THE THEORY OF EXPLOITATIVE TRADE AND INVESTMENT POLICIES: A REFORMULATION AND SYNTHESIS

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1. INTRODUCTION

Traditional doctrine has long held that free trade and free international mobility of capital are in some sense optimal. However, nations continue to pursue their own advantage, and at the same time it is recognized in the more sophisticated versions of classical doctrine that achievement of an international optimum would require not only a consensus as to the optimal distribution of output (say in the form of specification of an international social welfare function) but also a willingness to implement that consensus by undertaking lump-sum redistributions of income - itself a form of market interference. Even if such a utopian program were to be carried out, it would still be necessary at the negotiating stage for nations to know how much they stand to gain or lose from abandoning restrictionist policies.

In this paper I propose to examine a model first presented by Kemp [15], and subsequently further analyzed by Jones [12], of a two-country two-commodity two-factor (labor and capital) world in which capital is (physically) perfectly mobile and the "home" country pursues its national advantage and the "foreign" country reacts passively.[1] I use the term "exploitative" to describe such policies, to convey the idea that the rest of the world is exploited in the same sense that natural resources are, as well as to avoid the somewhat insidious term "optimal." The term will therefore cover not only the types of policies that might be pursued by advanced industrial nations vis-à-vis the underdeveloped countries, but equally well to describe the kinds of policies that might be pursued by the "periphery", to use Dr. Prebisch's term [21], to improve its relative position in the world or to counteract adverse tendencies, real or alleged, emanating from the industrial centers.

Elsewhere I have shown (see Chipman [3]) that if world output can be efficiently produced by both countries producing both commodities, then under a very mild condition which can always be expected to be fulfilled whenever production functions are of a smooth neo-classical kind and are not identical as between countries, the world production possibility frontier will have a flat segment corresponding to such output combinations. Furthermore, sufficient conditions were obtained for the existence of efficient world output combinations when both countries diversify; simpler and very appealing conditions for this result have since been obtained by Uekawa [28].[2]

The above result has interesting implications for the theory of exploitative trade and investment policies. Consider first a situation in which, on the contrary, at least one of the countries specializes, and suppose that commodity trade is free but capital movements are subject to restriction. If the home country is a lender and the foreign country specializes, it will be to the home country's advantage to induce its citizens to withdraw some of their capital from abroad, since this will have the effect of raising its rate of return; by taxing income from capital invested abroad, the lending country can induce the withdrawal and capture the resulting increase in the return on foreign investment. Likewise, if the home country is a borrower and the foreign country specializes, it will be to the home country's advantage to encourage the foreign country to withdraw some of its capital, since the resulting excess of capital in the lending country will lower its rate of return there, thus lowering the opportunity cost of borrowing capital; this allows the home country to tax the income from foreign capital invested on its soil by the amount of the drop in the profits abroad, and the tax also serves to induce foreigners to withdraw their capital.

The above analysis, first set forth by Kemp [13, 14], no longer holds if the exploited country diversifies. In such a case, and if commodity trade remains unrestricted, autonomous movements of capital into or out of the exploited country will not affect its rate of return there. The "optimal" policy for the exploiting country to pursue in such circumstances is one of non-interference with capital flows - always assuming absence of restrictions to commodity flows.

The rationale for taxing income from home capital invested abroad, or from foreign capital invested at home, is quite a different one when the foreign country diversifies. Its role is no longer to raise or lower the foreign rate of return (by driving capital out) and thus capture the differential, but rather to capture a differential in rates of return that results as a by-product of commercial policy. If the home country imposes a tariff on its import good (commodity 2), or a tax on its export good (commodity 1) which improves its terms of trade (the relative price of commodity 1), this will - by the Stolper-Samuelson theorem - either raise or lower the foreign rate of return on capital according as industry 1 in the foreign country is capital intensive or labor intensive. If the home country is a lender, then in the former case the tariff will have, in addition to the favorable terms-of-trade effect, the added advantage of raising foreign profit levels, which can be tapped if a tax if levied on investment income earned abroad; in the latter case, if the fall in foreign profits is not too great, it will be to the home country's advantage to subsidize foreign investment in order to capture the predominating commodity terms-of-trade effect, whereas if the decline in foreign profits is great enough to outweigh the terms-of-trade effect, the home country should subsidize imports and lower its terms of trade in order to capture the resulting increase in profits on its foreign investments. Likewise, if the home country is a borrower and industry 1 (its export industry) is labor intensive abroad, then a tariff (or export tax) which improves its terms of trade will have the additional effect of driving down profit rates abroad, making it possible to tax foreign profits earned in the home country by the amount of the decrease; however, if industry 1 is capital intensive abroad, foreign profit rates will be driven up and foreign capital withdrawn unless subsidized to stay; if the subsidy cost outweighs the direct terms-of-trade gain, it will be advantageous for the borrowing country to subsidize imports or exports and lower its terms of trade, so as to drive down foreign profits and make it possible to increase taxes on foreign income earned in the home country.[3]

The above discussion is all set out quantitatively in Table 1 of section 5.

The cases in which the tariff rate and tax rate on income from foreign investment have opposite sign have been termed "paradoxical" by Kemp [15]. However, casual observation suggests that they are by no means pathological. For instance, Chile and Venezuela have both traditionally been heavy borrowers of foreign capital, yet their export industries (copper and petroleum) are highly capital intensive not only in those countries themselves but in the United States as well. The possibility therefore cannot be excluded that commercial policy, by improving these countries' terms of trade, would also raise profit rates in the U. S. and thus raise the cost of borrowing, leading to withdrawal of foreign capital unless accompanied by remedial measures.

All the arguments of this paper apply equally well to a model in which labor, rather than capital, is the mobile factor (cf. Ramaswami [22]). However, in this case they carry the insidious implication that immigrants' income should be subject to a discriminatory tax (or subsidy), which would be unconstitutional in most countries if overtly pursued.

The remainder of the paper is organized as follows. Section 2 brings out into the open the important "reciprocity" relations of Samuelson [24] that underlie the theoretical analysis of the later sections, and develops a concept due to Samuelson which I describe as a "production function for foreign exchange." This is defined as the maximum obtainable value of output, at fixed world prices, expressed as a function of the country's endowments of labor and capital; the peculiar flat conic section of this production surface (depicted in Figures 1 and 2) is what leads to the results of special interest. Section 3 deals with the hypothesized passive behavior of the foreign country, and section 4 formulates the optimum problem for the home (exploiting) country. This leads finally to the analysis of exploitative tariff and tax policies in section 5.

2. A PRODUCTION FUNCTION FOR FOREIGN EXCHANGE

Let a country be endowed with non-negative amounts L and K (not both zero) of labor and capital respectively, and produce amounts y_1 and y_2 of two commodities in accordance with production functions

$$y_i = f_i(L_i, K_i) \qquad (i = 1,2) \tag{2.1}$$

which are assumed to be positively homogeneous of degree 1, twice continuously differentiable, with isoquants strictly convex to the origin.[4] Here, L_i and K_i represent the employment of labor and capital in industry i, assumed to satisfy

$$L_1 + L_2 \leqq L, \qquad K_1 + K_2 \leqq K \qquad (L_i \geqq 0,\ K_i \geqq 0)\ . \tag{2.2}$$

Thus, labor and capital are exogenously determined, inelastically supplied, and perfectly mobile as between industries. The production possibility set $\mathcal{Y}(L, K)$ is defined as the set of all non-negative output combinations (y_1, y_2) for which there exist input combinations (L_i, K_i) satisfying (2.1) and (2.2).

Suppose that world prices $p_1 \geqq 0$, $p_2 \geqq 0$ (not both zero) are given for the two commodities, and that any amounts can be imported or exported at these prices. Define the function[5]

$$\Pi(p_1, p_2, L, K) = \max\{p_1 y_1 + p_2 y_2 : (y_1, y_2) \in \mathcal{Y}(L, K)\}\ . \tag{2.3}$$

Since $\mathcal{Y}(L, K)$ is closed and bounded, this function is well defined. For each combination L, K of factors it defines the maximum value of output obtainable at the given world prices p_1, p_2. Under competitive conditions, this corresponds to the gross domestic product (GDP), and by hypothesis it is available for purchase of goods from abroad at prices p_1, p_2. For fixed p_1, p_2, the function Π may therefore be regarded as a production function for foreign exchange. We shall see presently that it has the general properties of production functions, but with one peculiar special feature.

Consider Figure 1. Isoquants are drawn corresponding to production of a dollar's worth of each commodity; if both prices are positive, these are the amounts of labor and capital satisfying $f_1(L, K) = 1/p_1$ and $f_2(L, K) = 1/p_2$ respectively. A dollar of foreign exchange can therefore be obtained from resources $V' = (L', K')$ if these are all allocated to industry 1 and $f_1(L', K') = 1/p_1$; or from resources $V'' = (L'', K'')$ if these are all allocated to industry 2 and $f_2(L'', K'') = 1/p_2$. Then for any λ_1, λ_2 with $0 \leqq \lambda_1 \leqq 1$ and $\lambda_1 + \lambda_2 = 1$, if resources

$$V = (L, K) = \lambda_1(L', K') + \lambda_2(L'', K'')$$

are available, and allocated to each industry according to

$$V_1 = (L_1, K_1) = \lambda_1(L', K'); \quad V_2 = (L_2, K_2) = \lambda_2(L'', K''),$$

then outputs in the respective industries will be (by homogeneity)

$$f_1(L_1, K_1) = \lambda_1 f_1(L', K') = \lambda_1/p_1$$

$$f_2(L_2, K_2) = \lambda_2 f_2(L'', K'') = \lambda_2/p_2,$$

yielding a value of

$$p_1 f_1(L_1, K_1) + p_2 f_2(L_2, K_2) = \lambda_1 + \lambda_2 = 1.$$

Given that

$$L_1 + L_2 = \lambda_1 L' + \lambda_2 L'' = L$$

$$K_1 + K_2 = \lambda_1 K' + \lambda_2 K'' = K,$$

it follows from the definition of Π that

$$\Pi(p_1, p_2, L, K) \geqq p_1 f_1(L_1, K_1) + p_2 f_2(L_2, K_2) = 1 .$$

Therefore the point $V = (L, K)$ is on or above (in the diagram it is on) the isoquant for the production of one dollar's worth of foreign exchange.

