SEASONALITY IN THE AUSTRIAN ECONOMY: COMMON SEASONALS AND FORECASTING[1]

Robert M. KUNST

Forschungsbericht/
Research Memorandum No. 305

September 1992

1 This paper was written while the author was visiting professor at the University of California San Diego.

ABSTRACT

Seasonal cointegration generalizes the idea of cointegration to processes with unit roots at frequencies different from 0. Here, also the dual notion of common trends, "common seasonals", is adopted for the seasonal case. Using a five-variable macroeconomic core system of the Austrian economy, it is demonstrated how common seasonals and seasonal cointegrating vectors look in practice. Statistical tests provide clear evidence on seasonal cointegration in the system. However, it is shown that accounting for seasonal cointegration does not necessarily entail increased predictive accuracy for individual series. For the "master series" of GDP, seasonal cointegration modeling is profitable for very short forecast horizons but for long horizons simple seasonal dummies appear to dominate.

ZUSAMMENFASSUNG

Saisonale Kointegration generalisiert die Idee der Kointegration auf Prozesse mit Einheitswurzeln an von Null unterschiedlichen Frequenzen. Hier wird auch der hiezu duale Begriff der gemeinsamen Trends auf den Saisonfall angewandt: "gemeinsame Saisonzyklen". Anhand eines fünfdimensionalen österreichischen Kernsystems mit makroökonomischen Zeitreihen wird demonstriert, wie solche saisonale Kointegrationsvektoren und gemeinsame Zyklen in der Praxis ausschauen. Obwohl jedoch statistische Tests klare Evidenz für das Auftreten von saisonaler Kointegration bringen, bleiben die Resultate eines Vergleichs prognostischer Fähigkeiten mit herkömmlichen Modellen hinter den Erwartungen zurück.

1. Introduction

The notion of cointegration was developed in the late 1970s [1] in conjunction with error correction models, the standard example relating changes in consumer expenditures to corresponding changes in personal disposable income but also to past deviations of actual consumption from its "long-run equilibrium" share of income. In that example, consumption and income were called cointegrated as the model

$$\Delta C_t = a_0 + a_1 \Delta Y_t + a_2 (C_{t-1} - \beta Y_{t-1}) + \varepsilon_t \quad (1)$$

implies integrated (difference-stationary) C and Y but stationary "error correction" or cointegrating variable $C-\beta Y$, provided certain parameter restrictions hold, such as $a_2 < 0$. Such models responded to the then frequent complaint that, while modeling in the original variables incurs strange and lavish lag structures, modeling in first differences lets all variables drift apart in longer-run simulations. The cointegrating model is the perfect amalgam of the two philosophies: it is essentially a model in differences but nevertheless C and Y are tied together in the long run, with the strength of the ties indicated by a_2.

Conceptions like cointegration, error correction, and common trends quickly spread into virtually all areas of applied economic research. [2] The tremendous success of the model is due partly to its power in resolving old empirical puzzles such as the static aggregate consumption function, partly to the concurrent surge of integrated-process models in the real business cycles literature. Since cointegration makes little sense in the classical world of economic processes fluctuating around fixed trend lines or curves, these "trend-stationary" models had to get out of the way to make place for a cointegrated universe. The battle between these two views of the world appears to continue but the defense lines of the advocates of trend stationarity are receding.

If the basic concept of cointegration is seen as a set of individually non-stationary processes on the one hand and a linear relation among several variables of the set removing the source of non-stationarity on the

---

[1] Historicists often cite Sargan (1964) as an early source for error correction models. The full consequences of the model, however, lay dormant for almost two decades

[2] The macroeconometric LIMA model by the Institute for Advanced Studies is also effectively using error correction designs for consumption as well as for investment and other aggregates.

other, several generalizations are of immediate attractiveness. Structural breaks, outliers, or seasonal patterns in time series are among the sources of non-stationarity encountered most frequently in practice. Up to now, however, such "common features" (Engle and Kozicki [1990]) did not generate an impact comparable to the now classical cointegration model.

Particularly with respect to seasonality, an eventual success of the "seasonal cointegration" model - viz., the coincidence of individually "seasonal unit roots" processes with a non-seasonal linear combination - would rely crucially on invalidating the "seasonal dummies", on empirical or on theoretical grounds. Contrary to the trend situation, advocates of fixed patterns have a firm grip on the battlefield of seasonality.

In consequence, neither of the two core articles on seasonal cointegration by Engle et al. (1989) and Hylleberg et al. (1990, HEGY) gives an exhaustive treatment of the phenomenon as such. Engle et al. (1989) report the apparition of seasonal cointegration in an empirical project and suggest some ideas how to cope with it. The follow-up paper by HEGY brings in all the required algebra and presents tests for seasonal integration in univariate problems. Nevertheless, it clings to the tradition of reducing multivariate cointegration problems to univariate residual testing (compare Engle and Granger [1987]).

The manuscript by Lee (1989) is remarkable for two reasons. Firstly, it actually indicates the solution for the problem of efficiently estimating seasonal cointegrating structures in Gaussian vector autoregressions. Secondly, it gives asymptotic distributions for the relevant test statistics. Building on Johansen (1988,1991), the seasonal cointegration problem is consistently treated in a multivariate vector autoregression framework.

A different way is adopted in the recent contribution by Joyeux (1992) who builds on frequency domain properties of cointegrated series at arbitrary frequencies and generalizes the problem to non-integer integration orders. Her suggested test relates to the frequency-domain cointegration test by Phillips and Ouliaris (1987) the same way that Lee's method relates to Johansen's cointegration test. While the merits of the former procedures are greater generality beyond the restriction of vector autoregressions, the merits of the latter approaches are greater clarity and simplicity and all-in-one testing and estimation.

The organization of this paper is as follows. Section 2 summarizes the main results concerning seasonal cointegration from the literature, particularly from HEGY and Lee (1989). It also mentions the main alternatives to the model. Section 3 takes a look at the seasonal counterpart of common trends, the "seasonal common factors". Section 4 focuses on the prediction of seasonal data in the presence of seasonal cointegration. Section 5 demonstrates the points of the previous sections on an exemplary macroeconomic system comprising five Austrian accounts series. Section 6 concludes.

2. Seasonal cointegration

The following will only be concerned with the case of quarterly observations. All results can be - and most already have been - extended to the monthly case or to other less customary frequencies. Moreover, it will be assumed that all series within an N-vector require seasonal differencing to obtain stationarity. In this case, the seasonally differenced series can be represented as a moving average process

$$(1-B^4)X_t = A(B)\varepsilon_t (2)$$

with B denoting the lag operator. X_t is said to "have unit roots at ±1 and ±i", the zeros of $\Delta_4 = 1-B^4$. The following argument holds as long as a rational function is assumed for A(B). Formally, Δ_4 can be inverted

$$X_t = \Delta_4^{-1}A(B)\varepsilon_t = (1-B)^{-1}(1+B)^{-1}(1+B^2)^{-1}A(B)\varepsilon_t \quad (3)$$

The isomorphism between the lag operator and a complex variable allows a representation in partial fractions

$$X_t = (C_1(1-B)^{-1}+C_2(1+B)^{-1}+(C_3+C_4B)(1+B^2)^{-1})A(B)\varepsilon_t \quad (4)$$

If N=1 and A(B)=1 then C_1=C_2=1/4 and C_3=1/2 and C_4=0. For N=1 and general polynomial A(B), this can be included in the partial fractions representation to yield

$$X_t = C_1Y_{1t} + C_2Y_{2t} + C_3Y_{3t} + C_4Y_{3t}-1 + p(B)q^{-1}(B)\varepsilon_t \quad (5)$$

with polynomials p(.) and q(.) and

$$Y_{1t} = \Sigma^{t}_{s=0}\, \varepsilon_{t-s},\ Y_{2t} = \Sigma^{t}_{s=0}\, (-1)^{s} \varepsilon_{t-s},\ Y_{3t} = \Sigma^{t/2}_{s=0}\, (-1)^{s} \varepsilon_{t-2s} \quad (6)$$

Hence, X_t can be decomposed into a random walk, two periodical "seasonal" processes - Y_2 with a semi-annual and Y_3 with an annual periodicity -, and a stationary remainder. All components are driven by the same innovations process ε_t, thus this decomposition is not what many economists have in mind when they want to decompose a process into "trend, season, and cycles", struggling to disentangle "permanent shocks" to trends from "transient shocks" to cycles or from "seasonal shocks".

