CONSUMPTION AND INFORMATION

A decision theoretic approach

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1. Introduction

The problem of information has been studied up to now only to a small extent in economic theory. Exceptions are for example Stigler's {1} work which discusses the uncertainty of buyers with respect to the prices of goods, Marschak's {2} paper whose topic is the selection of an optimal information system and Arrow's {3} attempt to determine the value of and the demand for information.

The present contribution includes one more attempt in order to generalize the neoclassical models to models with risk and incomplete information. The main topic is the development of a buyer's decision model which allows the derivation of some kind of an individual demand function for information. For didactical purposes the model will first developed under conditions of complete confidence into the information medium (chapt. 2) - and then the more general model - given partial confidence (chapt. 4). One can easily characterize the topic by two questions:

a) how much money should a "rational" individual invest in information and
b) which medium of information should the buyer consult.

In order to solve these problems the indivudual has to use all his past information to estimate the possible gain of information and the utility of the future decision which is influenced by the information obtained. For this question it is irrelevant whether his knowledge about the reality is correct or objectivly false. What counts is the fact that the individual acts as if it had knowledge about reality. In chapter 3 an example is constructed to present some insights into the decision process, while in the last two chapters the conclusions and suggestions for extending the model are discussed.

2. The model

2.1. Assumptions:

In order to make the model precise, we state a few assumptions. Consider the following initial situation:
An individual (buyer, consumer) faces a certain want, which can be satisfied by a number of physical properties of specific consumption goods. This physical properties are called "characteristics"[2).

All commodities which are known by the consumer and which presumable have these characteristics, consitute the set of his alternatives.

We are stating the essential assumptions:
A.1: The consumer chooses from a set of n alternatives (which are called "types") exactly that element which assures him the greatest (expected) utility.

Remark: A.1 represents the assumption that the consumer behaves "rationally".

A.2: The (expected) utility of an alternative (type) T_i depends only on the occurrence of m "characteristics", that means

$$U(T_i) = U(b_{i1}\, u(C_1),\, b_{i2}\, u(C_2), \ldots,\, b_{im}\, u(Cm)),$$

whereby

$$b_{ik} = \begin{cases} 1 & \text{if characteristic } C_k \text{ occurs in type } T_i \\ 0 & \text{otherwise} \end{cases}$$

$u(C_k)$ = utility of characteristic C_k.

Remark: It is worth noting that "general" characteristics - let's take e.g. the characteristic "comfort" in the case of buying a car - have to be split up into "elementary" characteristics - like "heating appliance", "reclining seats", "alarm light" etc, for which a yes-or-no-decision about their occurrence is possible.

A.3: There exists one and only one n.m-matrix Θ for the consumer; the elements of Θ are probabilities Θ_{ik}; $i = 1,\ldots n$; $k = 1,\ldots,m$.

Remark: The element Θ_{ik} represents the subjective probability (in terms of the consumer), that the event "characteristic C_k occurs in type T_i" takes place.

Remark: The matrix Θ can be identified with the a-priori-estimations for the occurrence of the several characteristics in the different types. In other words: Θ contains all past "information ". It needs not be mentioned that the closer Θ_{ik} lies to 1, the greater is the subjective certainty for the occurrence of the corresponding event.

<u>Definition 1:</u> By <u>information</u> we understand yes-or-no-statements about the occurrence of the events: "characteristic C_k exists in type T_i"

<u>A.4:</u> For the consumer it is possible to receive information from one of s different information-media (information-services, informants, etc) I_l; $l=1,\ldots,s$.
The provision of information from the medium I_l causes the (constant) costs $K_l \geqq 0$.

<u>A.5:</u> The consumer has complete confidence in all information media, that means that he acts as if statements of the media corresponded exactly to reality.

<u>A.6:</u> For the consumer, there exists a unique monotone increasing transformation from utility-into money units.