The converse is also true; that is, given any $V = (L, K)$ such that $\Pi(p_1, p_2, L, K) \geqq 1$, we can find $V' = (L', K')$ and $V'' = (L'', K'')$ such that for some λ_i with $0 \leqq \lambda_i \leqq 1$ and $\lambda_1 + \lambda_2 = 1$ we have

$$V = \lambda_1 V' + \lambda_2 V''$$

and

$$f_1(L', K') \geqq 1/p_1 , \quad f_2(L'', K'') \geqq 1/p_2 .$$

(If $p_1 = 0$, say, we can take $\lambda_1 = 0$ and set $L' = K' = \infty$.) For, suppose

$$\Pi(p_1, p_2, L, K) = \pi \geqq 1 .$$

Then by definition of Π, there are L_i and K_i satisfying

$$L_1 + L_2 = L, \quad K_1 + K_2 = K$$

such that

$$\Pi(p_1, p_2, L, K) = p_1 f_1(L_1, K_1) + p_2 f_2(L_2, K_2) .$$

Defining

$$\lambda_i = \frac{p_i f_i(L_i, K_i)}{\pi} \qquad i = 1,2$$

and

$$V' = (L', K') = \frac{\pi}{\lambda_1}(L_1, K_1); \quad V'' = (L'', K'') = \frac{\pi}{\lambda_2}(L_2, K_2) ,$$

we see that $\lambda_1 + \lambda_2 = 1$ and that V' and V'' satisfy the required conditions.

Summarizing, we can state this result formally as follows.

<u>Theorem 1</u>. The set $\{(L, K) : \Pi(p_1, p_2, L, K) \geqq 1\}$ is the convex hull of the sets $\{(L, K) : f_i(L, K) \geqq 1/p_i\}$, $i = 1, 2$.

In Figure 1, the isoquant $\Pi(p_1, p_2, L, K) = 1$ (for fixed p_1, p_2) is shown by the thick curve. Its special feature is the flat segment (indicated by a broken line in the figure) within the cone corresponding to diversification of production. Within this cone, labor and capital are perfect substitutes in the production of foreign exchange, and the wage-rental ratio (the slope of the isoquant) is invariant with respect to factor endowments. This property of the isoquants is, in fact, the essence of the factor price equalization theorem. The characterization given by Theorem 1 is perfectly general, and readily extended to the case of m factors and n products.

Suppose in Figure 1 that one imagines the quantity of labor to be fixed and the country's endowment of capital to be variable. Then as the endowment point moves upward from the horizontal axis along a vertical line, the output of commodity 2 will be zero and that of commodity 1 will increase until the diversification cone is reached; as the point continues upward inside the diversification cone, the output of commodity 1 will fall and that of commodity 2 will rise until, at the edge of the cone, the output of commodity 1 has reached zero; from then on it stays equal to zero, and the output of commodity 2 continues to increase. This is all depicted in Figure 2. The dark solid lines depict $p_1 y_1$ and $p_2 y_2$ as functions of K (for fixed p_1, p_2, and L); these are the <u>Rybczynski</u>

functions (cf. Rybczynski [23]), which are linear within the diversification zone. The function Π, regarded as a function of K with p_1, p_2, and L fixed, has the usual properties of production functions except for the flat segment (indicated by the broken line in Figure 2) corresponding to diversification. The slope $\partial\Pi/\partial K$ of this segment will correspond to the rental of capital in this region, which will be independent of the capital stock.

The diagrams correspond to the special case of absence of factor intensity reversal. If, for instance, the isoquants for the two commodities intersected twice instead of once, there would be two diversification cones, and two flat segments instead of one on the isoquants of the function Π. In this case, there will exist some prices p_1 and p_2 for which the product isoquants will meet at a point of tangency, and the diversification cone will degenerate to a ray. If the country's factor endowment vector should happen to be on this ray, outputs would no longer be unique. In Figure 2, this would show up in the linear segments of the Rybczynski functions becoming vertical; thus they would no longer be single-valued functions, but (multi-valued) correspondences. The country's production possibility frontier will in this case be flat; we shall see below that this is the only case (when there are two products and factors) in which it is flat, i.e., in all other cases the production possibility frontier will be strictly concave to the origin.

We now proceed to a more precise analysis of these questions.

The Lagrangean expression for problem (2.3) is

$$\mathcal{L}(L_1, K_1, L_2, K_2, w, r; p_1, p_2, L, K) =$$

(2.4) $$p_1 f_1(L_1, K_1) + p_2 f_2(L_2, K_2) + w(L - L_1 - L_2) + r(K - K_1 - K_2)$$

where w and r are Lagrangean multipliers. The Kuhn-Tucker necessary and sufficient[6] conditions are

(2.5)
$$\frac{\partial \mathcal{L}}{\partial L_i} = p_i \frac{\partial f_i}{\partial L_i} - w \leqq 0 ; \quad L_i (p_i \frac{\partial f_i}{\partial L_i} - w) = 0 ; \quad (i = 1,2)$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = p_i \frac{\partial f_i}{\partial K_i} - r \leqq 0 ; \quad K_i (p_i \frac{\partial f_i}{\partial L_i} - r) = 0 ; \quad (i = 1,2)$$

together with

(2.6)
$$\frac{\partial \mathcal{L}}{\partial w} = L - L_1 - L_2 \geqq 0 ; \quad w(L - L_1 - L_2) = 0$$

$$\frac{\partial \mathcal{L}}{\partial r} = K - K_1 - K_2 \geqq 0 ; \quad r(K - K_1 - K_2) = 0 .$$

(cf. Kuhn and Tucker [16].) Now from the strict convexity to the origin of the production isoquants we have $\partial f_i / \partial L_i > 0$ and $\partial f_i / \partial K_i > 0$ for $i = 1,2$, and since by hypothesis either $p_1 > 0$ or $p_2 > 0$ it follows from (2.5) that we must have $w > 0$ and $r > 0$. Therefore equality must hold in (2.6), i.e., both factors must be fully employed.

Define the <u>production possibility frontier</u> $\hat{\mathcal{Y}}(L, K)$ as the set of all (y_1, y_2) satisfying (2.3) for some semi-positive[7] prices p_1, p_2. This frontier is defined as <u>strictly concave to the origin</u>[3] whenever, for all (y_1^o, y_2^o) on the frontier, there exist prices $p_1^o \geqq 0$, $p_2^o \geqq 0$ (not both zero) such that

$$p_1^o y_1^o + p_2^o y_2^o > p_1^o y_1 + p_2^o y_2$$

for all $(y_1, y_2) \in \hat{\mathcal{Y}}(L, K)$ such that $(y_1, y_2) \neq (y_1^o, y_2^o)$.

As is well known, the set $\mathcal{Y}(L, K)$ is convex.[9] We shall now establish a necessary and sufficient condition for $\hat{\mathcal{Y}}(L, K)$ to be strictly concave to the origin, for all L, K.[10]

Theorem 2. In order that $\hat{\mathcal{Y}}(L, K)$ be strictly concave to the origin for all factor endowments L, K it is necessary and sufficient that

$$(2.7) \qquad \begin{vmatrix} \partial g_1/\partial w & \partial g_1/\partial r \\ \partial g_2/\partial w & \partial g_2/\partial r \end{vmatrix} \neq 0$$

for all $w > 0$, $r > 0$, where $g_i(w, r) = \min\{wL_i + rK_i : f_i(L_i, K_i) \geqq 1\}$.

Remark. By the envelope theorem of production theory,[11] $\partial g_i/\partial w$ and $\partial g_i/\partial r$ are the amounts of labor and capital used per unit of output of industry i, so at positive output and positive labor input levels, (2.7) can be written

$$(2.8) \qquad \begin{vmatrix} \frac{L_1}{y} & \frac{K_1}{y_1} \\ \frac{L_2}{y_2} & \frac{K_2}{y_2} \end{vmatrix} = L_1 L_2 \left(\frac{K_2}{L_2} - \frac{K_1}{L_1} \right) \neq 0 ,$$

yielding the familiar condition of non-reversal of factor intensity.

Proof of Theorem 2. The sufficiency is shown as follows. Let a point $y^o = (y_1^o, y_2^o)$ be on the production possibility frontier $\hat{\mathcal{Y}}(L, K)$; then there is a set $p^o = (p_1^o, p_2^o) \geq 0$ of prices such that $p_1^o y_1^o + p_2^o y_2^o \geqq p_1^o y_1 + p_2^o y_2$ for all $y = (y_1, y_2)$ in the production possibility set $\mathcal{Y}(L, K)$. Let $y^1 = (y_1^1, y_2^1)$ be on the production possibility frontier, and $y^1 \neq y^o$. We are to show that $p_1^o y_1^o + p_1^o y_2^o > p_1^o y_1^1 + p_2^o y_2^1$. Suppose not, i.e., let $p_1^o y_1^o + p_1^o y_2^o = p_1 y_1^1 + p_2^o y_2^1$. Let L_i^j, K_i^j be such as to satisfy

$$y_i^j = f_i(L_i^j, K_i^j) \quad \text{for} \quad i = 1,2; \quad j = 0, 1,$$

and denote

$$L_i^\theta = (1 - \theta)L_i^o + \theta L_i^1 \; ; \; K_i^\theta = (1 - \theta)K_i^o + \theta K_i^1 \qquad i = 1,2.$$

Then the point $(f_1(L_1^\theta, K_1^\theta), f_2(L_2^\theta, K_2^\theta))$ belongs to $\mathcal{Y}(L, K)$, hence

(2.9) $$p_1^o[f_1(L_1^o, K_1^o) - f_1(L_1^\theta, K_1^\theta)] + p_2^o[f_2(L_2^o, K_2^o) - f_2(L_2^\theta, K_2^\theta)] \geqq 0 .$$

On the other hand, by the strict convexity to the origin of the production isoquants we have

(2.10) $$f_i(L_i^\theta, K_i^\theta) \geqq f_i(L_i^o, K_i^o) \qquad i = 1, 2$$

with equality holding if and only if

(2.11) $$(L_i^1, K_i^1) = \lambda_i(L_i^o, K_i^o) \quad \text{for some} \quad \lambda_i > 0 .$$

It follows from (2.9) and (2.10) that (2.11) holds whenever $p_i > 0$.