It is immediate from (5) that $C_2=0$ would cause 1+B to cancel from A(B) in (2) and one of the basic seasonal cycles to be absent. It should also be mentioned that the general solution of the difference equation (2) admits four arbitrary seasonal constants as an added deterministic part of (5).

If N>1 and all C_i are NxN-matrices, then

$$X_t = C_1 Y_{1t} + C_2 Y_{2t} + C_3 Y_{3t} + C_4 Y_{3t-1} + Q(B)\varepsilon_t \quad (7)$$

with Q(B) being the ratio of two matrix polynomial operators, obeying to the usual roots restrictions for stationary ARMA processes. If any of the Ci matrices is zero, this implies the absence of certain unit roots in X. In detail, $C_1=0$ means that X has no random walk component and thus $(1+B)(1+B^2)$ suffices to stationarize X, contrary to the initial assumption. $C_2=0$ means that the quick seasonal component is absent and thus $(1-B)(1+B^2)$ suffices to render X stationary, again at odds with the assumptions. $C_3=C_4=0$ means that the full four-quarters cycle Y_{3t} plays no role and $1-B^2$ would stationarize X, letting the process consist of a random-walk trend, a semi-annual cycle, and short-run dependency. Interpretations of individual $C_3=0$ or $C_4=0$ are less straightforward.

Eventual rank deficiencies in these matrices directly bring in events of cointegration. Suppose C_1 is of rank N-1. Then there is a non-trivial vector β with $\beta' C_1=0$. But then

$$\beta' X_t = \beta' C_2 Y_{2t} + \beta' C_3 Y_{3t} + \beta' C_4 Y_{3t-1} + \beta' Q(B)\varepsilon_t \quad (8)$$

Hence, the 1-dimensional process $\beta'X$ can be written as the sum of non-stationary cycles and a stationary ARMA process but does no more contain any random walk component. Hence, β is a cointegrating vector in the classical sense or rather "a cointegrating vector at frequency zero" or a $CI_0(1,1)$ vector.

In general, the nullity of C_1 equates the number of linearly independent $CI_0(1,1)$ vectors or, in other words, the cointegrating rank at frequency 0. Likewise, any β vector in the nullspace of C_2 admits a representation like

$$\beta'X_t = \beta'C_1Y_{2t} + \beta'C_3Y_{3t} + \beta'C_4Y_{3t-1} + \beta'Q(B)\varepsilon_t \quad (9)$$

Thus, $\beta'X$ is the sum of an N-dimensional random walk, of two non-stationary cycles with annual periodicity and different phase, and a stationary ARMA component. Any dependence on semi-annual cycles has ceased and β can be regarded as cointegrating at frequency π, in short as $CI_\pi(1,1)$. Analogously, the nullity of C_2 is the cointegrating rank at π.

For the frequency $\pi/2$, the situation becomes more complicated because of polynomial cointegrating vectors (PCIV) of the form $\beta_1+\beta_2B$. Such PCIV succeed in canceling any influence of annual cycles from $\beta_1X_t + \beta_2X_{t-1}$ but are not in the nullspace of any of the matrices C_3 or C_4. If either $C_3=0$ or $C_4=0$, the nullspace of the other one of the two contains the $CI_{\pi/2}(1,1)$ vectors in strict analogy to the frequencies 0 and π. The occurrence of PCIV is a direct consequence of the solution of $1+B^2$ being a pair of complex conjugates and of the spectral density being forced into symmetry around π (or 0) by discrete time. [3]

All these features can perhaps be analyzed best in a vector autoregressive framework which has been just the idea taken up by Lee (1989) who extended Johansen's (1988,1991) ingenious maximum likelihood strategy for joint estimation and testing in cointegrated systems to the seasonal case.

Any vector autoregression can be written as (HEGY [1990])

$$\Phi(B)\Delta_4X_t = D_1Y_{1t-1} + D_2Y_{2t-1} + D_3Y_{3t-1} + D_4Y_{3t-2} + \varepsilon_t \quad (10)$$

[3] The term "aliasing" which is used in connection with complex unit roots commonly refers to the effects of continuous-time high-frequency cycles in discrete time. Here, however, the correspondence of -i to $3\pi/2$ and to cycles of a length of 4/3 quarters is merely algebraic and fictitious.

with $\Phi(B)$ being a finite-order polynomial matrix with all zeros of its determinant outside the unit circle and D_1, D_2 being matrices whose rank yields the number of linearly independent cointegrating relations at frequency 0 and π. Because of the transition to the "error correction" form with the focus on first differences of all variables, the meaning of the matrices has changed to "mirror images", the nullspaces of C_i matrices equaling the row spaces of D_i.[4] Moreover, Y_i now denote observed transforms of the original variables, in detail

$$Y_{1t} = (1+B+B^2+B^3)X_t \quad Y_{2t} = (1-B+B^2-B^3)X_t \quad Y_{3t} = (1-B^2)X_t \quad (11)$$

Nevertheless, the meaning of the components is comparable to those in (5), with Y_1 being trending but non-seasonal and Y_2 and Y_3 being non-trending and displaying seasonal cycles at certain frequencies only.

If D_2=D_3=D_4=0, the system (10) can be divided through the operator $1+B+B^2+B^3$ and, except for seasonal deterministics, Johansen's cointegration form is recovered. It is known that the maximum likelihood estimator of D_1 can be expressed as $\alpha\beta'$ with β consisting of all eigenvectors to all non-zero eigenvalues v of the generalized eigenvalue problem

$$|vS_{pp}-S_{p0}S_{00}^{-1}S_{0p}| = 0 \quad (12)$$

In (12), S_{00} and S_{pp} are empirical second moments matrices of ΔX_t and X_{t-1}, conditioned on short-run influences ΔX_{t-1}, ΔX_{t-2}, ..., ΔX_{t-p+1}, and S_{0p} and S_{p0} are similarly defined cross-moment matrices.[5] Exploiting the asymptotic independence between Y_{1t}, Y_{2t}, Y_{3t}, Lee (1989) accordingly suggests to calculate the maximum likelihood estimator for β in $D_1 = \alpha_1\beta_1'$ again from (12) but now conditioning Δ_4X_t as well as X_{t-1} on Y_{2t-1}, Y_{3t-1}, Y_{3t-2}, additional to the short-run influences Δ_4X_{t-1}, ..., Δ_4X_{t-p+4}. Similarly, solution of the problem

$$|vR_{pp}-R_{p0}R_{00}^{-1}R_{0p}| = 0 \quad (13)$$

---

[4] In detail, if D_i can be written as $\alpha\beta'$ with α, β having full column rank, then C_i is $\beta^{\dagger}R\alpha^{\dagger}$, with full-rank R. For i=1,2, this follows directly from generalizing Granger's representation theorem (see Engle and Granger [1987]).

