}}{r - \pi}\phi\tilde{E} \tag{B7}$$

$$\frac{\partial U}{\partial \pi} d\pi = \frac{[C_0 h(v) E^{\phi}]^{1-\frac{1}{\sigma}}}{r - \pi} \frac{\pi}{r - \pi} \tilde{\pi} \tag{B8}$$

Substituting (B5), (B6), (B7) and (B8) into (B4), we arrive at the following condition:

$$(1-\psi)\,(\tilde{C} + \frac{d\lambda}{C}) + \psi\tilde{G} + \phi\tilde{E} + \frac{\pi}{r-\pi}\tilde{\pi} = 0 \tag{B9}$$

where:

$$0 < \psi = \frac{h'v}{h} = \frac{\partial H}{\partial G}\frac{G}{H} < 1 \tag{B10}$$

and $d\lambda$ is the compensating variation. Rearranging terms in (B9), we derive for the marginal excess burden, $\tilde{\lambda} = \frac{d\lambda}{C}$:

$$\tilde{\lambda} = -\frac{\phi}{1-\psi}\tilde{E} - \frac{\psi}{1-\psi}\tilde{G} - \frac{1}{1-\psi}\frac{\pi}{r-\pi}\tilde{\pi} - \tilde{C} \tag{B11}$$

We now derive an alternative formulation for the excess burden in (B11). Substituting (II.1) into (II.13), we find for consumption:

$$s_c(\tilde{C} - \tilde{K}) = (\delta - s_s)\tilde{s} + (\gamma - s_a)\tilde{a} + (\alpha\gamma - \epsilon\eta)\tilde{P} - s_i\tilde{i} - s_g\tilde{g} \tag{B12}$$

Substituting (B12) into (B11) to eliminate consumption, we find:

$$\begin{aligned}\tilde{\lambda} = &-\frac{\phi}{1-\psi}\tilde{E} - \frac{\psi}{1-\psi}\tilde{G} - \frac{1}{1-\psi}\frac{\pi}{r-\pi}\tilde{\pi} \\ &-\frac{\delta - s_s}{s_c}\tilde{s} - \frac{\gamma - s_a}{s_c}\tilde{a} - \frac{\alpha\gamma - \epsilon\eta}{s_c}\tilde{P} + \frac{s_i}{s_c}\tilde{i} + \frac{s_g}{s_c}\tilde{g}\end{aligned} \tag{B13}$$

The coefficient for pollution is found by substituting (II.12) into (B13) to eliminate $\tilde{E}$. Then we use the relations between the shares in table II to rewrite the coefficient for abatement. By using (II.8) we can eliminate $\tilde{i}$. Finally, by using the relations between the shares and the definitions of the elasticities at the end of table II we can rewrite the coefficient for growth as in (2.2).

Appendix C: Solution of the model

This appendix solves the model of table II for the case that $\sigma_y = \sigma_m$ and that the government uses T_y to balance its budget (ex post).

By substituting (II.3) and (II.12) into (II.1) we can write output as:

$$\tilde{y} = [\gamma + \alpha\gamma - \epsilon\eta]\tilde{P} + \gamma\tilde{T}_a + \gamma\tilde{t}_p + \delta\tilde{s} \tag{C1}$$

Then we substitute (C1) into (II.4) to eliminate $\tilde{y}$, use (II.12) to eliminate $\tilde{E}$ and impose $\sigma_y = \sigma_m$. This yields:

$$\sigma_y\tilde{r} = [\gamma + \alpha\gamma - \epsilon\eta + (1-\sigma_y)\epsilon\eta]\tilde{P} + \gamma\tilde{T}_a + \gamma\tilde{t}_p + \delta\tilde{s} - \sigma_y\tilde{T}_y \tag{C2}$$

Substitution of (C2) into (II.2) to eliminate $\tilde{r}$ yields the following relation for pollution:

$$\Theta\tilde{P} = -\sigma_y\tilde{T}_y - [1-\sigma_y-\gamma]\tilde{T}_a - (1-\gamma)\tilde{t}_p + \delta\tilde{s} \tag{C3}$$

where:

$$\Theta = 1-\gamma-\alpha\gamma+\epsilon\eta+(1-\sigma_y)(\alpha-\epsilon\eta) > 0 \tag{C4}$$