Suppose one of the prices is zero, say $p_1^o = 0$. Then since $w > 0$ and $r > 0$ it follows from (2.5) that $(L_1^o, K_1^o) = (L_1^1, K_1^1) = (0, 0)$, hence $y_1^o = y_1^1 = 0$. Since $p_2^o > 0$, it follows that $y_2^o = y_2^1$, contradicting the assumption that $y^1 \neq y^o$. Therefore $p_1^o > 0$, $p_2^o > 0$, and from (2.11) we have

$$(L_1^1, K_1^1) = \lambda_1(L_1^o, K_1^o)$$
$$(L_2^1, K_2^1) = \lambda_2(L_2^o, K_2^o) .$$

Since both factors must be fully employed, we have

(2.12)
$$L_1^o + L_2^o = L \qquad K_1^o + K_2^o = K$$
$$\lambda_1 L_1^o + \lambda_2 L_2^o = L \qquad \lambda_1 K_1^o + \lambda_2 K_2^o = K .$$

Now $\lambda_1 \neq \lambda_2$, otherwise $\lambda_1 = \lambda_2 = 1$ and $y^1 = y^0$, contrary to hypothesis. From (2.12) we have

$$(2.13) \qquad (1 - \lambda_1, 1 - \lambda_2) \begin{bmatrix} L_1^o & K_1^o \\ L_2^o & K_2^o \end{bmatrix} = (0, 0) .$$

If one of the outputs is zero, say $y_1^o = 0$, then $(L_1, K_1) = (0, 0)$ and (2.13) implies $(L_2^o, K_2^o) = (0, 0)$ hence $y_2^o = 0$; but the origin cannot belong to $\hat{\mathcal{Y}}(L, K)$, so $y_1^o > 0$ and $y_2^o > 0$. Then (2.13) contradicts (2.8) and thus (2.7). This proves that the condition (2.7) is sufficient.

To prove that (2.7) is necessary, suppose the determinant in (2.7) to vanish for some $w^o > 0$, $r^o > 0$. Then for some $L^o \geqq 0$, $K^o \geqq 0$ (not both zero) we have

$$\frac{\partial g_i}{\partial w} = \lambda_i L^o, \quad \frac{\partial g_i}{\partial r} = \lambda_i K^o \qquad i = 1, 2$$

hence from (2.8) it follows that

$$\hat{\mathcal{Y}}(L^o, K^o) = \{(y_1, y_2) : \lambda_1 y_1 + \lambda_2 y_2 = 1\} .$$

Q.E.D.

Henceforth condition (2.7) will be assumed to hold. It follows from Theorem 1, therefore, that for each p_1, p_2 there will be unique y_1, y_2 maximizing (2.3) for given L, K. This defines the functions

$$(2.14) \qquad y_1 = Y_1(p_1, p_2, L, K), \qquad y_2 = Y_2(p_1, p_2, L, K)$$

and thus we have

$$(2.15) \qquad \Pi(p_1, p_2, L, K) = p_1 Y_1(p_1, p_2, L, K) + p_2 Y_2(p_1, p_2, L, K) .$$

If $L > 0$ then, owing to the condition $L_1 + L_2 = L$, either $L_1 > 0$ or $L_2 > 0$; if, say, $L_1 > 0$ then $w = p_1\, \partial f_1/\partial L_1$, from (2.5), hence w is uniquely determined. Similarly for r, if $K > 0$. If $L > 0$ and $K > 0$ we therefore have the functions

$$(2.16) \qquad w = W(p_1, p_2, L, K), \quad r = R(p_1, p_2, L, K) .$$

Finally, for given y_i, w, and r, the factor demands L_i and K_i are determined in accordance with (2.1) and (2.5), giving rise to the functions which we shall denote

$$(2.17) \qquad L_i = \tilde{L}_i(p_1, p_2, L, K), \quad K_i = \tilde{K}_i(p_1, p_2, L, K) \qquad i = 1, 2 .$$

Our aim in the rest of this section is to study the properties of the functions (2.14), (2.16), and (2.17) .

The functions (2.14), (2.16), (2.17) are continuous, and must by definition satisfy the following relations identically in the variables p_1, p_2, L and K:

$$(2.18a) \qquad \tilde{L}_1(p_1, p_2, L, K) + \tilde{L}_2(p_1, p_2, L, K) = L$$

$$(2.18b) \qquad \tilde{K}_1(p_1, p_2, L, K) + \tilde{K}_2(p_1, p_2, L, K) = K$$

$$(2.18c) \qquad Y_i(p_1, p_2, L, K) = f_i[\tilde{L}_i(p_1, p_2, L, K), \tilde{K}_i(p_1, p_2, L, K)] \quad (i = 1,2)$$

$$(2.18d) \qquad W(p_1, p_2, L, K) \geqq p_i \frac{\partial}{\partial L_i} f_i[\tilde{L}_i(p_1, p_2, L, K), \tilde{K}_i(p_1, p_2, L, K)] \quad (i = 1,2)$$

$$(2.18e) \qquad R(p_1, p_2, L, K) \geqq p_i \frac{\partial}{\partial K_i} f_i[\tilde{L}_i(p_1, p_2, L, K), \tilde{K}_i(p_1, p_2, L, K)] \quad (i = 1,2)$$

Moreover, in any neighborhood in which $\tilde{L}_i(p_1, p_2, L, K) > 0$ throughout the neighborhood, (2.18d) must hold with equality throughout the neighborhood;

similarly in any neighborhood in which $\tilde{K}_i(p_1, p_2, L, K) > 0$, (2.18e) must hold with equality there. Conversely, if (2.18d) holds with strict inequality in any neighborhood of (p_1, p_2, L, K), then $\tilde{L}_i(p_1, p_2, L, K) = 0$ identically in that neighborhood; likewise if (2.18e) holds with strict inequality in a neighborhood, $\tilde{K}_i(p_1, p_2, L, K) = 0$ identically there. These considerations give us what we need to establish the following theorem, which is a rigorous statement of a result first stated by Samuelson:[12]

<u>Theorem 3</u>. If (2.7) holds, then in any neighborhood of (p_1, p_2, L, K) in which each of the functions

$$\tilde{L}_i\,,\ \tilde{K}_i\,,\ W - p_i\,\frac{\partial f_i(\tilde{L}_i\,,\ \tilde{K}_i)}{\partial L_i}\,,\quad R - p_i\,\frac{\partial f_i(\tilde{L}_i\,,\ \tilde{K}_i)}{\partial K_i} \qquad (i = 1,2)$$

remains either positive or zero, the following properties hold:

$$(2.19) \qquad \frac{\partial Y_i}{\partial p_j} = \frac{\partial Y_j}{\partial p_i}\ ;\ \frac{\partial W}{\partial K} = \frac{\partial R}{\partial L}\ ;\ \frac{\partial Y_i}{\partial L} = \frac{\partial W}{\partial p_i},\quad \frac{\partial Y_i}{\partial K} = \frac{\partial R}{\partial p_i}\,.$$

<u>Proof</u>. The proof consists in establishing (1) that Π has continuous second-order partial derivatives in the prescribed neighborhoods, and (2) that

$$(2.20) \qquad \frac{\partial \Pi}{\partial p_i} = Y_i\ ;\ \frac{\partial \Pi}{\partial L} = W,\quad \frac{\partial \Pi}{\partial K} = R\ .$$

The conclusion (2.19) then follows immediately. Property (1) follows from a result of Alexandroff [1], and a detailed proof will not be indicated here. It remains to establish (2.20). We have from (2.15) and (2.18c),

$$(2.21) \qquad \frac{\partial \Pi}{\partial p_i} = Y_i + p_1\frac{\partial Y_1}{\partial p_i} + p_2\frac{\partial Y_2}{\partial p_i}$$

$$= Y_i + p_1\left[\frac{\partial f_1}{\partial L_1}\frac{\partial \tilde{L}_1}{\partial p_i} + \frac{\partial f_1}{\partial K_1}\frac{\partial \tilde{K}_1}{\partial p_i}\right] + p_2\left[\frac{\partial f_2}{\partial L_2}\frac{\partial \tilde{L}_2}{\partial p_i} + \frac{\partial f_2}{\partial K_2}\frac{\partial \tilde{K}_2}{\partial p_i}\right]$$

$$= Y_i + \left[p_1\frac{\partial f_1}{\partial L_1} - W\right]\frac{\partial \tilde{L}_1}{\partial p_i} + \left[p_1\frac{\partial f_1}{\partial K_1} - R\right]\frac{\partial \tilde{K}_1}{\partial p_i} + \left[p_2\frac{\partial f_2}{\partial L_2} - W\right]\frac{\partial \tilde{L}_2}{\partial p_i} + \left[p_2\frac{\partial f_2}{\partial K_2} - R\right]\frac{\partial \tilde{K}_2}{\partial p_i} + W\left[\frac{\partial \tilde{L}_1}{\partial p_i} + \frac{\partial \tilde{L}_2}{\partial p_i}\right] + R\left[\frac{\partial \tilde{K}_1}{\partial p_i} + \frac{\partial \tilde{K}_2}{\partial p_i}\right].$$

Now the last two terms on the right side of the third equality of (2.21) vanish upon differentiating both sides of the identities (2.18a) and (2.18b) with respect to p_i. The remaining four terms following Y_i are also seem to vanish: for instance, $(p_1\partial f_1/\partial L_1 - W)\partial\tilde{L}_1/\partial p_i$ vanishes since either $\tilde{L}_1 > 0$ and the term in parentheses vanishes by the equality of (2.5), or else $\tilde{L}_1 = 0$ identically in which case $\partial\tilde{L}_1/\partial p_i = 0$ as well. Thus,

$$(2.22)\qquad p_1\frac{\partial Y_1}{\partial p_i} + p_2\frac{\partial Y_2}{\partial p_i} = 0.$$

This establishes the first equality of (2.20). (Note that (2.22) simply states that the slope of the production possibility frontier $\hat{\mathcal{Y}}(L, K)$ at (p_1, p_2) is the price ratio p_1/p_2.)