[5] Relative to Johansen (1988), notation has been changed slightly for more convenience in the seasonal model.

with R_{pp} the conditional moments matrix of Y_{2t-1}, R_{00} the conditional moments matrix of Δ_4X_t, and R_{0p}, R_{p0} again conditional cross-moments matrices, conducting conditioning in all cases on Δ_4X_{t-1}, ..., Δ_4X_{t-p+4}, and Y_{1t-1}, Y_{3t-1}, Y_{3t-2}, yields an Nxr_2-matrix β_2. This matrix is the right factor in $D_2 = \alpha_2\beta_2$' whose rank is r_2, the number of cointegrating vectors at π.[6] In other words, β_1 and β_2 contain those vectors that maximize correlation between $\beta_1'Y_{1t-1}$ and $\beta_2'Y_{2t-1}$ versus Δ_4X_t by solving a canonical correlation problem. One looks for linear combinations of the de-seasonalized but trending Y_1 and of the trend-free but only partially de-seasonalized Y_2 which correlate well with stationary Δ_4X_t. The canonical eigenvector problem generates as side-products the required loading matrices α_1 and α_2 and a sequence of test statistics which help in determining r_1 and r_2, the ranks of D_1 and D_2.

In order to determine D_4, D_3 is assumed as 0 and the canonical problem of Δ_4X_t versus Y_{3t-2} is solved after conditioning on the short run and on Y_{1t-1}, Y_{2t-1}. A similar procedure could be adopted to determine D_3 after restricting D_4=0 but the synchronous annual fluctuations are generally preferred to the asynchronous ones. Actually, free estimation and secondary transformation in real-world system usually renders comparatively small values for the D_3 entries, so the synchronicity assumption can be sustained in empirical situations.

Again, all results can be generalized to the monthly case or to other periodicities of empirical relevance. For monthly data, all roots except those at -1 (frequency π) and +1 (frequency 0) are again subject to PCIV, hence easy results can only be obtained under synchronicity assumptions. Apart from the Nyqvist frequency π (2 months) and 0, relevant frequencies are $5\pi/6$ (2.4 months), $2\pi/3$ (3 months), $\pi/2$ (4 months), $\pi/3$ (6 months), $\pi/6$ (1 year). Other relevant periodicities may include 5 for trading days and daily observations and 6 for bimonthly observations.

3. Common seasonal factors

Beveridge and Nelson (1981) as well as Stock and Watson (1988) suggested what is now commonly

[6] The solution is non-unique but the space spanned by the column vectors of β_2 is and a solution can be fixed by normalization.

known as the Beveridge-Nelson decomposition for trending systems, with A(B) again permitted to be a rational function:

$$(1\text{-}B)X_t = A(B)\varepsilon_t = A(1)\varepsilon_t + (1\text{-}B)A^*(B)\varepsilon_t \quad (14)$$

Based on Granger's representation theorem, it can be shown (see Johansen [1991]) that A(1) is a matrix of rank n-k if k is the cointegrating rank and is expressed as $\beta^{\dagger}R\alpha^{\dagger\prime}$ where † indicates the orthogonal complement, α and β are error correction loading matrix and cointegrating matrix, respectively, and R is some full-rank (n-k)x(n-k)-matrix. Consequently, the integral of the lower-dimensional $\alpha^{\dagger\prime}\varepsilon_t$ could be called the "common trends" of the system as the first term in (14) only depends on it and only this term is responsible for the overall trending behavior. (Definition A)

The reliance on the analytically simpler vector autoregressions as well as the inherent arbitrariness of innovation coordinates in multivariate systems insinuates to assign the expression "common trends" directly to $\alpha^{\dagger\prime}X_t$ (Definition B) [7] Due to an inherent duality, $\alpha^{\dagger}$ is calculated easily as the orthogonal complement of the factor loadings of the error correction variables in the vector autoregressive representation. Note that, in particular, $\alpha^{\dagger\prime}X_t$ does not depend on any past error correction.

In their canonical analysis, Box and Tiao (1977) implicitly conceived $\beta^{\dagger\prime}X$ instead of $\alpha^{\dagger\prime}X$ as the "common trends" (Definition C). The later definition, however, seems to have the more straightforward interpretation and thus has had more impact on the literature (see, however, Kasa (1992) for an application of definition C).

Suppose now that the data-generating process can be described by four unit roots and some corresponding cointegrating structures

$$\Phi(B)\Delta_4 X_t = D_1 Y_{1t-1} + D_2 Y_{2t-1} + D_3 Y_{3t-1} + D_4 Y_{3t-2} + \varepsilon_t \quad (15)$$

with the rank-deficient D_i matrices being decomposable into $D_i = \alpha_i\beta_i'$.

[7] This is the definition used explicitly by Johansen (1992) and as "common I(1) factors" by Gonzalo and Granger (1991).

Multiplication of (15) with the orthogonal complement of α_1 from the left cancels the first term. But this term is the one responsible for integratedness at the zero frequency and the random-walk-like trending behavior.

Hence, $\alpha_1^{\dagger\prime}X$ describes an $(N-r_1)$-dimensional process whose seasonal differences do not depend on $\beta_1'Y_{1t-1}$. Consequently, the first differences of the seasonally averaged $\alpha_1^{\dagger\prime}X$ do not depend on any frequency-zero error correction variables, a property reminiscent of the definition of common trends in the classical Definition B.

Now let equation (15) be premultiplied with $\alpha_2^{\dagger\prime}$. The second term cancels out, thus $\alpha_2^{\dagger\prime}X$ is an $(N-r_2)$-dimensional process whose seasonal differences do not depend on $\beta_2'Y_{2t-1}$. Hence, sums (or averages) of successive observations from the $(N-r_2)$-dimensional process $\alpha_2^{\dagger\prime}X$, cleaned from the irrelevant unit roots by filtering through $(1-B)(1+B^2)$, do not depend on lagged error correction factors at frequency π. In this regard, the process comes closer to an $(N-r_2)$-dimensional random jump process

$$Z_t + Z_{t-1} = \varepsilon_t \qquad (16)$$

than the N-dimensional Y_{2t} whose spectrum has a singularity at π but whose averages $Y_{2t} + Y_{2t-1}$ generate a spectral matrix at π with a rank deficiency. In accordance with the current notion of common trends, $\alpha_2^{\dagger\prime}Y_2 = (1-B+B^2-B^3)\alpha_2^{\dagger\prime}X$ will be called the vector of "common semi-annual seasonals" of the system. Similarly, assuming $D_3 = 0$, $\alpha_4^{\dagger\prime}Y_3 = (1-B^2)\alpha_4^{\dagger\prime}X$ can be called the "common annual seasonals" of X.

Just as common trends are typically quite trending, the common seasonals display their respective basic frequency vividly. The result that $\alpha^{\dagger}$ is the solution to the "dual" canonical eigenvector problem

$$|\nu S_{00}-S_{0p}S_{pp}^{-1}S_{p0}| = 0 \quad (17)$$

(see Johansen [1992]) in a vector autoregression with only unit roots at +1 permitted, naturally carries over to the four-roots case, where additional conditioning on Y_{2t-1}, Y_{3t-1}, Y_{3t-2} and solution of (17) yields $\alpha_1^{\dagger}$ as a matrix comprising those eigenvectors that correspond to the zero eigenvalues. In empirical applications, the eigenvectors corresponding to the $N-r_1$ smallest roots determine estimates of the common trends.