2.2. Structure of the Model

2.2.1. Embedding the Model into the decision theoretic framework[3)]

The constituents for decision theoretic models are:

a) The set of actions. In our model this corresponds to the set of types $\{T_1, \ldots, T_n\}$

b) The set of "states of the world"[4)] $\{S_1, \ldots, S_q\}$. In the presented model the "states of the world" are clearly given by matrices S_j, the elements b^j_{ik} of which are 0 or 1. That means:

$$S_j = \begin{array}{cccccccc} C_1 & C_2 & \ldots\ldots & C_k & \ldots\ldots & C_m & \\ b^j_{11} & b^j_{12} & \ldots\ldots & b^j_{1k} & \ldots\ldots & b^j_{1m} & T_1 \\ \cdot & & & & & \cdot & \cdot \\ \cdot & & & & & \cdot & \cdot \\ & & & b^j_{ik} & & & T_i \\ \cdot & & & & & \cdot & \cdot \\ b^j_{n1} & b^j_{n2} & & b^j_{nk} & & b^j_{nm} & T_n \end{array}$$

Remark: If one wishes to compute the number q of possibilities of the "states of the world" in our case, one has to start with a pure n.m null-matrix, permute the first row (with 0's and 1's), then the second row and so on. The element b^j_{ik} then is the element in the i-th row, k-th column in the j-th permutation-matrix. Thus it is evident that the set of states consists of all n.m - matrices S_j with elements 0 or 1. Therefore this set contains $2^{n.m}$ elements, in other words it has the power (cardinal number) $2^{n.m}$. Hence the index j varies from 1 to $q=2^{n.m}$.

c) The set of consequences $\{e_{11},\ldots\ldots,e_{nq}\}$.
It exists a mapping $e:\{(T_i,S_j),\ i=1,\ldots,n;\ j=1,\ldots,q\} \to$ $\to \{e_{ij},\ i=1,\ldots n;\ j=1,\ldots q\}$, such that to every ordered pair (T_i,S_j) there is associated a unique consequence e_{ij}. In our model the consequence e_{ij} is given by the i-th row-vector of matrix S_j.

d) A preference relation on the set of consequences.
In the presented model the preference relation[5] is represented by a utility function u, which relates uniquely with every consequence e_{ij} a utility $U_{ij}=u(e_{ij})$. The utility function u is defined as function of the occurence of the characteristics C_k resp. their utilities $u(C_k)$:

$$U_{ij} = u(e_{ij}) = u(b^j_{i1}u(C_1),\ldots\ldots,b^j_{im}u(C_m))\ .^{6)}$$

e) In the case of risk[7] we have to complete our model with a probability distribution $\{p_1,\ldots,p_q\}$ on the set of states. The probabilities p_j can be obtained in different ways. We have decided to take an intuitive approach and we want to emphasize that this is only <u>one</u> out of a number of possible ways to define the probability distribution.[8]
For our task it seemed to be reasonable to use the following way of computation of the p_j's:
The problem is to find a mapping $P:\{S_j, j=1,..,q\} \rightarrow \{p_j, j==1,..,q\}$, such that to every state S_j there is associated one and only one probability p_j. Recalling the fact that the p_j's are subjective probabilities[9] we have to start from the consumer's subjective knowledge about "reality", i.e. from his matrix Θ of past information (see A.3). It seems logical to define the mapping P in such a manner that the "closer" a state S_j lies to the consumer's idea of real state, Θ, the greater is the corresponding probability p_j. To be more precise let's take some distance function between Θ and S_j, say $d(\Theta,S_j)$, and define:

$$\bar{p}_j = d^{-1}(\Theta,S_j) \qquad j=1,..,q \quad .$$

To get a probability distribution one has to standardize and hence the p_j's are given by:

$$p_j = \bar{p}_j / \sum_j \bar{p}_j \quad .$$

<u>Example:</u> Suppose p_j is defined by the following formula:

$$\text{(E1)} \qquad p_j = \left(\sum_{l=1}^{q} \sum_{i=1}^{n} \sum_{k=1}^{m} |\theta_{ik} - b_{ik}^{l}|\right)^{-1} \cdot \left(\sum_{i=1}^{n} \sum_{k=1}^{m} |\theta_{ik} - b_{ik}^{j}|\right)$$

$$j=1,\ldots,q \quad .$$

We have to show that the p_j's form a probability distribution:

- On the right side of (E1) only absolute values are summed and multiplied. Hence $p_j \geqq 0$ for all j.
- To check the standardization to 1 we have to sum up:

$$\sum_{j=1}^{q} p_j = \frac{\sum_{j=1}^{q} \sum_{i=1}^{n} \sum_{k=1}^{m} |\theta_{ik} - b_{ik}^{j}|}{\sum_{l=1}^{q} \sum_{i=1}^{n} \sum_{k=1}^{m} |\theta_{ik} - b_{ik}^{l}|} = 1 \ .$$

Note that in chapter 3 (numerical example) this method for constructing the p_j's is used.

