TARGETING THE EXCHANGE RATE UNDER INFLATION

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Forschungsbericht/
Research Memorandum No. 299

May 1992

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Abstract

The purpose of this paper is to implement empirically a variant of the new theory of exchange rate targeting, suitable for high inflation small open economies. We formulate an expectations induced relationship between the exchange rate and the fundamental subject to random shocks and target zone constraints on rates of depreciation. The empirical analysis provides estimates for the key parameters of the exchange rate dynamic equation, and thereby identities the unique roles played by policy variables and market fundamentals in foreign exchange markets.

Kurzübersicht

Dieser Beitrag führt eine empirische Anwendung der neuen Theorie der Wechselkursfestsetzung für hoch inflationäre kleine, offene Volkswirtschaften durch. Es wird eine erwartungsinduzierte Beziehung zwischen dem Wechselkurs und seinen fundamentalen Bestimmungsfaktoren formuliert, gegeben zufällige Schocks und die durch das Wechselkursziel bestimmten Beschränkungen der Abwertungsrate. Die empirische Analyse liefert Schätzungen für die wesentlichen Parameter der Wechselkursgleichung und identifiziert so die Rolle der Politikvariablen sowie der fundamentalen Faktoren auf Märkten für ausländische Zahlungsmittel.

I. INTRODUCTION

The recent literature on exchange–rate behavior in a target zone has been able to formulate tractable models which, subject to random shocks, can separate out the effects of the fundamentals from policy rules. The target zone models, in which ceilings are placed on the level of the exchange rate, predicts that such policies are able to stabilize the exchange rate, at least temporarily.[1] The empirical validity of the basic target zone model has been tested with mixed results, primarily for low inflation countries, such as the EMS and other European countries.[2]

It is commonplace in inflationary episodes that the authorities set upper limits on exchange rate depreciation as a monetary anchor to slow inflation. These episodes could be extremely useful as a proving ground for the target zone model. The model, however, has not yet been implemented to high inflation data sets.

A useful working assumption under inflationary conditions is that the intervention limits are placed on rates of change, rather than on levels of the exchange rate. In this context it proves tractable to assume (and test) that the log–differential of the fundamental (rather than the log–level of the fundamental) follows a regulated Brownian motion. Accordingly, in this paper, in order to explain exchange rate behavior under high inflation, a variant of the target zone model is developed along these lines.

The paper implements the variant of the target zone model, and provides an empirical analysis of exchange rate policies for Israel, characterized by high and variable inflation rates, intensive foreign exchange market interventions, and volatile interest differentials. The assumption of strict regulations of the rate of change of the fundamental,

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[1] See, for example, Krugman (1989, 1991), Flood and Garber (1989) and Svensson (1989).

[2] See, for example, Bertola and Svensson (1990), Flood, Mathieson and Rose (1990), and Rose and Svensson (1991).

which underlies the behavior of exchange rate depreciations, is suitable for the Israeli high inflation period 1978–1985. Indeed, the empirical results tend to support the model's predictions.

The paper is organized as follows. Section II develops the target zone model with depreciation ceilings. Section III implements the model to estimate the (undeclared) depreciation ceilings, as perceived by the economic agents and Section IV concludes with suggested extensions.

II. THE ANALYTICAL FRAMEWORK

Consider the standard log–linear model of exchange–rate behavior.[3] The exchange rate is determined by:

$$s_t = v_t + \delta \frac{E_t[ds_t]}{dt}; \quad \delta > 0, \tag{1}$$

where

s_t = exchange rate

v_t = fundamental

δ = interest–rate (semi) elasticity of money demand and

E_t = expectations operators (conditioned on period t information).

The exchange–rate behavior could be derived from a standard specification for the demand for money, $m = p = c + \alpha y - \delta i + \epsilon$ and the uncovered interest parity condition

[3] See Krugman (1989), and Svensson (1989).