The second and third equalities of (2.20) are obtained in similar fashion, so it suffices to establish the second. We have from (2.15), and (21.8c),

$$(2.23)\qquad \frac{\partial \Pi}{\partial L} = p_1\left[\frac{\partial f_1}{\partial L_1}\frac{\partial \tilde{L}_1}{\partial L} + \frac{\partial f_1}{\partial K_1}\frac{\partial \tilde{K}_1}{\partial L}\right] + p_2\left[\frac{\partial f_2}{\partial L_2}\frac{\partial \tilde{L}_2}{\partial L} + \frac{\partial f_2}{\partial K_2}\frac{\partial \tilde{K}_2}{\partial L}\right]$$

$$= \left[p_1\frac{\partial f_1}{\partial L_1} - W\right]\frac{\partial \tilde{L}_1}{\partial L} + \left[p_1\frac{\partial f_1}{\partial K_1} - R\right]\frac{\partial \tilde{K}_1}{\partial L} + \left[p_2\frac{\partial f_2}{\partial L_2} - W\right]\frac{\partial \tilde{L}_2}{\partial L} + \left[p_2\frac{\partial f_2}{\partial K_2} - R\right]\frac{\partial \tilde{K}_2}{\partial L} + W\left[\frac{\partial \tilde{L}_1}{\partial L} + \frac{\partial \tilde{L}_2}{\partial L}\right] + R\left[\frac{\partial \tilde{K}_1}{\partial L} + \frac{\partial \tilde{K}_2}{\partial L}\right].$$

The first four terms on the right of the second equality (2.23) vanish by the preceding reasoning. Differentiating both sides of the identities (2.18a) and (2.18b) with respect to L, we see that $\partial\tilde{L}_1/\partial L + \partial\tilde{L}_2/\partial L = 1$ and $\partial\tilde{K}_1/\partial L + \partial\tilde{K}_2/\partial L = 0$, establishing the second equality of (2.20).

Q.E.D.

The functions Y_i of (2.14) are the <u>Rybczynski functions</u> (cf. Rybczynski [23]); for fixed world prices p_1, p_2, they specify the form of output variations as functions of factor endowments. The functions W and R of (2.16) may be called the <u>Stolper-Samuelson functions</u> (cf. Stolper and Samuelson [27]), since they specify, for fixed factor endowments, the form of factor price variations as functions of variations in commodity prices. Denoting the labor-output and capital output coefficients in industry i, as functions of the wage rate and rental of capital, by $a_i(w, r)$ and $b_i(w, r)$ respectively, we have

$$(2.24)\qquad \begin{aligned} a_1(w, r)y_1 + a_2(w, r)y_2 &= L_1 + L_2 = L \\ b_1(w, r)y_1 + b_2(w, r)y_2 &= K_1 + K_2 = K \end{aligned}$$

and since

$$(2.25)\qquad \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} \partial g_1/\partial w & \partial g_2/\partial w \\ \partial g_1/\partial r & \partial g_2/\partial r \end{bmatrix},$$

and this matrix is non-singular by (2.7), we can solve (2.24) to obtain

$$(2.26)\qquad \begin{aligned} Y_1(p_1, p_2, L, K) &= a^1(p_1, p_2, L, K) . L + a^2(p_1, p_2, L, K) . K \\ Y_2(p_1, p_2, L, K) &= b^1(p_1, p_2, L, K) . L + b^2(p_1, p_2, L, K) . K \end{aligned}$$

where

$$(2.27)\qquad \begin{bmatrix} a^1 & a^2 \\ b^1 & b^2 \end{bmatrix} = \begin{bmatrix} a_1(W, R) & a_2(W, R) \\ b_1(W, R) & b_2(W, R) \end{bmatrix}^{-1}.$$

In accordance with Theorem 1, when $y_1 > 0$ and $y_2 > 0$, the functions a^i and b^i are independent of L and K. Moreover, since the functions $\partial g_i/\partial w$ and $\partial g_i/\partial r$ are homogeneous of degree zero in w and r, the functions a^i and b^i are homogeneous of degree zero in p_1 and p_2, i.e., they depend just on the ratio p_1/p_2. Thus, for given p_1/p_2, the functions Y_1 and Y_2 are linear within the cone of diversification. We therefore have the result that when the country diversifies,

$$\frac{\partial W}{\partial L} = \frac{\partial^2 \Pi}{\partial L^2} = 0 , \quad \frac{\partial W}{\partial K} = \frac{\partial^2 \Pi}{\partial L \partial K} = \frac{\partial R}{\partial L} = 0 , \quad \frac{\partial R}{\partial K} = \frac{\partial^2 \Pi}{\partial K^2} = 0 , \tag{2.28}$$

and, by (2.19) and (2.26),

$$\frac{\partial R}{\partial p_1} = \frac{\partial Y_1}{\partial K} = a^2 ; \quad \frac{\partial R}{\partial p_2} = \frac{\partial Y_2}{\partial K} = b^2 , \tag{2.29}$$

which depend on p_1/p_2 but are independent of L and K. Since Y_1 and Y_2 are positive within the diversification cone, the functions a_i and b_i are also positive there, hence the diagonal elements a^1 and b^2 of the inverse matrix have opposite sign to the off-diagonal elements a^2 and b^1; moreover, since W and R are homogeneous of degree 1 in p_1 and p_2, we have by Euler's theorem

$$\begin{aligned} p_1 a^1 + p_2 b^1 &= p_1 \frac{\partial W}{\partial p_1} + p_2 \frac{\partial W}{\partial p_2} = W \\ p_1 a^2 + p_2 b^2 &= p_1 \frac{\partial R}{\partial p_1} + p_2 \frac{\partial R}{\partial p_2} = R . \end{aligned} \tag{2.30}$$

It follows that one of the elasticities $(p_i/R)\partial R/\partial p_i$ is negative and the other greater than unity - which is, in essence, simply the Stolper-Samuelson theorem (cf. Chipman [2]). Thus we have either

(2.31a) $$\frac{\partial R}{\partial p_1} < 0 \quad \text{and} \quad \frac{\partial R}{\partial p_2} > \frac{R}{p_2}$$

or

(2.31b) $$\frac{\partial R}{\partial p_1} > \frac{R}{p_1} \quad \text{and} \quad \frac{\partial R}{\partial p_2} < 0 \ .$$

On the other hand, in a region in which a country specializes, say in commodity 2, we have $Y_1 = 0$ identically in a neighborhood, whence by (2.19)

(2.32) $$\frac{\partial R}{\partial p_1} = \frac{\partial Y_1}{\partial K} = 0; \quad \frac{\partial R}{\partial p_2} = \frac{\partial Y_2}{\partial K} = \frac{\partial f_2}{\partial K} = \frac{R}{p_2} \ .$$

These relations are basic to the analysis that follows.

3. THE OFFER FUNCTION WHEN CAPITAL IS MOBILE

Consider a country capable of producing two products with labor and capital, when the quantity L of labor is fixed but the quantity K of capital is variable, capital being internationally mobile. Assume that the inhabitants have titles of ownership to a fixed amount C of capital at home and abroad. Let prices p_1 and p_2 be determined on world markets. Then we wish to determine the country's excess demand for each commodity as a function of p_1, p_2, and K.

Let there be N consuming units, and let the νth unit have command of L^* units of labor and title to C^ν units of the world capital stock, where

$$(3.1) \qquad \sum_{\nu=1}^{N} L^\nu = L, \qquad \sum_{\nu=1}^{N} C^\nu = C.$$

The νth unit's income will be given by

$$(3.2) \qquad I^\nu(p_1, p_2, L, K) = L^\nu W(p_1, p_2, L, K) + C^\nu R(p_1, p_2, L, K)$$

and its demand function for the ith product will be

$$(3.3) \qquad x_i^\nu = h_i^\nu[p_1, p_2, I^\nu(p_1, p_2, L, K)] \equiv X_i^\nu(p_1, p_2, L, K) .$$

It will be assumed that each commodity has positive marginal utility, hence demand is finite only if all prices are strictly positive; moreover, under this assumption all income will be spent, so the equation

$$(3.4) \qquad p_1 X_1^\nu(p_1, p_2, L, K) + p_2 X_2^\nu(p_1, p_2, L, K) = I^\nu(p_1, p_2, L, K)$$

holds identically. Defining

$$(3.5) \qquad X_i(p_1, p_2, L, K) = \sum_{\nu=1}^{N} X_i^\nu(p_1, p_2, L, K); \ I(p_1, p_2, L, K) = \sum_{\nu=1}^{N} I^\nu(p_1, p_2, L,$$

we obtain by summing (3.4) over the consuming units and using (3.1), (3.2), and (3.5),

$$(3.6) \quad p_1X_1(p_1, p_2, L, K) + p_2X_2(p_1, p_2, L, K) = I(p_1, p_2, L, K)$$

$$= L\,W(p_1, p_2, L, K) + C\,R(p_1, p_2, L, K)\ .$$

Now from (2.5) and Euler's theorem we have

$$(3.7) \quad L_i W + K_i R = p_i \left(L_i \frac{\partial f_i}{\partial L_i} + K_i \frac{\partial f_i}{\partial K_i}\right) = p_i f_i$$

hence

$$(3.8) \quad p_1Y_1(p_1, p_2, L, K) + p_2Y_2(p_1, p_2, L, K) = \Pi(p_1, p_2, L, K)$$

$$= L\,W(p_1, p_2, L, K) + K\,R(p_1, p_2, L, K)\ .$$

Defining excess demand for the ith commodity (quantity imported if positive, exported if negative) by

$$(3.9) \quad z_i = Z_i(p_1, p_2, L, K) = X_i(p_1, p_2, L, K) - Y_i(p_1, p_2, L, K)\ ,$$

we have from (3.6) and (3.8) the basic identity

$$(3.10) \quad p_1Z_1(p_1, p_2, L, K) + p_2Z_2(p_1, p_2, L, K) = (C - K)\,R(p_1, p_2, L, K)\ .$$

From (3.6) and (3.8) this yields

$$(3.11) \quad I(p_1, p_2, L, K) = \Pi(p_1, p_2, L, K) + (C - K)R(p_1, p_2, L, K),$$

i.e., the excess of gross national product over gross domestic product is equal to the deficit in the balance of payments on current account.