To summarize we consider the decision matrix DM of our model:

$$\begin{array}{lccccc}
 & S_1 & \cdots & S_j & \cdots & S_q \\
 & p_1 & \cdots & p_j & \cdots & p_q \\
T_1 & U_{11} & \cdots & U_{1j} & \cdots & U_{1q} \\
\cdot & \cdot & & \cdot & & \cdot \\
T_i & U_{i1} & \cdots & U_{ij} & \cdots & U_{iq} \\
\cdot & \cdot & & \cdot & & \cdot \\
T_n & U_{n1} & \cdots & U_{nj} & \cdots & U_{nq}
\end{array} \qquad \text{(DM)}$$

2.2.2. The expected value of perfect information

In this section a concept for the computation of the (monetary) value of perfect information will be provided. We start with a well-known definition:

Definition 2: The expression

$$\sum_{j=1}^{q} U_{ij} \cdot p_j \qquad (1)$$

is said to be the expected utility of type T_i.

Remark: Sometimes[10)] the term (1) is also called "Bernoulli-Utility", "Neumann-Morgenstern-Utility", "Risk-Utility", "linear utility" etc.

In some situations it may be sufficient to use the expected utility for consumption decisions. Suppose but the consumer wants to gather information (in the sense of definition 1). The problem is to decide whether information gathering is presumable profitable. To solve this,one can imagine the following information-decision process:
The information given by a information medium is: "The real state of the world is characterized by matrix S_r". This answer is expected with a (subjective!) probability p_r by the individual. After obtaining this information he chooses the maximal element out of the r-th column in his decision matrix (DM).Thus we give the following

<u>Definition 3</u>: The <u>expected utility in the case of perfect information</u> is given by

$$\sum_{j=1}^{q} \max_{i=1,..,n} \{U_{ij}\} p_j \qquad (2) \quad .$$

It seems to be reasonable to balance the maximal expected utility without information provision against expected utility in the case of perfect information. The following definition is due to this task.

<u>Definition 4</u>: The difference D between expression (2) and the maximum of (1),

$$D = \sum_{j=1}^{q} \max_{i}\{U_{ij}\}.p_j - \max_{i}\{\sum_{j=1}^{q} U_{ij}p_j\} \qquad (3) \quad ,$$

is said to be the <u>expected value of perfect information</u>.

<u>Proposition 1</u>: The expected value of perfect information is always nonnegative.

Proof: It is immediately evident that

$$\max_{i=1,n} \{\sum_{j=1}^{q} U_{ij}p_j\} \leqq \sum_{j=1}^{q} \max_{i=1,n} \{U_{ij}p_j\} \quad , \text{ q.e.d.}$$

Remark: In term of the consumer proposition 1 states that information provision always can be expected to be profitable (expressed in utility units).

Now the consumer is capable to compute the monetary value $M(D)$ of the expected value of perfect information (see A.6!). According to his individual utility-money transformation he relates with every utility amount D a unique monetary value $M(D)$, and thus it's possible to compare the money amount $M(D)$ with the costs K_1 of the various information media.

Definition 5 : The set

$$I_p = \{I_1 : M(D) \geq 0\}$$

is said to be the set of profitable information media.

So we find an easy answer to the initial question whether information should be provided.
If I_p is an empty set ($I_p = \emptyset$), no profitable information medium exists and it is not economically reasonable to request informatiom. The consumer decides according to the Bernoulli-principle (A.1) on the basis of his a-priori-information.
If I_p is nonempty ($I_p \neq \emptyset$) the rational consumer chooses the cheapest I_1 of the set of profitable information media.

3. Numerical Example

An Individual is looking for an appartment. The relevant characteristics for his choise are:

C_1 : balcony

C_2 : quiet surroundings

He finds two alternatives, that means two appartments, offered in a newspaper advertisement. The first appartment lies in a upper-class district of town whereas the other one is situated in an lower-class district. Therefor his a-priori-estimaticns for the occurence of C_1 and C_2 in appartment T_1 resp. T_2 are given by the matrix

$$\Theta = \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.35 \end{bmatrix}$$

expressed in words this means that -for example- the event "appartment T_2 has quiet surroundings" has a (subjective) probability of 0.35.