$i = i^* + \{E_t[ds_t]\}/dt$. With this specification the fundamental is given by:

$$v_t = m_t - c + q_t - p^* - \alpha y_t + \delta i_t^* - \epsilon_t; \quad \alpha > 0 \tag{2}$$

where

m	= money supply
q	$= s + p^* - p$, real exchange rate
p	= domestic price level
p^*	world price level
c	= a constant of the money demand equation
α	= output elasticity of money demand
y	= domestic output
i	= domestic rate of interest
i^*	= world rate of interest, and
ϵ	= money demand (or supply) disturbance.

As usual, equations (1)–(2) imply that the exchange rate (measured as the price of foreign currency in terms of domestic currency) depends positively on the supply of money, the world rate of interest, and the real exchange rate, while at the same time it depends negatively on the domestic output and money demand shocks. Crucially, the exchange rate also depends positively on the expected rate of depreciation, $\{E_t[ds_t]\}/dt$. This feature makes the present specification a genuinely forward looking model.

A forward solution to equation (1) (assuming no bubbles) implies that:

$$s_t = (\tfrac{1}{\delta}) \int_t^{\infty} E_t v_{\psi}\, e^{-\frac{\psi - t}{\delta}}\, d\psi. \tag{3}$$

Thus, the current spot exchange rate can be viewed as the discounted sum of present, and expected future, values of the fundamentals.

We assume that the rate of change of the fundamental, f, follows a Brownian motion:

$$f_t = \frac{dv_t}{dt} \tag{4}$$

and

$$df_t = \mu dt + \sigma dz \tag{5}$$

where z is a Weiner process with $E[dz] = 0$ and $E[(dz)^2] = dt$. The terms μ and σ respectively denote the drift and instantaneous standard deviation of the rate of change of the fundamental.

Integrating (4), and taking expectations, yields:

$$E_t v_{\psi} = v_t + \int_t^{\psi} E_t f_{\theta} d\theta \tag{6}$$

while, substituting (6) into (3) yields:

$$s_t = \frac{1}{\delta} \int_t^{\infty} \{v_t + \int_t^{\psi} E_t f_{\theta} d\theta\} e^{-\frac{\psi - t}{\delta}}\, d\psi. \tag{7}$$

Since the rate of change of the fundamental, f, follows a Brownian motion we can write the second term on the right–hand–side of (7) (which consists of future values of f) as a function, h(f), of the current value of f. That is:

$$h(f_t) = \frac{1}{\delta}\int_t^{\infty} \{ E_t f_\theta d\theta\} e^{-\frac{\psi - t}{\delta}} d\psi . \tag{8}$$

Observe that the first term on the right–hand–side of (7) reduces to v_t. We thus have:

$$s_t = v_t + h(f_t). \tag{9}$$

Differentiating (9), and applying Ito's Lemma yields:

$$E_t ds_t = dv_t + h'(f_t)\mu dt + \tfrac{1}{2}h''(f_t)\sigma^2 dt. \tag{10}$$

Substituting (10) into (1), using $dv_t = f_t dt$, then yields:

$$s_t = v_t + \frac{\delta}{dt}\{f_t dt + h'(f_t)\mu dt + \tfrac{1}{2}h''(f_t)\sigma^2 dt] . \tag{11}$$

Equations (11) and (9) amount to a deterministic differential equation for $h(f_t)$, namely:

$$h(f) = f\delta + h'(f)\mu\delta + h''(f)\,\frac{\delta}{2}\,\sigma^2 . \tag{12}$$

The standard solution of (12) is given by:

$$h(f) = f\delta + \delta^2\mu + AE^{\tau f} + BE^{\beta f} \tag{13}$$

where

$\tau > 0,\ \beta < 0$ are the roots of the characteristic equation associated with (12), given by:

$$\frac{\delta}{2}\sigma^2\tau^2 + \delta\mu\tau - 1 = 0\ . \tag{14}$$

The term $\delta f + \delta^2\mu$ in equation (13) represents the specific solution while the second term, $Ae^{\tau f} + BE^{\beta f}$, represents the homogenous solution of the differential equation. The coefficients A and B represent the constants of integration. Substituting equation (13) into equation (9) yields the final form solution for the exchange rate:

$$s_t = v_t + \delta f_t + \delta^2\mu + Ae^{\tau f} + Be^{\beta f}\ . \tag{15}$$