In section 5 it will turn out to simplify matters to consider the special case in which all consumers have identical homothetic utility functions. In that case, they have identical demand functions, and (3.3) and (3.5) become

$$(3.12)\qquad x_i = h_i[p_1, p_2, I(p_1, p_2, L, K)] = X_i(p_1, p_2, L, K).$$

Making use of (3.11) and (2.20) we obtain

$$(3.13)\qquad \frac{\partial X_i}{\partial K} = \frac{\partial h_i}{\partial I}\left[\frac{\partial \Pi}{\partial K} - R + (C-K)\frac{\partial R}{\partial K}\right] = \frac{\partial h_i}{\partial I}(C-K)\frac{\partial R}{\partial K}.$$

In particular, if $\partial R/\partial K = \partial^2\Pi/\partial K^2 = 0$, i.e., if the country diversifies, then $\partial X_i/\partial K = 0$, that is, movements of capital have no effect on income hence none on demand. This result is perfectly general, and does not depend on the assumption that consumers have identical homothetic utility functions; for from (3.2), (2.20), and (2.28) we have, when the country diversifies,

$$(3.14)\qquad \frac{\partial I^\nu}{\partial K} = L^\nu\frac{\partial^2\Pi}{\partial L\partial K} + C^\nu\frac{\partial^2\Pi}{\partial K^2} = 0,$$

hence from (3.3),

$$(3.15)\qquad \frac{\partial X_i}{\partial K} = \sum_{\nu=1}^{N}\frac{\partial X_i^\nu}{\partial K} = \sum_{\nu=1}^{N}\frac{\partial h_i^\nu}{\partial I^\nu}\frac{\partial I^\nu}{\partial K} = 0.$$

4. THE PROBLEM FORMULATED

Let there be two countries, a home (exploiting) country and a foreign (exploited) country. Following the notation of Kemp [15] and Jones [12], variables referring to the foreign country will have asterisks attached, and those without asterisks will refer to the home country.

It is given that the foreign excess demands are determined by the functions

(4.1) $$z_i^* = Z_i^*(p_1^*, p_2^*, L^*, K^*) \qquad (i = 1,2)$$

which are assumed to satisfy the budget equation

(4.2) $$p_1^* Z_1^* (p_1^*, p_2^*, L^*, K^*) + p_2^* Z_2^* (p_1^*, p_2^*, L^*, K^*) = (C^* - K^*)R^*(p_1^*, p_2^*, L^*, K$$

identically in the variables p_1^*, p_2^*, and K^* (see (3.10)). The quantity C^* of world capital owned by its inhabitants is assumed to remain constant throughout. The total world stock of capital will be denoted by $\underset{\sim}{K}$; thus

(4.3) $$K + K^* = C + C^* = \underset{\sim}{K} .$$

The home country is assumed to have a Samuelsonian social utility function $U(x_1, x_2)$ which is determined by distributing domestic incomes in such a way as to maximize a social welfare function (cf. Samuelson [25]). If factor supplies are immobile, this could in principle be accomplished by means of a system of sliding income tax rates and subsidies, fine-tuned so as to be adjusted to changes in domestic prices and incomes. It should be emphasized that the determination of such tax rates is likely to be a far more complicated matter than the determination of the tariff rates and tax rates on foreign capital (or on domestic capital invested abroad) needed to effect the exploitative trade and investment policies. The function U will be assumed to be continuously differentiable and strictly increasing in both arguments.

The problem we consider is therefore posed as follows:

$$\text{Maximize} \quad U(x_1, x_2)$$

subject to

$$(p_1) \qquad f_1(L_1, K_1) - x_1 - Z_1^*(p_1^*, p_2^*, L^*, K^*) \geqq 0$$

$$(p_2) \qquad f_2(L_2, K_2) - x_2 - Z_2^*(p_1^*, p_2^*, L^*, K^*) \geqq 0$$

$$(w) \qquad L - L_1 - L_2 \geqq 0$$

$$(r) \qquad \underset{\sim}{K} - K^* - K_1 - K_2 \geqq 0$$

$$(s) \qquad \underset{\sim}{K} - K^* \geqq 0 .$$

We form the Lagrangean function

$$(4.4) \qquad \mathcal{L}(x_1, x_2, L_1, L_2, K_1, K_2, p_1^*, p_2^*, K^*; p_1, p_2, w, r, s)$$
$$= U(x_1, x_2) + p_1[f_1(L_1, K_1) - x_1 - Z_1^*(p_1^*, p_2^*, L^*, K^*)]$$
$$+ p_2[f_2(L_2, K_2) - x_2 - Z_2^*(p_1^*, p_2^*, L^*, K^*)]$$
$$+ w(L - L_1 - L_2)$$
$$+ r(\underset{\sim}{K} - K^* - K_1 - K_2)$$
$$+ s(\underset{\sim}{K} - K^*) .$$

Differentiating (4.4) partially with respect to the respective variables, we obtain the necessary conditions

(x_1) $$\frac{\partial U}{\partial x_1} - p_1 \leqq 0$$

(p_1^*) $$p_1 \frac{\partial Z_1^*}{\partial p_1^*} + p_2 \frac{\partial Z_2^*}{\partial p_1^*} \geqq 0$$

(K^*) $$p_1 \frac{\partial Z_1^*}{\partial K^*} + p_2 \frac{\partial Z_2^*}{\partial K^*} + r + s \geqq 0$$

(L_1) $$p_1 \frac{\partial f_1}{\partial L_1} - w \leqq 0$$

(K_1) $$p_1 \frac{\partial f_1}{\partial K_1} - r \leqq 0 \ .$$

Each of these fourteen inequalities becomes an equality if the variable indicated on the left is positive; this is to be understood whenever any of them is referred to as a "condition".

Conditions (L_1), (K_1), (w), and (r) are the necessary and sufficient conditions for the solution of the problem (2.3), hence may be eliminated and replaced by

$$\begin{aligned} & w = W(p_1, p_2, L, \underset{\sim}{K} - K^*); \quad y_1 = f_1(L_1, K_1) = Y_1(p_1, p_2, L, \underset{\sim}{K} - K^*) \\ (4.5) \quad & \\ & r = R(p_1, p_2, L, \underset{\sim}{K} - K^*); \quad y_2 = f_2(L_2, K_2) = Y_2(p_1, p_2, L, \underset{\sim}{K} - K^*) \ . \end{aligned}$$

Since the foreign utility function U^* is assumed to be strictly increasing in both arguments, no solution is possible with zero prices p_1^*, p_2^*; and similarly for home prices p_1, p_2. Thus, conditions (p_1) and (p_1^*) must be equalities. In order to simplify the problem at this stage, we shall put aside the cases in which $K^* = 0$ (i.e., all the world stock of capital is located in the home country) or $K^* = \underset{\sim}{K}$ (all of it is located in the foreign country). These possibilities must, of course, be allowed for and included in a classification of all possible solutions, but this will not be considered in the present paper. We therefore will restrict attention at present to the case in which the solution has the property

(4.6) $$0 < K^* < \underset{\sim}{K} .$$

The first of these inequalities implies that equality holds in condition (K^*), and the second states that strict inequality holds in condition (s), hence $s = 0$.

We have eliminated conditions (L_i), (K_i), (w), (r), and (s), and replaced the inequality signs in (p_i^*) and (K^*) by equality signs (with $s = 0$). It remains to simplify conditions (p_i) and (x_i). Taking account of (4.2) and the fact that equalities hold in (p_1) and (p_2), we have, with the aid of (4.5),

(4.7) $$p_1x_1 + p_2x_2 = \Pi(p_1, p_2, L, \underset{\sim}{K} - K^*) - p_1Z_1^*(p_1^*, p_2^*, L^*, K^*) - p_2Z_2^*(p_1^*, p_2^*, L^*, K^*)$$
$$= \tilde{I}(p_1, p_2, p_1^*, p_2^*, K^*).$$

The second equation of (4.7) defines the home country's GNP (in domestic prices) as a function $\tilde{I}$ of home and domestic prices and capital stock in the foreign country; it consists of GDP (the function Π), valued in domestic prices, plus the deficit in the home country's balance of payments on current account. From (4.7) and conditions (x_i) we have

(4.8) $$x_i = h_i[p_1, p_2, \tilde{I}(p_1, p_2, p_1^*, p_2^*, K^*)] \equiv \tilde{X}_i(p_1, p_2, p_1^*, p_2^*, K^*) \quad (i = 1,2).$$

Our fourteen conditions have now been reduced to the two conditions (4.8), the two conditions (p_i^*), and condition (K^*) - five in all. There are five unknowns: p_1, p_2, p_1^*, p_2^*, and K^*. Now it is immediately verified that the conditions remain invariant with respect to multiplication of either pair (p_1, p_2) or (p_1^*, p_2^*) by a positive constant. Therefore one price can be fixed arbitrarily in each country, or alternatively some other normalization can be adopted such as requiring their sum to be constant, for example. It is customary to suppose that the home country will impose a tariff on its imports but leave exports unrestricted; it turns out that there is a slight gain in cleanness of mathematical notation if instead we suppose the distortion to be introduced in the domestic price of the home country's export good,[13] which we may think of as commodity 1.[14] We shall therefore adopt the convention that

$$p_2 = p_2^* = 1 . \tag{4.9}$$

We are therefore left with the following three equations in the three unknowns p_1, p_1^*, and K^* :

$$p_1 \frac{\partial}{\partial p_1^*} Z_1^*(p_1^*, 1, L^*, K^*) + \frac{\partial}{\partial p_1^*} Z_2^*(p_1^*, 1, L^*, K^*) = 0 \tag{4.10a}$$

$$p_1 \frac{\partial}{\partial K^*} Z_1^*(p_1^*, 1, L^*, K^*) + \frac{\partial}{\partial K^*} Z_2^*(p_1^*, 1, L^*, K^*) + R(p_1, 1, L, \underset{\sim}{K} - K^*) = 0 \tag{4.10b}$$

$$Y_1(p_1, 1, L, \underset{\sim}{K} - K^*) - \tilde{X}_1(p_1, 1, p_1^*, 1, K^*) - Z_1^*(p_1^*, 1, L^*, K^*) = 0 . \tag{4.10c}$$

They may be interpreted quite simply.[15] Equation (4.10a) states that the domestic relative price of commodity 1 in the home country should be equal to the negative of the slope of the foreign offer curve; (4.10b) can be interpreted as saying that the domestic rental of capital should be equal to the marginal productivity of foreign investment; finally (4.10c) expresses the equilibrium condition of demand and supply.