By substituting $\tilde{y}$ and $\tilde{a}$ from (C1) and (II.3) into the government budget constraint (II.11), we find another expression for $\tilde{P}$ and $\tilde{T}_y$ in terms of the exogenous variables:

$$\begin{aligned}[T_p^* - T_a s_a + (\gamma+\alpha\gamma-\epsilon\eta)T_y]\tilde{P} + (1-T_y)\tilde{T}_y &= -[T_p^* - T_a s_a + T_y\gamma]\tilde{t}_p \\ &\quad + [s_a - \gamma T_y]\tilde{T}_a + [s_s - \delta T_y]\tilde{s} + s_g\tilde{g}\end{aligned} \tag{C5}$$

Relations (C3) and (C5) form two linear equations in two endogenous variables. Rewriting them in matrix notation yields:

$$\begin{pmatrix} 1-T_y & T_p^* - T_a s_a + T_y(\gamma+\alpha\gamma-\epsilon\eta) \\ \sigma_y & 1-\gamma-\alpha\gamma+\epsilon\eta + (1-\sigma_y)(\alpha-\epsilon\eta)\end{pmatrix}\begin{pmatrix}\tilde{T}_y \\ \tilde{P}\end{pmatrix} =$$

$$\begin{pmatrix} -(T_p^* - T_a s_a) - T_y\gamma & s_a - T_y\gamma & s_s - T_y\delta & s_g \\ -(1-\gamma) & \gamma-(1-\sigma_y) & \delta & 0\end{pmatrix}\begin{pmatrix}\tilde{t}_p \\ \tilde{T}_a \\ \tilde{s} \\ \tilde{g}\end{pmatrix} \tag{C6}$$

Solving this linear system by inverting the matrix at the left hand side of (C6), we find:

$$\Delta^* \begin{pmatrix} \tilde{T}_y \\ \tilde{P} \end{pmatrix} = \begin{pmatrix} 1-\gamma-\alpha\gamma+\epsilon\eta + (1-\sigma_y)(\alpha-\epsilon\eta) & -(T_p^*-T_a s_a) - T_y(\gamma+\alpha\gamma-\epsilon\eta) \\ -\sigma_y & 1-T_y \end{pmatrix} * $$
$$\begin{pmatrix} -(T_p^*-T_a s_a)-\gamma T_y & s_a - T_y\gamma & s_s - T_y\delta & s_g \\ -(1-\gamma) & \gamma-(1-\sigma_y) & \delta & 0 \end{pmatrix} \begin{pmatrix} \tilde{t}_p \\ \tilde{T}_a \\ \tilde{s} \\ \tilde{g} \end{pmatrix} \tag{C7}$$

where:

$$\begin{aligned} \Delta^* &= \sigma_y[1-\gamma-\alpha\gamma+\epsilon\eta - T_y - T_p^* + T_a s_a] + (1-\sigma_y)(1-T_y)(1-\gamma)(1+\alpha) \\ &= \sigma_y[\beta + (\delta - s_s) + (\epsilon\eta - \alpha\gamma) - s_g] + (1-\sigma_y)(1-T_y)(1-\gamma)(1+\alpha) \end{aligned} \tag{C8}$$

The determinant in (C8) should be positive for the equilibrium to be stable. (C7) yields the reduced form equations for the output tax and pollution:

$$\begin{aligned} \Delta^*\tilde{T}_y &= [1-\gamma-\alpha\gamma+\epsilon\eta + (1-\sigma_y)(\alpha-\epsilon\eta)](s_a\tilde{T}_a + s_s\tilde{s} + s_g\tilde{g}) \\ &- [T_y+T_p^*-T_a s_a + T_y(1-\sigma_y)(\alpha-\epsilon\eta)](\gamma\tilde{T}_a + \delta\tilde{s} - (\alpha\gamma-\epsilon\eta)\tilde{t}_p) \\ &- (1-\sigma_y)[T_p^*-T_a s_a + T_y(\gamma+\alpha\gamma-\epsilon\eta)][(\alpha-\epsilon\eta)\tilde{t}_p - \tilde{T}_a] \end{aligned} \tag{C9}$$

$$\begin{aligned} \Delta^*\tilde{P} &= -\Delta^*\tilde{t}_p + (1-\sigma_y)(1-T_y)[(1-\gamma)\alpha\tilde{t}_p - (1-\gamma)\tilde{T}_a + \delta\tilde{s}] \\ &+ \sigma_y[(\gamma-s_a)\tilde{T}_a + (\delta-s_s)\tilde{s} - (\alpha\gamma-\epsilon\eta)\tilde{t}_p - s_g\tilde{g}] \end{aligned} \tag{C10}$$