The questions to be answered is: What amount of utility is he able to gain if he gets perfect information about the ocurrence of C_1 and C_2 in the alternatives T_1 and T_2?

(The utilities of C_1 and C_2 be $u(C_1)=2$, $u(C_2)=5$).

The expected value of perfect information is given by expression (3):

$$D = \sum_{j=1}^{q} \max_{i=1,n} \{U_{ij}\} p_j - \max_{i=1,n} \{ \sum_{j=1}^{q} U_{ij} p_j \}$$

To begin with the computation of the U_{ij}'s we first consider the possible states of the world $S_j, j=1(1)q,\ q=2^{2 \cdot 2}=16$. (2 characteristics, 2 alternatives).

$$S_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_4 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$S_5 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots$$

$$S_9 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots$$

$$S_{13} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad S_{16} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

To get U_{ij}'s we have to make another assumption about the form of the utility function $U\ (e_{ij})$. For convenience this functions be linear:

$$U_{ij} = \sum_{k=1}^{2} b^j_{ik} \cdot u(C_k) = b^j_{i1} \cdot u(C_1) + b^j_{i2} \cdot u(C_2) \quad .$$

The factors b^j_{ik} are defined (as in chapter 2) as elements of the matrix S_j.

Thus U_{ij}'s are:

$U_{11} = 0$	$U_{12} = 5$	$U_{13} = 2$	$U_{14} = 7$	$U_{15} = 0$	$U_{16} = 5$	$U_{17} = 2$	$U_{18} = 7$
$U_{19} = 0$	$U_{1,10} = 5$	$U_{1,11} = 2$	$U_{1,12} = 7$	$U_{1,13} = 0$	$U_{1,14} = 5$	$U_{1,15} = 2$	$U_{1,16} = 7$
$U_{21} = 0$	$U_{22} = 5$	$U_{23} = 0$	$U_{24} = 0$	$U_{25} = 5$	$U_{26} = 5$	$U_{27} = 5$	$U_{28} = 5$
$U_{29} = 2$	$U_{2,10} = 5$	$U_{2,11} = 2$	$U_{2,12} = 2$	$U_{2,13} = 7$	$U_{2,14} = 7$	$U_{2,15} = 7$	$U_{2,16} = 7$

The next step would be to cumpute the probability distribution for the states S_j. The probabilities p_j are determined by the following formula (cf.chapter 2):

$$p_j=\left(\sum_{l=1}^{16}\sum_{i=1}^{2}\sum_{k=1}^{2}|\theta_{ik}-b_{ik}^{l}|\right)^{-1}\cdot\left(\sum_{i=1}^{2}\sum_{k=1}^{2}|\theta_{ik}-b_{ik}^{j}|\right)$$

To demonstrate the application of the above formula let's take for instance p_{10} :

$$p_{10}=\left(\sum_{l=1}^{16}\sum_{i=1}^{2}\sum_{k=1}^{2}|\theta_{ik}-b_{ik}^{l}|\right)^{-1}\cdot\left(\sum_{i=1}^{2}\sum_{k=1}^{2}|\theta_{ik}-b_{ik}^{10}|\right) .$$

$$(b_{ik}^{10}) = S_{10} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \qquad \sum_{i=1}^{2}\sum_{k=1}^{2}|\theta_{ik}-b_{ik}^{10}| = 2.35 .$$

$$\left(\sum_{l=1}^{16}\sum_{i=1}^{2}\sum_{k=1}^{2}|\theta_{ik}-b_{ik}^{l}|\right)^{-1}= 0.033 \quad => \quad p_{10} = 0.077 .$$

Consequently we have determined the factors of expression (3) and we only need to carry out the computations for the expected value of perfect information D.

j	P_j	$U_{1j}P_j$	$U_{2j}P_j$	$\max\limits_i \; U_{ij} \; .P_j$
1	.064	0	0	0
2	.057	.285	0	.285
3	.044	.088	0	.088
4	.037	.259	0	.259
5	.074	0	.370	.370
6	.067	.335	.335	.335
7	.054	.108	.270	.270
8	.047	.329	.235	.329
9	.084	0	.168	.168
10	.077	.385	.154	.385
11	.064	.182	.128	.128
12	.057	.399	.114	.399
13	.094	0	.658	.658
14	.087	.475	.609	.609
15	.074	.148	.518	.518
16	.067	.469	.469	.469
	1.000	3.384	4.028	5.270