From here on, the analysis can proceed in either of two directions. First of all we can classify the types of solutions to (4.10) according as the foreign country specializes or diversifies, and examine the tax and tariff structure that would be required in each case. This is the procedure followed by Kemp [15] and Jones [12]. By itself, such an analysis is not a sufficient guide for policy purposes, however, - even in principle - since we do not know in advance which solution (if either - keeping in mind the possibilities excluded by (4.6)) will prevail. Secondly, therefore, we need to find some conditions that will determine the nature of the solution. Since conditions (4.10) are necessary, and not necessarily sufficient, this entails (1) establishing the existence of a solution to the problem itself, and (2) finding conditions that will guarantee (4.6) and exclude one of the remaining two possibilities (specialization and diversification). This program will form the subject of a future paper, and will not be taken up here; in the following section, therefore, we confine ourselves to describing the types of solutions that may be expected to occur.

5. EXPLOITATIVE TAX AND TARIFF POLICIES

To gain insight into the nature of the problem, it is helpful to consider the home country's indirect social utility function V defined by

$$(5.1) \qquad V(p_1, p_2^*, K^*) = U[Y_1(p_1, 1, L, \underset{\sim}{K} - K^*) - Z_1^*(p_1^*, 1, L^*; K^*), \; Y_2(p_1, 1, L, \underset{\sim}{K} - K^*) - Z_2^*(p_1^*, 1, L^*; K^*)].$$

This definition incorporates the market clearing conditions (p_i) of the previous section, as well as the conditions (4.5) of efficient production. As is to be expected, therefore, we obtain from conditions (x_i) and (2.22)

$$(5.2) \qquad \frac{\partial V}{\partial p_1} = p_1 \frac{\partial Y_1}{\partial p_1} + \frac{\partial Y_2}{\partial p_1} = 0.$$

Likewise we have

$$(5.3) \qquad \frac{\partial V}{\partial p_1^*} = -p_1 \frac{\partial Z_1^*}{\partial p_1^*} - \frac{\partial Z_2^*}{\partial p_1^*}$$

which when set $= 0$ becomes (4.10 a). Defining the function $\tilde{R}$ by

$$(5.4) \qquad \tilde{R}(p_1, K^*) = R(p_1, 1, L, \underset{\sim}{K} - K^*)$$

and making use of (2.20), we obtain

$$(5.5) \qquad \frac{\partial V}{\partial K^*} = -p_1 \frac{\partial Z_1^*}{\partial K^*} - \frac{\partial Z_2^*}{\partial K^*} - \tilde{R}$$

which when set $= 0$ becomes (4.10 b).

Differentiating the identity (4.2) with respect to p_1^*, we may express (5.3) in the form

$$(5.6) \qquad \frac{\partial V}{\partial p_1^*} = (p_1^* - p_1)\frac{\partial Z_1^*}{\partial p_1^*} + Z_1^* + (K^* - C^*)\frac{\partial R^*}{\partial p_1^*} .$$

Likewise, differentiating (4.2) with respect to K^* and adopting the assumption that consumers in the foreign country have identical homothetic utility functions,[16] we may with the help of formula (3.13) put (5.5) into the form

$$(5.7) \qquad \frac{\partial V}{\partial K^*} = (R^* - \tilde{R}) - (p_1^* - p_1)\frac{\partial Y_1^*}{\partial K^*} + \left[p_1\frac{\partial h_1^*}{\partial I^*} + \frac{\partial h_2^*}{\partial I^*}\right](K^* - C^*)\frac{\partial R^*}{\partial K^*} .$$

Before proceeding to discuss optimal tax and tariff formulas, it is instructive to consider the optimal direction of change starting from a position of unrestricted free trade in which $p_1^* = p_1$ and $r^* = r$. Then the first term on the right in (5.6) vanishes, hence it will be in the interest of the home country to take measures to improve its terms of trade if $Z_1^* = Y_1 - X_1 > 0$ (i.e. if it is exporting commodity 1) and if at the same time it is a lender and commodity 1 is produced and is capital intensive in the foreign country (i.e., $\partial R^*/\partial p_1^* = \partial Y_1^*/\partial K^* > 0$). Under these circumstances, in addition to the traditional terms-of-trade effect, the home country gains the increased profits from its investments abroad, which it can capture by imposing a tax to maintain the differential $r^* - r > 0$. However, if the home country is a lender and $\partial R^*/\partial p_1^* = \partial Y_1^*/\partial K^* < 0$, i.e., commodity 1 is produced and is labor-intensive abroad, then the fall in profits on foreign investment might offset the terms-of-trade effect and there is a no priori presumption as to which effect is stronger. On the other hand if the foreign country specializes in commodity 2 (hence $\partial R^*/\partial p_1^* =$

$\partial Y_1^*/\partial K^* = 0$), then the terms-of-trade effect is the only one to consider.

If the home country is a borrower which exports commodity 1, then an increase in its terms of trade p_1^* will have an unambiguously favorable effect if the foreign country specializes in commodity 2, or if it produces commodity 1 and commodity 1 is labor intensive in the lending country. The increase in p_1^* will lead to a fall in profits in the foreign country, making it possible for the home country to tax foreign investment income by a corresponding amount. If production of a commodity is labor intensive (resp. capital intensive) at home if and only if it is labor intensive (resp. capital intensive) abroad, and if the lending country exports its capital-intensive product and the borrowing country exports its labor-intensive product, then the traditional rule will hold that either country stands to gain from an improvement in its commodity terms of trade. But the rule need not hold: if a borrowing country is exporting a capital-intensive product, a reduction in its terms of trade could have the additional effect of lowering foreign profit rates to such a degree that the resulting differential in marginal productivities of capital in the two countries - which it could collect by taxing income from foreign-owned capital - would more than make up for the direct effect of the worsened terms of trade.

Formula (5.7) may be analyzed in similar fashion. At an initial position of unrestricted free trade, the first two terms on the right vanish, and the bracketed term is equal to unity. (5.7) then reduces to

$$(5.8) \qquad \frac{\partial V}{\partial K^*} = (K^* - C^*)\,\frac{\partial^2 \Pi^*}{\partial K^{*2}} ,$$

a formula which goes back to Kemp [14]. If the home country is a lender and the foreign country specializes in one of the commodities, then $\partial V/\partial K^* < 0$, i.e., the home country should withdraw some of its capital from the foreign country. This was suggested by MacDougall [18] and established by Kemp [13]. If the foreign country diversifies, however, then $\partial V/\partial K^* = 0$, and there are no first-order gains to be had from restriction of foreign investment. Differentiating (5.7) and evaluating the result at $p_1^* = p_1$, we find that when $\partial^2\Pi^*/\partial K^{*2} = 0$,

$$(5.9) \qquad \frac{\partial^2 V}{\partial K^{*2}} = -\frac{\partial \tilde{R}}{\partial K^*} = \frac{\partial R}{\partial K} = \frac{\partial^2 \Pi}{\partial K^2} \leqq 0.$$

This is negative if the home country specializes, and zero if the home country diversifies. Suppose now that free commodity trade is imposed, and (5.6) is replaced by the condition $p_1^* = p_1$; and suppose that the foreign country diversifies. Then if the home country specializes, the optimal policy with respect to capital movements is one of non-interference; on the other hand, if the home country diversifies, it is a matter of indifference whether capital movements are interfered with or not. Interference with capital flows in such circumstances (when free commodity trade is imposed) is justified only when the foreign country specializes, because only in that case can the home country affect profit rates in the foreign country. On the other hand, changes in commodity terms of trade can affect foreign profit rates, so these conclusions concerning interference with capital flows no longer follow when interference with trade flows is also allowed.

Formula (5.8) may be explained in the following terms. If the home country is a lender and the foreign country specializes, then a withdrawal of capital from the foreign country raises its marginal product there, making it possible for the home country to tap this increase by taxing its citizens' foreign investment income. If the home country is a borrower and the foreign country specializes, by inducing a withdrawal of foreign capital it brings about a drop in the marginal productivity of capital abroad, allowing it to tax the profits on the remaining foreign capital by the amount of the discrepancy in the marginal productivity of capital in the two countries.

Thus, if the exploited country specializes, it will be in the interest of the country that adopts the exploitative investment policy to move in the direction of autarky on capital account. The outcome is qualitatively the same, regardless of which country adopts the policy (and collects the taxes), namely, that the total amount of foreign lending (or borrowing) should be reduced.