In strict analogy to the seasonal cointegration problem, the conditional canonical correlations betwixt $\Delta_4 X_t$ and Y_{2t-1} can be solved to yield $N-r_2$ eigenvectors of the zero (empirically, near-zero) roots. Assuming influences from Y_3 as absent, a similar problem can be solved for $\Delta_4 X_t$ and Y_{3t-2}. The resulting matrices of stacked eigenvectors naturally determine the system's common seasonals. In consequence, a common seasonal can be viewed as a variable whose respective de-seasonalizing averages such as $1+B$ or $1+B^2$ have as little correlation as possible with past system variables, approximating the behavior of pure seasonal unit root processes such as (16) where such transformations are unpredictable white noise.

4. Forecasting seasonal systems

Although seasonality deserves some interest on its own rights, empirical economists are rather reluctant to focus on it directly. In fact, more publications have been concerned with the effects of seasonality and of handling seasonality on other features, such as the construction of leading indicators or frequency-zero cointegration and persistency evaluation, than with seasonality itself. Adopting this point of view for the moment, it still appears interesting to know whether seasonal cointegration can actually be exploited for prediction. Nevertheless, I will concentrate on predicting seasonal series which are considered as true series rather than on seasonally adjusted indicators or annual aggregates.

Assuming that the true N-variate data-generating process features seasonal cointegration, it can be written as in (15)

$$\Phi(B)\Delta_4 X_t = D_1 Y_{1t-1} + D_2 Y_{2t-1} + D_3 Y_{3t-1} + D_4 Y_{3t-2} + \varepsilon_t$$

with rank-deficient matrices Di. Ignoring seasonal cointegration then is equivalent to restricting $D_2=D_3=D_4=0$. In practice, such specification will typically be obtained by seasonally averaging individual series and applying the standard cointegration model to this adjusted system. Such approach again is similar to explicitly seasonally adjusting individual series e.g. by Census X-11, as many seasonal adjustment procedures approximately contain the factor $1+B+B^2+B^3$ (see Wallis [1982]). In both cases - averaging or Census X-11 - eventual misspecification due to violation of the condition $D_2=D_3=D_4=0$ can incur increased lag orders in the empirically determined polynomial $\Phi(.)$.

Setting aside the problem of lag order identification, misspecification of seasonal cointegration causes an additional prediction error that is proportional to stationary seasonal error correction variables such as $\beta_2'Y_{2t}$. Just as in the frequency-zero cointegration problem, losses are asymmetric, as spuriously assuming seasonal cointegration necessarily causes an additional error proportional to e.g. Y_{2t} which is a highly seasonal but non-trending non-stationary variable.

Explicitly allowing for deterministic seasonals in the model brings in a further variety of misspecification errors. Deterministic seasonal models such as

$$\Delta X_t = a_1 Q_{1t} + a_2 Q_{2t} + a_3 Q_{3t} + a_4 Q_{4t} + \varepsilon_t \quad (18)$$

with Q_{it} being dummy variables for the respective quarters, generate repetitious seasonal patterns. More general lag structures than (18) replicate the main feature of prediction based on this class of models, viz. a tendency to return to an average seasonal pattern encountered in the sample.

Forecasting properties under the assumption of stochastic seasonality caused by seasonal unit roots are pronouncedly different. In the purest model

$$\Delta_4 X_t = \varepsilon_t \quad (19)$$

it is the last seasonal pattern rather than the average one which is extrapolated into the future. While again short-run lag structures may blur this distinction, the basic fact remains that deterministic models favor the average pattern and stochastic models the most recent one. Hence, in a world of ever-changing seasonality, the benefits of stochastic modeling ought to be clearly visible.

Although no coercive evidence can be provided by case studies, this paper evaluates forecasting performances in two model systems based on the following specifications :

a)A cointegrating vector autoregression of seasonally averaged variables. This approach approximates formal seasonal adjustment of individual series.

b) A seasonally cointegrating vector autoregression.

c) A vector autoregression with seasonal dummies included.

Whereas model b) encompasses model a), with respect to model c) there is a non-nested situation. A basic lag order of 5 for vector autoregression in levels e.g. was taken as indicating one conditioning lag of $\Delta_4 X_t$ for a) and b) but four lags for c), thus allowing a more flexible short-run lag structure.

5. A model case of seasonal cointegration

Up to now, most reported applications of seasonal cointegration technology have been restricted to systems of two variables. In particular with respect to common trend and common seasonality features, however, slightly larger systems are more telling. In this paper, an Austrian macroeconomic core system [8] will illustrate the points of the previous sections. The system is similar to those used in Kunst (1991), except for the fact that the real interest rate has been dropped because of its less satisfactory properties with respect to the overall system. [9] Thus, the system contains five variables: gross domestic product; private consumption; gross fixed investment; goods exports; and a wage rate. All variables are in logarithms of constant prices. Data series run from 1964 through 1990. Figure 1 contains graphics of the five time series.

Kunst (1991) reports that seasonality in the Austrian macroeconomy lies somewhere in between the extremes represented by Germany and the U.K., the former economy displaying clear evidence of stochastic seasonal fluctuations and seasonality in the latter one being near-deterministic. In the five-dimensional system considered here, the testing procedure by Lee (1989) detected three cointegrating vectors at the

[8] All data are taken from the WIFO database of the Vienna Institute for Economic Research.

Particularly, real interest rates turned out to be by far the least predictable series, hence their relative forecasting performance appears uninteresting.

zero frequency, four at π and two at $\pi/2$.

Univariate seasonal unit root tests ("HEGY" tests) reject unit roots in the exports series, an observation consistent with other comparable countries, including the aforementioned economies of Germany and the U.K. Hence, exports literally do not obey one of the basic assumptions of the model. Nevertheless, any series which does not have a certain unit root (e.g. -1) can be incorporated into the system, formally defined to be cointegrating with itself at the corresponding frequency (e.g. π) with the cointegrating vector being the corresponding unit vector.

At first, entirely stochastic models were fit to the data. No trend lines or seasonal constants were added. In accordance with the system lag order suggested by AIC, no lagged differences were added either. Table 1 shows those canonical vectors that have been found significant by the corresponding testing procedure.

Just as in the six-variable models of Kunst (1991), the vectors are less telling than would have been expected. The first vector at frequency $\pi/2$ and the third one at π are similar and relate seasonal fluctuations in the wage rate to the overall output. The second vector at $\pi/2$ approximates a unit vector with its only entry at the exports series, reflecting the non-seasonal nature of goods exports. Although both $\pi/2$ vectors are approximately contained in the space spanned by the π vectors, accurate "uniform seasonal cointegration" - i.e. vectors cointegrating at both seasonal frequencies - is not supported by the results and is rejected on statistical grounds.

Figure 2 demonstrates how seasonal cointegrating vectors operate. The picture relies on the first canonical vector at frequency π given in Table 1. That vector is designed to extract seasonality at π from the resulting series. The upper graph in Figure 2 shows how this semi-annual seasonality typically looks. Even if filtered through $1-B+B^2-B^3$ to get rid of the roots at 0 and $\pi/2$, i.e. trend and annual cycles, a conspicuous sawtooth pattern indicative of the π root remains in the GDP series. Similar patterns emerge from the other filtered series, maybe with the exception of exports. The lower part of Figure 2 shows what happens if $1-B+B^2-B^3$ is applied to the first component of β_2'X which yields β_2'Y_2 in the notation of (11). Even though this first column of β_2 has a large entry at the GDP series, its semi-annual seasonality is canceled by influences from the remaining series, in particular from consumption and wages.