The result therefore is that the expected value of perfect information for this decision problem (in utility units) is:

$$D = \sum_{j=1}^{16} \max_{i=1,n} \{U_{ij}\}p_j - \max_{i=1,n} \{ \sum_{j=1}^{16} U_{ij}p_j\} =$$

$$= 5.270 - 4.028 = \underline{\underline{1.242}} \; .$$

4. Generalisation of the model

In this chapter the model will be extended in so far as A 5 - complete confidence of the consumer in the informant - will be weakened. This is done by assuming, that the consumer has different confidence in different media of information. We modify the model in the following way:

4.1. Assumptions

The assumptions A 1, A 2, A 3, A 4 and A 6 remain unchanged.

Instead of A 5 we formulate

A 5^+: The consumer associates with each medium of information exactly one real number R_l whereby $0 \leqq R_l \leqq 1$ for all l.

R_l is a quantitative expression of the degree of confidence of the consumer in the informant I_l.

Remark: $R_l = 1$ complete confidence $R_l = 0$ no confidence at all.

4.2. The expected value of partial information

We are able to embed our generalized model in an anlogous way into the fundamental scheme of decision theory. Therefore we are discussing at this stage only the essential extensions of chapter two.

Definition 6: Under partial information we understand information - in the above sense -, which is considered only partially reliable by the receiver of the information.

Remark: Definition 6 of partial information corresponds to assumption A.5^+, whereby the coefficients R_1 express partial confidence in the information media.

Suppose now the consumer achieves information from medium I_1, to which he relates (A.5^+!) the confidence -coefficient R_1. What is the influence of that information?

To answer this question we use Bayes's formula[11] to show how the original probability distribution P is changed. Let's assume information medium I_1 had given the answer (the information) "State S_k is the true one". Thus by $P_j^k(1)$ we denote the probability of state S_1, provided medium I_1 gave information S_k. Application of the Bayes formula:

$$P_j^k(1) = P(S_j | S_k^1) = \frac{P(S_j) \,.\, P(S_k^1 | S_k)}{\sum\limits_{t=1}^{q} P(S_t) P(S_k^1 | S_t)}$$

and

$$P(S_k^1 | S_j) = \begin{cases} (1-R_1)/(q-1) & \text{for all } j \neq k \\ R_1 & j=k \end{cases}$$

<u>Definition 7</u>: The expression

$$\sum_{h=1}^{q} \max_{i=1,n} \{ \sum_{j=1}^{q} U_{ij} P_j^h(1) \} P_h \qquad (4)$$

is said to be the <u>expected utility under partial information (with regard to information medium I_1)</u>.

In order to explain definition 7 we want to stress that the only difference between the expected utility under partial information (4) and the expected utility under perfect information (2) lies in the sets to be maximized. In expression (2) among the U_{ij}'s the maximal element U_j^+ will be chosen, whereas in (4) the maximization will be done within a set of Bernoulli-utilities

$$\{ \sum_{j=1}^{q} U_{ij} P_j^h(1) \ , \ i=1,\ldots,n \},$$

so that the maximal element of this set is a expected utility itself.

<u>Lemma</u>: The expected utility (2) under perfect information is a special case of the expected utility (4) under partial information , namely in the case $R_1 = 1$, that means for a comletely reliable information medium I_1.

Proof:

Kronecker's δ: $\qquad \delta_{jh} = \{ \begin{matrix} 1 & \\ 0 & j \neq h \end{matrix}$

Now start from expression (4):

$$(4) = \sum_{h=1}^{q} \max_{i=1,n} \{ \sum_{j=1}^{q} U_{ij} P_j^h(1) \} p_h =$$

$$= \sum_h \max_i \{ \sum_j U_{ij} \frac{p_j(\delta_{jh}R_1 + (1-\delta_{jh})(1-R_1)/(q-1)}{\sum_{t=1}^{q} p_t(\delta_{th}R_1 + (1-\delta_{th})(1-R_1)/(q-1)} \} p_h$$

Suppose now $R_1=1$ and continue:

$$(4)_{(R_1=1)} = \sum_{h=1}^{q} \max_{i=1,n} \{ U_{ih} \frac{p_h}{p_h} \} p_h = \sum_{j=1}^{q} \max_{i=1,n} \{ U_{ij} \} p_j =$$

$$= (2) \qquad \text{q.e.d.}$$

(Note: $p_j \equiv P(S_j)$ for all j)

Definition 8: The expected value of partial information (with regard to information medium I_1) is given by D_1:

$$D_1 = \sum_{h=1}^{q} \max_{i=1,n} \{ \sum_{j=1}^{q} U_{ij} P_j^h(1) \} p_h - \max_{i=1,n} \{ \sum_{j=1}^{q} U_{ij} p_j \} \qquad (6)$$

Remark: The monetary value $M(D_1)$ of the expected value of partial information can be obtained in a similar way like in the case of perfect information (cf. A.5).