The economic interpretation of equation (15) is that the first term, $v + \delta f + \delta^2\mu$ (corresponding to the intervention–free equation of the exchange rate) represents the effect of the fundamental and the rate of change of the fundamental on the exchange rate. The second term, $Ae^{\tau f} + BE^{\beta f}$, represents the deviation of the exchange rate from the free float path, due to various policy interventions.

The hallmark of the exchange rate target zone model stems from the regulatory limits imposed on the stochastic process of the exchange rate (or the fundamental). Our view in this paper is that under high inflation monetary policy set limits on the <u>rate</u> of depreciation rather than the exchange rate <u>level</u>. Denoting the expected rate of depreciation of the exchange rate by x_t, equations (1) and (15) imply that:

$$x_t = \frac{1}{dt} E_t ds_t = f_t + \delta\mu + \frac{1}{\delta}[Ae^{\tau f_t} + Be^{\beta f_t}] . \tag{16}$$

The upper limit on the rate of change of the fundamental is the corresponding limit on the expected rate of depreciation, $\bar{x}$, namely:

$$x_t \leq \bar{x} . \tag{17}$$

Excluding bubbles, the expected rate of depreciation under the free float regime, x_F, is determined by:

$$x_{Ft} = \delta\mu + f_t . \tag{18}$$

Thus, under the free float the expected rate of depreciation depends positively on both the drift (multiplied by the interest rate semi–elasticity of the demand for money) and the realized value of the rate change of the fundamental.

Assuming no lower limit of the target zone implies that $B = 0$, hence the expected rate of depreciation under the managed exchange rate regime, x_M, is given by:

$$x_{Mt} = \delta\mu + f_t + \frac{1}{\delta} Ae^{\tau ft} . \tag{19}$$

If no realignment is expected the integration constant, A, can be solved from the so–called smooth pasting condition as $h'(\bar{f}) = 0$, which is also equivalent to $x'_M(\bar{f}) = 0$. The condition reflects the fact that there exists only one–sided risks in exchange rate fluctuations near the upper limit.

Differentiating equation (19) and evaluating the derivatives around the upper limit yields:

$$f = \bar{f};\ A = -\left(\frac{\delta}{\tau}\right)\exp^{-\tau\bar{f}} < 0. \qquad (20).$$

Since A is negative, equations (18) and (19) imply that the expected rate of depreciation under the managed exchange–rate regime will never exceed the free–float depreciation rate.

Substituting equation (20) into equation (19) yields the relationship between the upper barrier depreciation rate $\bar{x}$ and the upper bound on the rate of change of the fundamental $\bar{f}$:

$$\bar{x} = \delta\mu - \left(\frac{1}{\tau}\right) + \bar{f}. \qquad (21)$$

If there are expected regime changes we could explicitly model these as high–order realignments. Following Bertola and Cabalero we assume that the authorities intervene whenever the rate of change of the fundamental deviates from some benchmark growth rate, c, by a distance $\bar{g}$, (that is, $\bar{f} = c + \bar{g}$). The intervention may take two forms: the authorities either shift f back to c, with probability 1–p, or realign the target zone by shifting c to $c + \beta\bar{g}$, with probability p for some $B > 1$.

In the interior of the target zone the expected depreciation as a function of f and c, x(f; c), must fulfill:

$$x(f;\, c) = \delta\mu + f + \frac{1}{\delta}\, Ae^{\tau(f-c)} \,. \tag{22}$$

The no–arbitrage profits' condition at the point $f = c + \bar{g}$ implies

$$px(c + \beta\bar{g};\;\, c + \beta\bar{g}) + (1-p)x(c;c) = x(c + \bar{g};\, c) \,. \tag{23}$$

Substituting equation (22) into (23) yields:

$$A = \frac{(1-\beta p)\,\delta\bar{g}}{1 - e^{\tau\bar{g}}} \,. \tag{24}$$

Observe that if the realignment–probability p is smaller than the inverse of β then A must be negative, as in equation (20).