From the dual canonical problem, common trend and common seasonal factor vectors are obtained. From Section 2, it evolves that assuming ri cointegrating vectors at a certain frequency entails assuming nDri common trends respectively seasonals at that frequency. Hence, we have two common trends, one common seasonal at frequency π and two common seasonals at $\pi/2$. The resulting common seasonals look like stable sawtooth patterns and are therefore not shown.

For the seasonally cointegrating model in the forecasting comparison, $D_3=0$ was imposed while r_1 was set at 3, r_2 at 4 and r_4 at 2. The resulting seasonal error correction model looks

$$\Delta_4 X_t = \mu + \alpha_1 \beta_1' Y_{1t-1} + \alpha_2 \beta_2' Y_{2t-1} + \alpha_4 \beta_4' Y_{3t-2} + \varepsilon_t \quad (20)$$

with matrices α_i and β_i being dimensioned as $5xr_i$. All parameters α_i, β_i (but not r_i) were consecutively re-estimated and updated during the forecast interval. Figure 3 gives a graphical and cursory evaluation of the results for all five individaul series. Mean square errors for forecasts for the last 20 observations (5 years) are given, for one-step to eight-step forecasts. Due to the highly seasonal nature of the series and some inevitable bias effects during this rather short period, eight-step forecasts are not necessarily much worse than one-step forecasts.

Even though the overall result may be influenced by the particular forecasting period selected for the experiment, the interesting fact remains that the vector autoregression without seasonal cointegration dominates in three out of five cases and the dummy specification in the remaining two, for one-step as well as for eight-step forecasts.

For the GDP series, which may be considered the central and most important series of all, the model comprising seasonal cointegration comes a distant third after its competitors for all forecast horizons beyond four quarters. It has already been pointed out that the benefits from introducing cointegrating vectors in a forecasting experiment are asymmetric, with spurious seasonal cointegration vectors causing more harm than neglected ones. Hence, the unconvincing performance of the seasonal cointegration model may change if the assumed amount of cointegrating vectors (4 at π and 2 at $\pi/2$) is reduced. It should also be pointed out, however, that the differences in forecasting performance amongst the models are not pronounced enough to allow final conclusions.

In the demand aggregate series "private consumption" and "investment", dummies appear to underestimate the persistence of shocks to basic seasonal patterns. Purely deterministic seasonality would result formally in four seasonal cointegrating vectors at both seasonal frequencies, with the one remaining common seasonal factor comprising all deterministic cycles. Such an entirely deterministic view of the world would be corroborated by the statistical findings of comparatively many cointegrating vectors. It is, however, at odds with the outcome of the forecasting experiment regarding the demand aggregates and with a cointegrating dimension of only two at π. In summary, although stochastic seasonal fluctuations play a minor role in the Austrian macroeconomy, there is room for exploring their nature in order to improve on prediction.

6. Summary and Conclusion

Within the framework of cointegrating vector autoregressions, the point has been made that most familiar features, such as common trends, cointegrating vectors, error correction factor loadings, immediately carry over to the seasonal case if one accepts seasonal unit roots as a valid model for seasonal cycles. In particular, the counterpart of common trend, the common seasonal, was defined and evaluated for two macroeconomic systems. This common seasonal expresses a latent factor behind the seasonal features of all system variables.

A rudimentary forecasting experiment based on Austrian macroeconomic series did not succeed in recommending seasonal cointegration as a feature that can be used easily to improve prediction accuracy. A summary evaluation would suggest conducting classical cointegration analysis on data series which have been adjusted individually by seasonal filtering. This latter solution is not too different in spirit from a cointegration analysis on seasonally adjusted data, while the seasonally cointegrating model would rather correspond to attempts at "multivariate seasonal adjustment".

Certainly, more experience is needed to finally gauge the usefulness of the notion of seasonal cointegration. Based on artificial data, the merits of choosing the correct model can certainly be underscored, the same way that forecasting in the presence of cointegration was treated by Engle and Yoo (1987). Recently, economic theory has brought forth some work on seasonality (compare e.g. Chatterjee and Ravikumar (1992)) which partly matches the increased attention that empirics pay to periodic cointegration models (see Osborn (1990) and Franses (1991)) as an alternative to both the deterministic and the seasonal unit roots model. More results in both areas are to be expected soon and will permit further insight into the true nature of seasonality.

Acknowledgement

Credits to Pierre Siklos for the basic GAUSS routine which was used, partly in modified versions, to estimate seasonal cointegrating vectors and common seasonals. The author wishes to thank Clive Granger for some comments on a previous draft of this paper.

TABLE 1: Cointegrating vectors within the Austrian macroeconomic systems at frequencies 0, π/2, π.

frequency	GDP	Cons.	Inv.	Exports	Wages
0	1.86	1.45	-0.81	-0.62	-0.42
	-1.66	1.50	0.03	0.60	-0.56
	-0.11	-0.34	-0.19	-0.08	0.36
π/2	5.97	-1.33	-1.08	-0.54	-3.66
	0.04	0.27	-0.06	-2.43	1.03
π	-8.32	3.94	1.23	0.78	-2.38
	1.10	0.74	-1.66	1.37	1.31
	-5.55	0.85	0.18	-0.44	2.31
	4.50	2.24	-2.02	-0.51	-0.41

TABLE 2: Common trend and seasonal common factor vectors for the Austrian macroeconomic system.

frequency	GDP	Cons.	Inv.	Exports	Wages
0	4.37	3.10	-0.87	0.81	-3.71
	-13.40	5.38	2.40	1.15	1.53
π/2	3.53	-2.54	2.16	-0.86	0.54
	5.28	3.09	-2.00	0.01	3.48
π	2.85	-2.22	-1.45	-0.62	-4.38

Literature

Beveridge and Nelson (1981) "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of Business Cycle". Journal of Monetary Economics 7, 151-174.

Box, G.E.P. and Tiao (1977) "A canonical analysis of multiple time series". Biometrika 64, 355-365.

Chatterjee, S. and B. Ravikumar (1992) "A neoclassical model of seasonal fluctuations". Journal of Econometrics 29, 59-86.

Engle, R.F. and C.W.J. Granger (1987) "Co-integration and error correction : Representation, estimation and testing". Econometrica 55, 251-276.

Engle, R.F., C.W.J. Granger and J.J. Hallman (1989) "Merging Short- and Long-Run Forecasts: An Application of Seasonal Cointegration to Monthly Electricity Sales Forecasting". Journal of Econometrics 40, 45-62.

Engle, R.F., C.W.J. Granger, S. Hylleberg, and H.S. Lee (1990) "Seasonal cointegration: The Japanese consumption function 1961.1-1987.4". Paper presented at the Econometric Society World Conference, Barcelona, Spain.

Engle, R.F. and S. Kozicki (1990) "Testing for Common Features". Discussion Paper 90-23, University of California, San Diego.

Engle, R.F. and B.S. Yoo (1987) "Forecasting and Testing in co-integrated systems". Journal of Econometrics 35, 143-159.

Franses, P.H. and T. Kloek (1991) "A Periodic Cointegration Model of Consumption in Austria and Japan". Paper presented at the ESEM 1991, Cambridge.

Gonzalo, J. and C.W.J. Granger (1991) "Estimation of Common Long-Memory Components in Cointegrated Systems". Discussion Paper 91D33, University of California, San Diego.

Hylleberg, S., R.F. Engle, C.W.J. Granger and B.S. Yoo (1990) "Seasonal Integration and Cointegration". Journal of Econometrics 44, 215-238.

Johansen, S. (1988) "Statistical Analysis of Cointegration Vectors". Journal of Economic Dynamics and Control 12, 231-254.

Johansen, S. (1991) "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models". Econometrica 59, 1551-1580.