This appears to be in conflict with the analysis presented by MacDougall [18], who argued that Australia stood to gain from increased capital imports from Britain. The difference, however, is in MacDougall's premises, which are that the capital inflow is autonomous and that Australia reacts passively. The appropriate indirect social utility function for this case would then be

$$(5.10) \qquad W(p_1, K) = U[Y_1(p_1, 1, L, K) + Z_1(p_1, 1, L, K), Y_2(p_1, 1, L, K) + Z_2(p_1, 1, L, K)]$$

and from (2.20), (3.10), and conditions (x_i) of section 4, we obtain

$$(5.11) \qquad \frac{\partial W}{\partial K} = -(K - C)\frac{\partial^2 \Pi}{\partial K^2} ,$$

which is MacDougall's measure of Australia's gain per unit of capital inflow,[17] and is positive if Australia is a borrower and specializes. The conclusion is correct, but is based on the supposition that British investors would actually be willing to supply the additional capital, even though this would lower the return on their investments abroad. On the contrary, they would have to be induced by a subsidy, resulting in a loss to Australia as measured by (5.8).

Let us now consider the tax structure required to sustain an optimal solution, obtained by setting the expressions in (5.6) and (5.7) equal to zero. Ruling out the case in which $z_1^* = z_2^* = 0$ at the optimum, which would entail $K^* - C^* = 0$ and $p_1^* = p_1$, $r^* = r$, we may assume $z_i^* \neq 0$ for some i, and hence for $i = 1$ without loss of generality. If commodity 1 is exported by the home country and an export tax of $100\tau\%$ is imposed, it will satisfy

(5.12) $$p_1^* = (1 + \tau)p_1.$$

If, instead, commodity 1 is imported and a tariff of $100\tau'\%$ is imposed, it will satisfy

(5.12') $$p_1 = (1 + \tau')p_1^*.$$

We shall adopt the convention that

(5.13) $$(1 + \tau)(1 + \tau') = 1,$$

so that if $\tau < 0$, this means either that exports of commodity 1 are subsidized at a rate of $-100\tau\%$, or that there is a duty of $100\tau'\%$ on imports of commodity 1, where $\tau' = -\tau/(1 + \tau)$.[18]

If the home country is a lender, and its government taxes its residents' income from foreign investment at a rate of $100t\%$, this tax rate will satisfy

(5.14) $r = (1 - t)r^*$.

If, instead, the home country is a borrower, and its government taxes the income on capital owned by non-residents at a rate of 100t' %, this tax rate will satisfy

(5.14') $r^* = (1 - t')r$.

As in the previous case, we adopt the convention that

(5.15) $(1 - t)(1 - t') = 1$

so that a negative tax rate of 100t % imposed by a borrowing country should be interpreted as a positive tax rate of 100t' % on income from foreign capital, where $t' = -t/(1 - t)$.

An actual subsidy of -100t % (where $t < 0$) on home income from capital invested abroad would entail $r = (1 - t)r^* > r^*$, which would encourage movement of foreign capital to the home country unless it was accompanied by a tax of at least 100 t' %, where $t' = -t/(1 - t) > 0$, on foreign income from capital invested at home. With this proviso, a negative t could be interpreted as a subsidy to the home country's lending abroad. Likewise, a subsidy on foreign capital invested at home of -100t' % (where $t' < 0$) would entail $r^* = (1 - t')r > r$, and this would encourage movement of domestic capital abroad unless accompanied by a tax of at least 100t %, where $t = -t'(1 - t') > 0$, on income from home capital invested abroad.

Equating (5.6) and (5.7) to zero and substituting (5.12) and (5.14) we obtain, upon factoring out $p_1^* z_1^* \neq 0$ and $r^* > 0$ and making use of the condition $\partial R^*/\partial p_1^* = \partial Y_1^*/\partial K^*$ from (2.19), the equations

$$(5.16) \quad \begin{aligned} &-\frac{\tau}{1+\tau}\eta^* + 1 + \varepsilon^* = 0 \\ &t - \frac{\tau}{1+\tau}\gamma^* + \iota^*\delta^* = 0 \end{aligned}$$

where η^* is the elasticity of the foreign country's excess demand for imports defined (following Marshall [19, pp. 337-8]) by

$$(5.17) \quad \eta^* = -\frac{p_1^*}{Z_1^*}\frac{\partial Z_1^*}{\partial p_1^*} ,$$

where

$$(5.18) \quad \iota^* = p_1 \frac{\partial h_1^*}{\partial I^*} + \frac{\partial h_2^*}{\partial I^*} ,$$

and where, following Jones [12] and Kemp [15],[19]

$$(5.19) \quad \gamma^* = \frac{p_1^*}{R^*}\frac{\partial R^*}{\partial p_1^*} , \quad \delta^* = \frac{K^* - C^*}{R^*}\frac{\partial R^*}{\partial K^*} , \quad \varepsilon^* = \frac{K^* - C^*}{Z_1^*}\frac{\partial R^*}{\partial p_1^*} .$$

Now, from the assumption that consumers in the foreign country have identical homothetic utility functions, it follows that their demand functions have unitary income elasticity, i.e., $\partial h_i^*/\partial I^* = h_i^*/I^* > 0$, hence neither good is inferior, so $\partial Z_1^*/\partial p_1^* < 0$. Equating (5.6) to zero and using (5.17) and (5.19) we see that <u>if the home country exports commodity 1</u>, i.e., $z_1^* > 0$, then[20]

$$(5.20) \quad \eta^* > 0 \quad \text{and} \quad \eta^* - 1 - \varepsilon^* = -\frac{p_1}{Z_1^*}\frac{\partial Z_1^*}{\partial p_1} > 0 ,$$

whereas these two inequalities are reversed if the home country imports commodity 1. In either case, $\eta^* \neq 0$ and $\eta^* - 1 - \varepsilon^* \neq 0$ and we can solve (5.16) to obtain

$$\text{(5.21a)} \qquad \tau = \frac{1 + \varepsilon^*}{\eta^* - 1 - \varepsilon^*}$$

$$\text{(5.21b)} \qquad t = \frac{1 + \varepsilon^*}{\eta^*} \gamma^* - \iota^* \delta^*$$

as well as

$$\text{(5.21a')} \qquad \tau' = - \frac{1 + \varepsilon^*}{\eta^*}$$

$$\text{(5.21b')} \qquad t' = \frac{\eta^* \iota^* \delta^* - (1 + \varepsilon^*)\gamma^*}{\eta^*(1 + \iota^* \delta^*) - (1 + \varepsilon^*)\gamma^*} \; .$$

These are the formulas developed by Kemp [15] and Jones [12], generalized to cover all cases. They are, of course, subject to the usual cautions concerning their proper interpretation, since the functions defined by (5.17), (5.18), (5.19) depend on p_1, p_1^*, and K^*. Thus, the optimal τ and t cannot be computed in general without knowing the optimal values of these three variables.[21] Equivalently, formulas (5.21) are based on the first two equations of (4.10), whereas a complete solution requires taking account of all three.

If the optimal solution finds the home country exporting commodity 1 and the foreign country specialized in the production of commodity 2, then $\partial R^*/\partial p_1^* = \partial Y_1^*/\partial K^* = 0$ in accordance with (2.32), whence $\gamma^* = \varepsilon^* = 0$ and (5.21a) reduces to the well-known optimal tariff formula for the traditional case of factor immobility (cf., e.g., Johnson [11]). It follows from (5.20) that when $z_1^* > 0$, an optimal solution requires $\eta^* > 1$, so $\tau > 0$. Since neither good is inferior, $\iota^* > 0$, so $t > 0$ if and only if the home country is a lender; if it is a borrower, we have $t' > 0$. Note that the result does not depend on whether commodity 2 is imported or exported; the latter possibility holds when the home country is a borrower. This may also be seen by considering the case in which the home country exports both commodities and the foreign country is specialized in the production of commodity 1. Then $\partial R^*/\partial p_1^* = R^*/p_1^*$ in accordance with (2.32), hence from (5.19) and (4.2) we have

$$(5.22) \qquad 1 + \varepsilon^* = \frac{1}{Z_1^*}\left[Z_1^* + (K^* - C^*)\frac{R^*}{p_1^*}\right] = -\frac{Z_2^*}{p_1^* Z_1^*} < 0$$

so that $\tau < 0$. In accordance with our convention (5.13) (see footnote 18), this means that a tax of $100\tau'$ % should be levied on the export of commodity 2, which is the same result as the above with the numbering of commodities interchanged.

If the home country imports both commodities, which is possible when it is a creditor, the foreign country must obviously diversify. If it imports one commodity and exports the other, and the foreign country specializes, we come back to the case just considered. It remains, therefore, only to consider the cases in which the foreign country diversifies.

When the optimal solution finds the foreign country diversifying, various "paradoxical" cases can, at least on the face of it, arise, as first noticed by Kemp [15]. In this case we have $\delta^* = 0$. Following Jones [12], denote

$$(5.23) \qquad \mu^* = \frac{(K^* - C^*)R^*}{p_1^* Z_1^*} ,$$

so that

$$(5.24) \qquad \varepsilon^* = \mu^* \gamma^* .$$

Since $\delta^* = 0$, (5.21b) and (5.21a') yield

$$(5.25) \qquad t = -\tau' \gamma^* = \frac{\tau}{1 + \tau} \gamma^* .$$

Thus, τ and t have the same sign if and only if $\gamma^* > 0$. If the home country exports commodity 1, then it follows from (5.20) and (5.21a) that $\tau > 0$ if and only if $\varepsilon^* > -1$; moreover, the sign of μ^* is the same as that of $K^* - C^*$. This together with (5.24) gives us what we need to ascertain the possible[22] patterns of tariff and tax rates and the conditions that give rise to them; these were obtained by Jones [12, pp. 14-15], and are

TABLE 1. EXPLOITATIVE TARIFF AND TAX PATTERNS WHEN HOME COUNTRY EXPORTS COMMODITY 1 AND FOREIGN COUNTRY DIVERSIFIES

	$K^* - C^* > 0$ ($\mu^* > 0$)	$K^* - C^* < 0$ ($\mu^* < 0$)
$(K^* - C^*) \frac{\partial R^*}{\partial p_1^*} > 0$ ($\varepsilon^* > 0$)	$\tau > 0, t > 0$	$\tau > 0, t' > 0$
$0 < Z_1^* + (K^* - C^*) \frac{\partial R^*}{\partial p_1^*} < Z_1^*$ ($-1 < \varepsilon^* < 0$)	$\tau > 0, t < 0$	$\tau > 0, t' < 0$
$Z_1^* + (K^* - C^*) \frac{\partial R^*}{\partial p_1^*} < 0$ ($\varepsilon^* < -1$)	$\tau < 0, t > 0$	$\tau < 0, t' > 0$

displayed in Table 1. As Jones observed, it is not possible for τ and t both to be negative when the home country is a creditor, nor for τ and t' both to be negative if the home country is a debtor.