To summarize, the equilibrium expected rate of depreciation is decomposed into a term governed by the behavior of the random fundamental factors and another term which captures the effect of the depreciation ceiling and the probability of realignment. This closed–form solution enables us to separate out empirically the deviations of the depreciations under a managed exchange–rate regime from the corresponding rates of change under the free float exchange–rate regime.

III. THE EMPIRICAL APPLICATION

We now implement a variant of the exchange rate target zone theory on monthly data from Israel: 1978:08 – 1985:06. Estimation is carried out in three stages:

First, we estimate a standard demand for money equation, assuming unbiased expectations:

$$m_t - s_t = c + \alpha y_t - \delta(s_{t+1} - s_t) + w_t \tag{25}$$

where w is a stochastic residual (equalling to $\epsilon - q$ in equation (2)).

Second, using the estimate of δ, we derive the entire time series for the fundamental (v) and the drift (μ) and variability (σ) parameters of the rate of change of the fundamental (f). Observe that the variability measure, σ, represents the standard error of the drift–adjusted periodic changes in f. At this stage we can also calculate from equation (10) the root of the characteristic equation, τ, from equation (10).[4] The estimated is:

$$\hat{\tau} = \frac{[-\hat{\delta}\hat{\mu} + (\hat{\delta}^2\hat{\mu}^2 + 2\hat{\delta}\hat{\sigma}^2)^{\frac{1}{2}}]}{\hat{\delta}\hat{\sigma}^2} \tag{26}$$

where a "hat" indicates an estimate.

Third, using $\hat{y}$ and $\hat{f}$ we estimate the intervention parameter, A, from the following equation:

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[4] See equation (14).

$$s_t = \hat{v}_t + \hat{\delta}\hat{f}_t + \hat{\delta}^2\hat{\mu} + A e^{\hat{\tau}\hat{f}} + u_t \tag{27}$$

where μ_t is a stochastic residual. Note that although equation (15) is deterministic in equation (24) we allow a stochastic deviation from trend to capture errors in measurements or missing variables.

The variables used in the empirical application are the monetary aggregate M_2, the Israeli sheqel – U.S. dollar exchange rate, and the index of industrial production (as an output proxy). Assuming rational expectations, the expected depreciation is constructed from the actual depreciation pushed one month ahead. The demand for money is estimated by using instrumental variables for y_t and $E(s_{t+1} - s_t)$.

Tables 1 and 2 describe the empirical findings. The first equation in Table 1 reports the estimates for the money demand equation. The quasi–slow market clearing formulation is made here due to the nonsynchronization of the money supply and price variables within a month, in which the rate of inflation within a month is substantial. The estimated output elasticity is $\alpha = 6.34$, and the estimated interest rate semi–elasticity is $\delta = -12.1$. Both estimates lie within the confidence intervals of previous estimations of the demand for money in Israel.[5] The time series for the fundamental, u, is derived, according to equation (1), by using the estimate of the interest semi–elasticity of the demand for money.

Given the time series the main characteristics of the growth rate $f = dv/dt$ are derived for two subperiods 1978:06 – 1983:009, and 1983:11 – 1985:06. The two periods are different in both the mean and standard deviation of the rate of inflation. They are

[5] See, for example, Leiderman and Marom (1985).

TABLE 1: ESTIMATION[1]

Demand for Money[2]

$$m_t - s_t = \underset{(4.49)}{-2.959} + \underset{(30.40)}{0.858}(m_{t-1} - s_{t-1}) + \underset{(4.98)}{0.901}y_t \underset{(5.42)}{-1.725}(s_{t+1} - s_t) + \underset{(2.74)}{0.205}D_1$$

$R^2 = 0.87$, D.W. = 1.57.