By substituting (C10) into (II.3) we find for abatement:

$$\begin{aligned} \Delta^*\tilde{a} &= \Delta^*\tilde{T}_a + (1-\sigma_y)(1-T_y)[(1-\gamma)\alpha\tilde{t}_p - (1-\gamma)\tilde{T}_a + \delta\tilde{s}] \\ &+ \sigma_y[(\gamma-s_a)\tilde{T}_a + (\delta-s_s)\tilde{s} - (\alpha\gamma-\epsilon\eta)\tilde{t}_p - s_g\tilde{g}] \end{aligned} \tag{C11}$$

Substituting the reduced form equations for pollution and abatement from (C10) and (C11) into (II.1), we can derive the reduced form equation for output.

Substituting (C3) into (II.2) to eliminate $\tilde{P}$, we find for the interest rate:

$$\Theta\tilde{r} = -(\alpha\gamma-\epsilon\eta)\tilde{t}_p - [1+\alpha(1-\sigma_y)]\tilde{T}_y$$
$$+ [\gamma+(1-\sigma_y)\epsilon\eta]\tilde{T}_a + \frac{1}{\sigma_y}[1+\alpha(1-\sigma_y)]\delta\tilde{s} \qquad (C12)$$

Hence, pollution taxes reduce the interest rate if $\alpha\gamma\text{-}\epsilon\eta > 0$, while higher output taxes are always associated with a lower interest rate. In order to find the reduced form for the interest rate we substitute (C9) into (C12). After tedious algebra, we find:

$$\Delta^{*}\tilde{r} = \sigma_y[(\gamma-s_a)\tilde{T}_a - (\alpha\gamma-\epsilon\eta)\tilde{t}_p + (\delta-s_s)\tilde{s} - s_g\tilde{g}]$$
$$+ (1-\sigma_y)(1-T_y)\alpha\gamma[\alpha\tilde{t}_p - \tilde{T}_a] + (1-\sigma_y)[\alpha(\gamma-s_a)-(\alpha\gamma-\epsilon\eta)][\tilde{T}_a + \tilde{t}_p] \qquad (C13)$$
$$+ (1-\sigma_y)(1+\alpha)(\delta-s_s)\tilde{s} + \frac{1}{\sigma_y}(1-\sigma_y)(1-T_y)\delta\tilde{s} - (1-\sigma_y)(1+\alpha)s_g\tilde{g}$$

The reduced form for the growth rate is closely related to (C13) (see (II.8) and (II.9)). Similarly, the reduced form for investment is derived from (C13) and (II.8). To arrive at a reduced form equation for consumption, we substitute (C11) for abatement and the reduced forms for output and investment into the goods market equilibrium (II.13). Finally, the reduced form for the marginal excess burden can be derived by substituting the reduced forms for pollution and growth, as well as the reduced form for abatement into (2.2). If $\sigma_y = 1$, the reduced form for the marginal excess burden reduces to (3.8).

Pollution taxes

In order to gain more insight in the effects of pollution taxes on the interest rate, we derive another expression for $\tilde{r}$. By substituting (II.5) into (II.7) to eliminate $\tilde{d}$, we find for profits:

$$s_w\tilde{w} = (1-T_y)[\delta\tilde{s} + \gamma\tilde{T}_a - \tilde{T}_y - \alpha\gamma\tilde{t}_p - \epsilon\eta\tilde{P} - \beta\tilde{r}] \qquad (C14)$$