Definition 9: By the set of feasible media of information we mean in the sequel the set I_f:

$$I_f = \{ I_1 : M(D_1) - K_1 \geq 0 \} .$$

<u>Proposition 2</u>: D_l, the expected value of partial information is always nonnegativ:

Proof: Suppose l fixed, but arbitrary. Therefore we write for $P_j^h(l)$: P_j^h.

Now let $U_i(h) = \sum_{j=1}^{q} U_{ij} P_j^h . P_h$ for $i=1,..,n$; $h=1,..,q$

Then:

$$\sum_{h=1}^{q} \max_{i=1,n} \{ \sum_{j=1}^{q} U_{ij} P_j^h \} p_h = \sum_h \max_i \{ U_i(h) \} \geqq$$

$$\geqq \max_i \{ \sum_h U_i(h) \} = \max_i \{ \sum_h \sum_j U_{ij} P_j^h p_h \} =$$

$$= \max_i \{ \sum_h \sum_j U_{ij} P(S_j | S_h) P(S_h) \} =$$

$$= \max_i \{ \sum_h \sum_j U_{ij} P(S_h | S_j) P(S_j) \} =$$

$$= \max_i \{ \sum_j U_{ij} P(S_j) . \sum_h P(S_h | S_j) \} = \max_i \{ \sum_j U_{ij} p_j \} .$$

(Note:

$$\sum_{h=1}^{q} P(S_h | S_j) = 1 \quad \text{for all } j$$)

q.e.d.

Proposition 3: The expected value of partial information (with regard to an arbitrary information medium I_l), D_l, is always smaller or equal to the expected value of perfect information D.

Proof: Again l be fixed, but arbitrary, therefore $P_j^h(l) = P_j^h$. Remember also that $p_j \equiv P(S_j)$ for all j. It has to be shown that $D_l \leqq D$:

$$\sum_h \max_i \{\sum_j U_{ij} P_j^h\} p_h \leqq \sum_h \sum_j \max_i \{ U_{ij} P_j^h\} p_h =$$

$$= \sum_h \sum_j \max_i \{U_{ij}\} P(S_j | S_h) P(S_h) =$$

$$= \sum_h \sum_j \max_i \{U_{ij}\} P(S_h | S_j) P(S_j) =$$

$$\Rightarrow \sum_j \max_i \{U_{ij}\} p_j \cdot \sum_h P(S_h | S_j) = \sum_j \max_i \{U_{ij}\} p_j$$

q.e.d.

Remark: By propositions 1 - 3 follows that the expected value of partial information lies always between 0 and D, the expected value of perfect information. The consequence is that, given a fixed matrix Θ, one can associate with every parameter R_l a unique expected value of partial information D_l and further a unique amount of money $M(D_l)$, corresponding to the individual utility-money transformation, which lies between 0 and M(D).

Corrolary: I_f, the set of feasible information media is subset of I_p, the set of profitable information media.

Proof: Considering assumption A.6, definitions 5 and 8, one can easily see the truth of this statement by application of proposition 3.

5. Conclusions

In this part we summarize once more the decision process of a consumer as it was developed in chapter 2 and 4. According to the complexity of the whole procedure it seems to be appropriate to visualize it by a flow chart. First a short explanation:

<u>Step 1</u>: Assume all informants I_l to be completely reliable (R_l=1 for all l). Then compute the expected value of perfect information D and its monetary value M(D). Next form the set I_p, the profitable informants. If $I_p = \emptyset$, there exists no medium of information whose consultation is expected to be an economic gain. Therefore no information will be required. If $I_p \neq \emptyset$ go to

<u>Step 2</u>: Now find out the subset of feasible information media, I_f. If $I_f=\emptyset$ there are no feasible information media and the consumer does not invest in further information. If I_f is not empty the consumer chooses (according to A.1) the informant I_{l+}, for which holds

$$M(D_{l+}) - K_{l+} = \max_{I_l \varepsilon I_f} \{M(D_l) - K_l\} .$$

Flow chart of the consumer's decision process

input:

$T_1, \ldots, T_n$	alternatives
$C_1, \ldots, C_m$	characteristics
Θ	n.m matrix of past information
$R_1, \ldots, R_s$	confidence parameters
$K_1, \ldots, K_s$	costs of information provision
$u(C_1), \ldots, u(C_m)$	utilities of characteristics
$M(D), M(D_1)$	utility-money transformation

1st step:

assume $R_1 = 1$ for all l

compute D and M(D)

form $I_p = \{I_1 : M(D) - K_1 \geq 0\}$

$I_p = \emptyset$?

jes

no

2nd step:

compute D_1 and $M(D_1)$ for $I_1 \varepsilon I_p$

form $I_f = \{I_1 : M(D_1) - K_1 \geq 0\}$

$I_f = \emptyset$?

no

jes

output:

optimal decision:
Decide on a-priori-information Θ and do not invest in further information

optimal decision:
If $M(D_1) - K_1 = 0$ for all $I_1 \varepsilon I_f$, then take (e.g.) for informant the maximal reliable one (R_1^{max}), otherwise buy information from informant I_{1+}, for which holds:

$$M(D_{1+}) - K_{1+} = \max_{I_1 \varepsilon I_f} \{ M(D_1) - K_1 \}$$

6. Future Aspects

There is no need for special reflections in order to see that the above model is extendable and applicable in many ways. Nevertheless we want to mention very briefly some aspects for future development.

- The model has purelystatic or rather comparative static character. One possible extension is for example to formulate the model in a dynamic form and to investigate a series of decision processes over a period of time. In this case, one has e.g. the possibility to include the costs of lags associated with the time needed for information provision. It is also reasonable to study learning effects with respect to:
    - choice of relevant characteristics
    - adaption of a-priori-probabilities θ_{ik}
    - adaption of the confidence parameters R_1

- Another generalization is possible by introducing changes in the structure of the information-provision-process:
  - The different information media are not able to give information about all alternatives, but only about one or some alternatives.
  - There is the possibility to get different quantities of information at corresponding costs (cf. Shannon's

measure of information, problems of stochastic programming)

- A further application is given in the field of political science, For instance to simulate voting decisions under different stages of information.

- An important problem for further research will lie in the solution of the corresponding aggregation problem in order to construct an aggregate demand function for information.

7. Footnotes

1) The authors are grateful to Professor Bradford, Professor Chipman and Professor Cornwall for helpful comments on earlier versions of the paper. They are also indebted to Abdur Rahman, Uwe Schubert and Gerhart Schwödiauer for their critical suggestions.

2) Lancaster {4} developed a complete theory of consumer's behaviour based on "characteristics" in his recent book. See especially pp. 6-8

3) See for example {5} pp. 13-21 or {6} pp. 7-14

4) See {5} pp. 8-10

5) A preference relation defined on the set of consequences is a relation R for which for any e_1, e_2, e_3 which are elements of the set of consequences, the following properties hold:

a)	$e_1 R e_1$	reflexivity
b)	$e_1 R e_2$ & $e_2 R e_3 \Rightarrow e_1 R e_3$	transitivity
c)	$e_1 R e_2$ or $e_2 R e_1$	comparability

"R" should be read as "preferred or indifferent" .

6) This function need not be linear.

7) A situation of risk is characterized by the fact that the actor is able to assign certain probabilities to each state of the world. This definition was introduced by F.H.Knight {7}.

8) There are two ways in order to justify this procedure.
   a) To investigate the predictive power of this approach empirically, or
   b) to find certain plausible axioms from which this approach can be derived logically.

9) See e.g. Savage {5}, who called it "personal probability" (chapt.3 pp.27-55 and chapt.4, pp. 56-68).

10) See {6} p. 61, footnotes 1,2

11) See e.g. {5} p.45

8. References

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{2} J.Marschak, "Economics of Information Systems", Journal of the American Statistical Association, vol.66,1971, pp.192-219

{3} K.J.Arrow, "The value of and the demand for information", Essays in the Theory of Risk Bearing, Amsterdam-London, North Holland 1970, pp.267-278

{4} K.Lancaster, Consumer Demand, New York - London, Columbia University Press, 1971

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