Johansen, S. (1992) "Estimating Systems of Trending Variables". Mimeo, University of Copenhagen.

Joyeux, R. (1992) "Tests for Seasonal Cointegration Using Principal Components". Journal of Time Series Analysis 13, 109-118.

Kasa, K. (1992) "Common Stochastic Trends in International Stock Markets". Journal of Monetary Economics 29, 95-154.

Kunst, R.M. (1991) "Seasonal Cointegration in Macroeconomic Systems: Case Studies for Small and Large European Economies". To appear in Review of Economics and Statistics.

Lee, H.S. (1989) "Maximum Likelihood Inferences on Cointegration and Seasonal Cointegration". Discussion paper, University of California, San Diego.

Osborn, D.R. (1990) "The Implications of Periodically Varying Coefficients for Seasonal Time-Series Processes" Journal of Econometrics 48, 373-384.

Phillips, P.C.B. and S. Ouliaris (1988) "Testing for cointegration using principal components methods". Journal of Economic Dynamics and Control 12, 205-230.

Sargan, J.D. (1964) "Wages and Prices in the United Kingdom: a Study in Econometric Methodology" in: Econometric Analyses for National Economic Planning, ed. by P.E. Hart, G. Mills, J.N. Whittaker.

Butterworths, London.

Stock, J.H. and M.W. Watson (1988) "Testing for Common Trends". Journal of the American Statistical Association 83, 404, 1097-1107.

Wallis, K.F. (1982) "Seasonal Adjustment and Revision of Current Data: Linear Filters for the X-11 Method" Journal of the Royal Statistical Society A 145, 74-85.

FIGURE 1. Austrian macroeconomic time series.

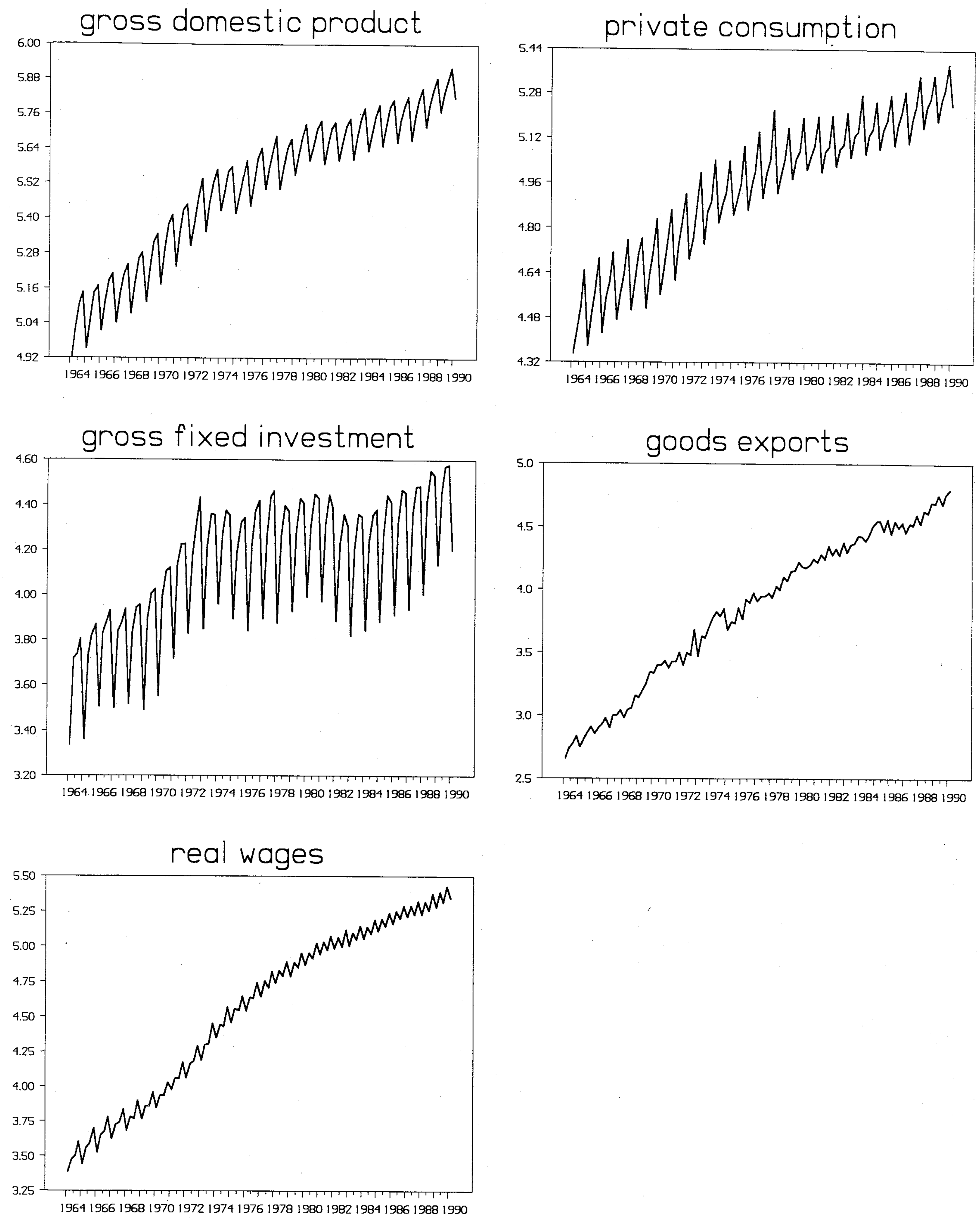
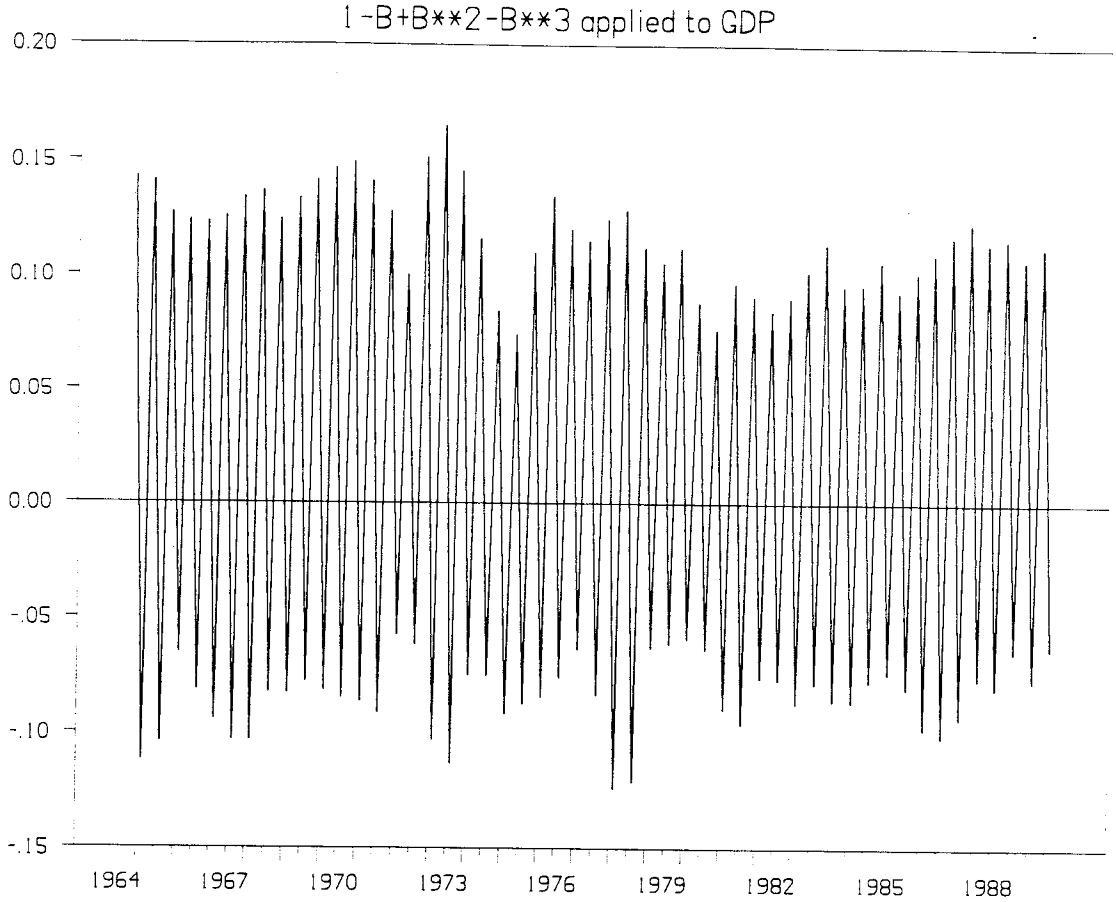