Rates of Change of the Fundamental

Period	Drift (μ)	Variability (σ)	Root (τ)
1978:06-1983:09	-0.0030	0.07673	14.54
1983:11-1985:06	0.0077	0.1538	6.683

[1] t-ratios are reported in parentheses.

[2] To correct for simultaneous equation bias we use the following instruments: y_{t-1}, y_{t-2} $(s_t - s_{t-1})$, $(s_{t-1} - s_{t-2})$, $(s_{t-2} - s_{t-3})$, $(m_{t-1} - s_{t-1})$ and a dummy variable for the stock market crash of October 1983.

TABLE 2: ESTIMATION [1]

Reduced-form exchange-rate equation[2]

$$s_t - v_t - \delta f_t - \delta^2 \mu_i = c - Ae^{\tau f_t}$$

Period	A	c	$\bar{g}$	$\bar{x}$
1978:06-1983:09	- 0.019 (7.39)	0.071 (16.79)	0.048	0.092
1983:11-1985:06	- 0.055 (9.25)	0.138 (5.50)	0.068	0.169

R^2 =0.76, D.W. = 1.88

[1] t-ratios are reported in parentheses.

[2] The estimates for the constant term and the coefficient A are statistically different across the two periods.

separated by the stock market crash of October 1983.[6] Table 1 reports on a significant rise in both the drift and variability of the rate of change of the fundamental from the first to the second period. Thus, the actual expected rate of depreciation is described by our mean reverting process without regime shifts within each period. Whether, the retrieved series of f, the realized rate of change of the fundamental, is controlled at some upper boundary or instead follows some linear process, such as an I(1), could be checked for each period. Doing so, we were indeed unable to reject this assumption.

The estimates in Tables 1 and 2 can be used to calculate the fitted free float expected rates of depreciation, x_F, and the corresponding managed depreciation, x_M, and the upper–bound expected depreciation rate, $\bar{x}$. The free–float rate is calculated from equation (18) using the estimates of δ, μ, and f. The managed rate is obtained from equation (19).

Table 2 reports on the estimates of the reduced–form exchange rate equation for each period (see equation (9)). As the mean reverting process of our model indicates (for a low realignment probability) the coefficient A is negative.

We determine the upper bound on the expected depreciation, $\bar{x}$, so as to yield the most credible regime (i.e., the one with the lowest probability of realignment), in each period. This means that $\bar{x}$ is the highest possible depreciation so that $\bar{x} \leq x_M$ (see estimates in Table 2). The intervention benchmark rate, $\bar{g} + c$, is derived as follows. The rate c corresponds to the constant term in Table 2. From the relationship

$$(28) \qquad \bar{x} = \delta\mu + \bar{g} + c + \frac{1}{\delta} A e^{\tau \bar{g}}$$

[6] For a description of monetary developments underlying the behavior of the fundamental see Bank of Israel Annual Report, various issues.

we retreive the parameter $\bar{g}$, which is also reported in the Table. Interestingly, the rise in inflation from the first to the second period is associated with a corresponding rise in both the upper boundary on expected depreciation and the benchmark intervention range, $\bar{g}$ + c. The latter increased from about 12 percent per month to about 21 percent per month.

If one assumes that the intervention–adjustment factor, β, is equal to the ratio of the mean inflation in the second and first periods the probability of realignment in the first regime is less than one half.

V. CONCLUDING REMARKS

The paper implements the target zone theory to inflationary episodes, with policy interventions in the form of depreciation ceilings. Incorporating expected regime switching,the empirical analysis yields plausible estimates of the parameters of the exchange rate equation, and upper limits on rates of depreciation. We verify that these limits, often undeclared, are perceived by market agents as adjustable but associated with a relatively low realignment probabilty.

The target zone theory at the present stage of its development, however, provides no normative guide for the design of a utility–based exchange rate policy. Should policymakers attempt to stabilize the exchange rate at a cost of increasing interest rate variability? Should they employ a sterilized or unsterilized intervention policy? Empirical assessment of these issues must await further advancement in theory.

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