We concentrate on the effects of pollution taxes and assume that $\tilde{s} = \tilde{g} = \tilde{T}_a = 0$. Substituting (C9) and (C10) into (C14) to eliminate $\tilde{T}_y$ and $\tilde{P}$ and rearranging terms, we find the following expression for the interest rate:

$$(1-T_y)\beta\tilde{r} = (\alpha\gamma - \epsilon\eta)\tilde{P} - s_w\tilde{w} \qquad (C15)$$

The first term at the RHS of (C15) represents the substitution effect. The second term represents a shift effect. The reduced form for profits is derived by substituting (C13) into (C15).

$$\Delta^{*}\tilde{w} = -(\alpha\gamma - \epsilon\eta)\tilde{t}_p - \frac{\alpha\gamma}{\delta-\alpha\gamma}(1-\sigma_y)[\alpha\gamma - \epsilon\eta + \alpha r s_k]\tilde{t}_p \qquad (C16)$$

The effect of pollution taxes on profits is determined by a substitution-effect (first term at the RHS of (C16)) and a shift effect (second term at the RHS of (C16)).

Appendix D: Pollution Permits

This appendix solves the model of table II if the level of pollution, P, rather than the pollution tax, T_p, is an exogenous policy variable. Furthermore, T_p does not represent a tax but the shadow price of pollution. Accordingly, the model differs from table II as, on the one hand, T_p does not involve costs for firms in the form of tax payments while, on the other hand, it does not yield public revenues. In particular, the term T_pP disappears from equations (I.3) and (I.13) in table I for dividends and the government budget constraint, respectively. For table II it involves the following changes:

Dividends

$$s_d\tilde{d} = -s_i\tilde{i} + (1-T_y)[\delta\tilde{s} + \gamma\tilde{T}_a - \tilde{T}_y + (\alpha\gamma - \epsilon\eta)\tilde{P}] \tag{II.5'}$$

Government budget constraint

$$(1-T_y)\tilde{T}_y + T_y\tilde{y} = (1-T_a)s_a\tilde{T}_a + T_a s_a\tilde{a} + s_g\tilde{g} + s_s\tilde{s} \tag{II.11'}$$

Assuming that $\tilde{s} = \tilde{g} = \tilde{T}_a = 0$ and $T_a = T_y$, substitution of (II.5') into (II.7) yields for profits:

$$s_w\tilde{w} = -(1-T_y)[\tilde{T}_y - (\alpha\gamma - \epsilon\eta)\tilde{P} + \beta\tilde{r}] \tag{D1}$$

Substituting (II.11') into (D1) to eliminate $\tilde{T}_y$, we arrive at the following expression for $\tilde{r}$:

$$(1-T_y)\beta\tilde{r} = (\alpha\gamma - \epsilon\eta)\tilde{P} - s_w\tilde{w} \tag{D2}$$

Expression (D2) is equivalent to (C15) in appendix C. Hence, this relationship holds, irrespective whether pollution taxes or pollution permits are adopted as environmental policy instruments.
In order to find the reduced form for the interest rate, we substitute (II.3) into (II.2) to eliminate $\tilde{t}_p$. This yields the following relation between $\tilde{a}$, $\tilde{r}$ and $\tilde{P}$.

$$\sigma_y\tilde{r} = \alpha(1-\sigma_y)\tilde{P} + \tilde{a} \tag{D3}$$

By substituting (II.11') into (II.4) to eliminate $\tilde{T}_y$ and combining this result with (II.1) to eliminate $\tilde{y}$, we find another expression between abatement, the interest rate, and pollution:

$$\sigma_y(1-T_y)\tilde{r} - (1-T_y)\gamma\tilde{a} = [(1-T_y)(1-\sigma_y)\alpha\gamma + \sigma_y(\alpha\gamma - \epsilon\eta)]\tilde{P} \tag{D4}$$

We now substitute $\tilde{a}$ from (D3) into (D4) to arrive at the reduced form for the interest rate:

$$\tilde{r} = \frac{\alpha\gamma - \epsilon\eta}{(1-T_y)(1-\gamma)}\tilde{P} \tag{D5}$$

The reduced form for growth is found by means of (II.8) and (II.9). Combining (D5) with (D2), we derive the reduced form equation for profits:

$$s_w\tilde{w} = \frac{\delta}{\beta+\delta}(\alpha\gamma - \epsilon\eta)\tilde{P} \tag{D6}$$