FIGURE 2. Semiannual cycles in Austrian GDP and in a seasonal EC factor.



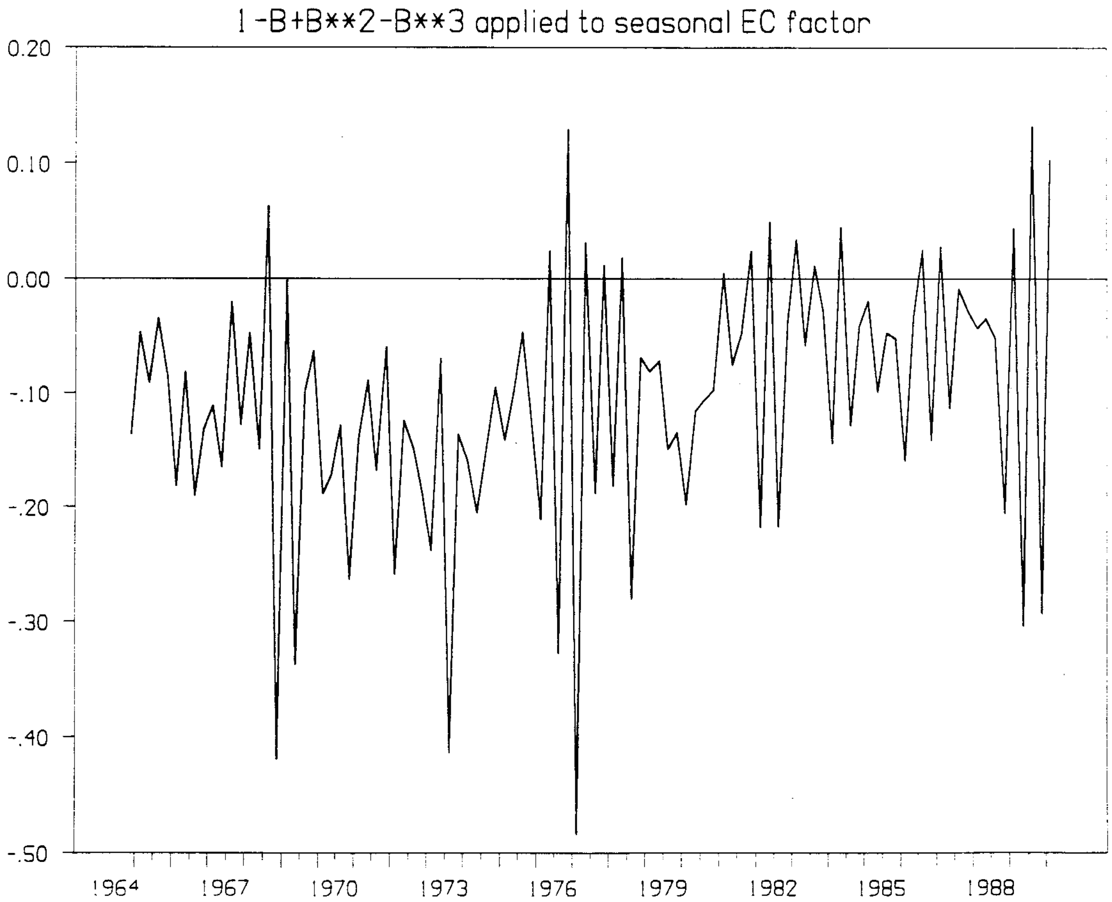


FIGURE 3. Relative forecasting performance of VAR models with and without seasonal cointegration.

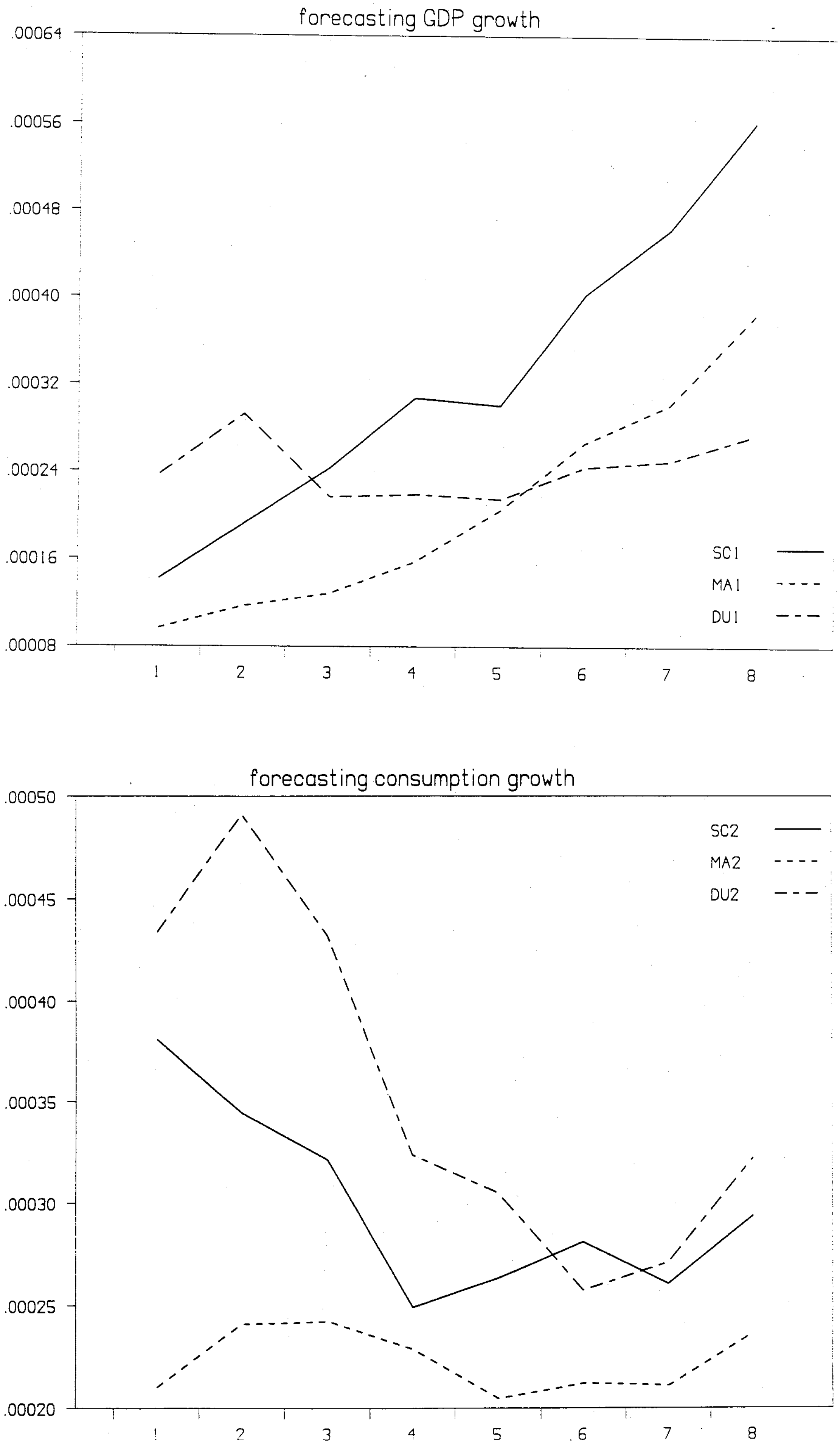
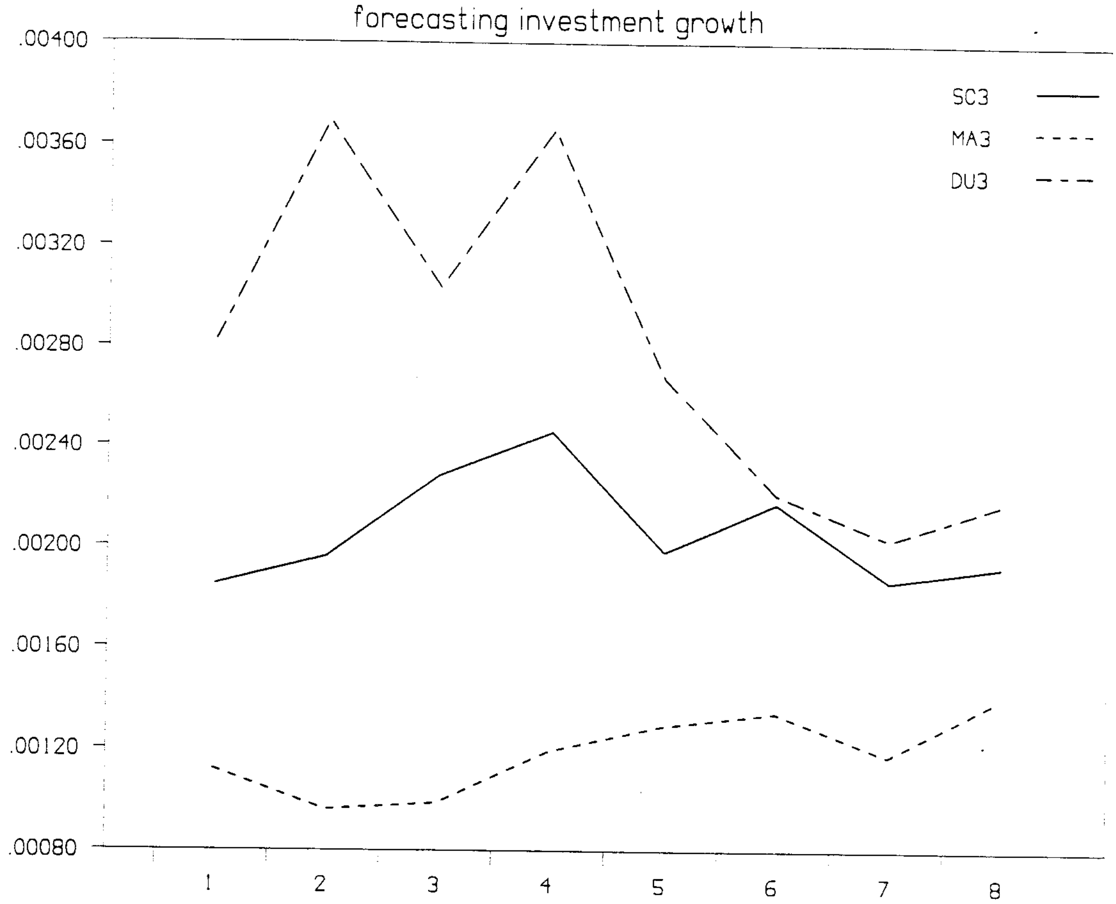


FIGURE 3 (continued).



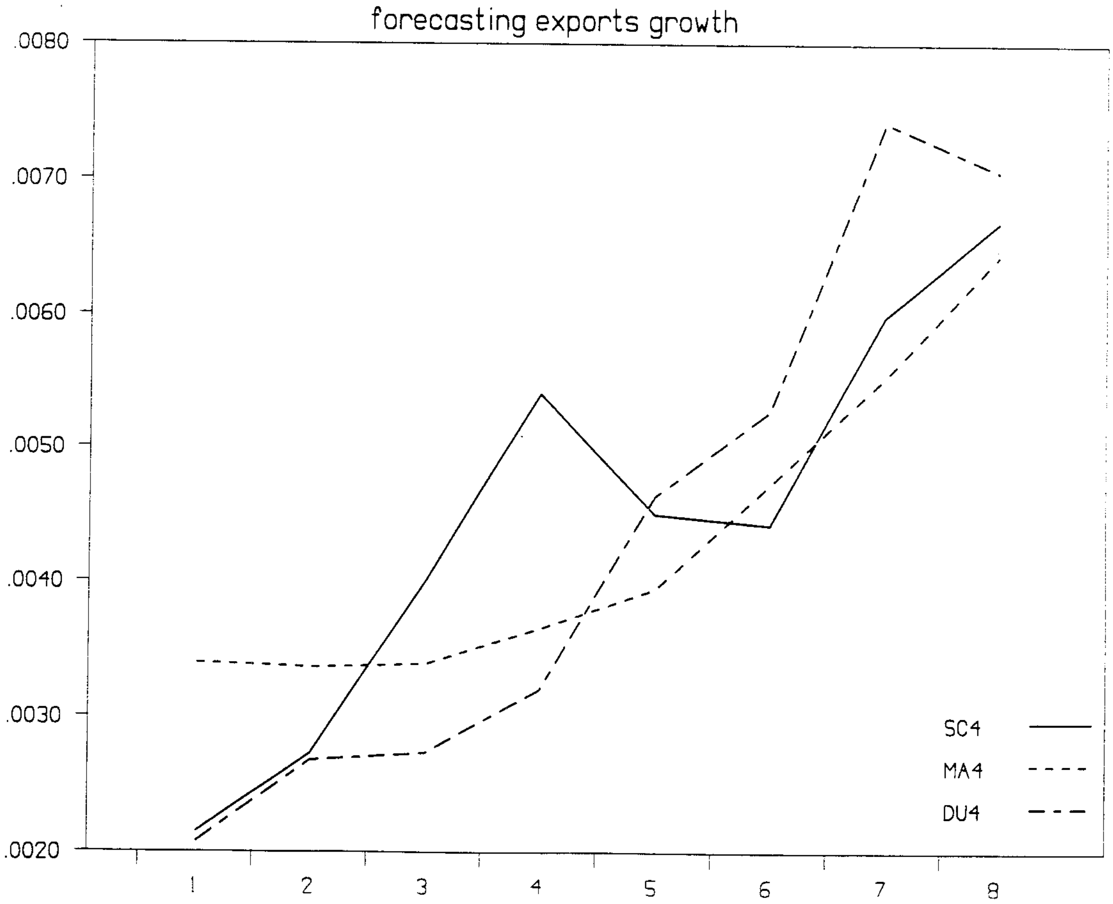


FIGURE 3 (continued).