The conditions given in Table 1 can be interpreted very simply by referring back to (5.6) and (5.7). From an initial situation of unrestricted free trade, we have

$$(5.26) \qquad \left.\frac{\partial V}{\partial p_1^*}\right|_{p_1^* = p_1} = Z_1^* + (K^* - C^*) \frac{\partial R^*}{\partial p_1^*} \quad ,$$

giving the expression in the left margin of Table 1. The first term, Z_1^*, measures the benefit to the home country through commodity trade of an improvement in its terms of trade; the second term, $(K^* - C^*)\, dR^*/dp_1^*$, measures the benefit through the increase in the return on home capital invested abroad (if $K^* - C^* > 0$) or through the decrease in the cost of borrowing from abroad (if $K^* - C^* < 0$). As observed earlier, when the foreign country diversifies we have

$$(5.27) \qquad \left.\frac{\partial V}{\partial K^*}\right|_{p_1^* = p_1,\ r^* = r} = 0 \quad ,$$

so the rationale for the home country taxing income from capital invested by one country in another is to capture the effect of changes in the terms of trade on profit rates in the foreign country. If the commodity and capital effects are both positive, so should be the tariff and tax rates. If the total effect is positive but outweighed by the commodity effect, the tariff should be positive and the tax negative. If the capital effect is negative and so strong that the total effect is negative, the tariff should be negative and the tax positive. What Table 1 tells us is that the same criteria hold for optimal tariffs and taxes as for incipient ones. In order to establish the existence of optimal tariff and tax patterns corresponding to each of the six cases of Table 1, it is therefore sufficient to establish (a) the existence of an initial free-trade equilibrium with the appropriate sign patterns for $\partial V/\partial p_1^*$ and with diversification of production, and (b) that the optimum is reached within the regime of diversification.[23]

Table 1 does not exclude the possibility that the home country exports both commodities; however, this is compatible only with two of the entries. Clearly, the possibility exists only when the home country is a debtor. Now consider the case $\tau > 0$, $t' < 0$; this requires $K^* - C^* < 0$ and $\partial R^*/\partial p_1^* > 0$, and from (2.31b) it then follows that $\partial R^*/\partial p_1^* > R^*/p_1^*$. Thus, from the budget equation (4.2) we have

$$(5.28) \qquad 0 < Z_1^* + (K^* - C^*)\frac{\partial R^*}{\partial p_1^*} < Z_1^* + (K^* - C^*)\frac{R^*}{p_1^*} = -\frac{Z_2^*}{p_1^*} ,$$

from which it follows that at the optimum for which $\tau > 0$ and $t' < 0$ we must have $z_2^* < 0$, i.e., the home country must import commodity 2. Therefore if the home country exports both commodities, the optimal tax rate on foreign profits earned in the home country must be positive. In this case, a negative τ will be interpreted as a tax at the rate $\tau' = -\tau/(1 + \tau)$ on the exports of commodity 2 (see footnote 18).

The only cases excluded from Table 1 are those in which the home country imports both commodities. For the sake of symmetry, therefore, we shall display the configuration of tariff and tax rates corresponding to all cases in which commodity 1 is imported; this is displayed in Table 2. In accordance with the convention (5.13), a negative export tax rate τ

TABLE 2. EXPLOITATIVE TARIFF AND TAX PATTERNS WHEN HOME COUNTRY IMPORTS COMMODITY 1 and FOREIGN COUNTRY DIVERSIFIES

	$K^* - C^* > 0$ $(\mu^* < 0)$	$K^* - C^* < 0$ $(\mu^* > 0)$
$(K^* - C^*) \frac{\partial R^*}{\partial p_1^*} < 0$ $(\varepsilon^* > 0)$	$\tau' > 0,\ t > 0$	$\tau' > 0,\ t' > 0$
$z_1^* < z_1^* + (K^* - C^*) \frac{\partial R^*}{\partial p_1} < 0$ $(-1 < \varepsilon^* < 0)$	$\tau' > 0,\ t < 0$	$\tau' > 0,\ t' < 0$
$z_1^* + (K^* - C^*) \frac{\partial R^*}{\partial p_1^*} > 0$ $(\varepsilon^* < -1)$	$\tau' < 0,\ t > 0$	$\tau' < 0,\ t' > 0$

on commodity 1 will be interpreted as a tariff at the rate $\tau' = -\tau/(1 + \tau)$ on imports of commodity 1, and a positive τ (negative τ') will be interpreted as a tariff at the rate τ on commodity 2 if it is imported.

If the home country imports both commodities, it must of course be a creditor. An argument similar to the above shows that the entry $\tau' > 0$, $t < 0$ of Table 2 must also be excluded. This case requires $K^* - C^* > 0$ and $\partial R^*/\partial p_1^* > 0$, hence $\partial R^*/\partial p_1^* > R^*/p_1^*$, so the budget equation (4.2) yields

$$(5.29) \qquad 0 > z_1^* + (K^* - C^*) \frac{\partial R^*}{\partial p_1^*} > z_1^* + (K^* - C^*) \frac{R^*}{p_1^*} = - \frac{z_2^*}{p_1^*} ,$$

whence at the optimum for which $\tau' > 0$ and $t < 0$ we must have $z_2^* > 0$, i.e., the home country must export commodity 2. Consequently, if the home country imports both commodities, the optimal tax rate on the profits from its investments abroad must be positive; and there will be a positive tariff on the imports of one or the other of the commodities.

The preceding analysis has been based on the assumption that both instruments τ and t are used. However, it is well worth while investigating the consequences of not using the second. Space does not permit more than a brief outline of the principal problems. First we may observe that (4.10a) still holds, but (4.10b) is replaced by the condition

$$R^*(p_1^*, 1, L^*, K^*) = R(p_1, 1, L, \underset{\sim}{K} - K^*). \tag{5.30}$$

Consequently, formula (5.21a) still holds but (5.21b) is replaced by $t = 0$. Now assume that both countries diversify, and suppose for example that industry 1 is relatively capital intensive in each of them; then if a tariff has the "normal" effect of increasing p_1^* (the "terms of trade") and reducing p_1 ("protecting" industry 2), then it must raise r^* and lower r, in violation of (5.30). It follows that if industry 1 is relatively capital intensive in both countries (or relatively labor intensive in both countries), p_1 and p_1^* must move in the same direction; thus, the terms of trade are "improved" if and only if the "wrong" industry is protected. The sign patterns are readily determined by applying the implicit function theorem to the set of four equations in footnote 3, and we find that

$$\frac{dp_1}{d\tau} = \frac{a_2^*\Delta}{J(\tau)} p_1 \ , \quad \frac{dp_1^*}{d\tau} = \frac{a_2\Delta^*}{J(\tau)} p_1 \ , \tag{5.31}$$

and

(5.32) $$\frac{dr}{d\tau} = - \frac{a_2 a_2^*}{J(\tau)} p_1 \ , \ \frac{dw}{d\tau} = \frac{a_2^* b_2}{J(\tau)} p_1 \ , \ \frac{dw^*}{d\tau} = \frac{a_2 b_2^*}{J(\tau)} p_1 \ ,$$

where (see (2.25))

(5.33) $$\Delta = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \ , \quad \Delta^* = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \ ,$$

and

(5.34) $$J(\tau) = a_2\Delta^* - (1 + \tau)a_2^* \Delta \ .$$

We find readily (cp. Chipman [3, p. 209]) that $J(\tau) = [a_2 a_1^* - (1 + t)a_2^* a_1]/r$, hence starting at unrestricted free trade equilibrium, if industry 1 is relatively capital intensive in both countries, a tariff will increase the terms of trade if and only if $a_1/a_2 < a_1^*/a_2^*$, i.e., if and only if the home country uses relatively less labor per unit of output in industry 1 than in industry 2, as compared with the foreign country. The opposite condition holds if industry 1 is relatively labor intensive in both countries. Under the conditions corresponding to the first two rows of Table 1, in which $Z_1^* + (K^* - C^*)\ \partial R^*/\partial p_1^* > 0$ hence $\tau > 0$, there is perhaps a weak a priori presumption that the tariff will "improve" the terms of trade.

Something definite can at least be said. At a second-best "optimum" when the foreign country diversifies, we have $\partial V/\partial K^* = (p_1^* - p_1)\ \partial Z_1^*/\partial K^* = -(p_1^* - p_1)\ \partial Y_1^*/\partial K^*$ from (5.7), so when $\tau > 0$, K^* should be increased or decreased according as industry 1 is relatively labor intensive or capital intensive in the foreign country. Thus, a borrowing country exporting commodity 1 which is relatively labor intensive in the foreign country, and importing commodity 2 which is relatively capital intensive in the foreign country, will want to drive some of the foreign capital out so as to lower foreign output (hence increase foreign imports) of commodity 1 and raise foreign output (hence increase foreign exports) of commodity 2; in this way it can collect more revenue from its export tax or import duty, as the case may be.[24] The essence of the matter is that by driving capital out, the home country will be more dependent upon trade, hence better able to exploit its monopoly position in trade. If, on the other hand, its export good is relatively capital intensive abroad and its import good relatively labor intensive abroad, then it will be in the borrowing country's interest to attract still more foreign capital, because this will now lower foreign output of commodity 1 and raise foreign output of commodity 2, thus increasing foreign imports and exports of these commodities, and permitting more duties to be collected. This time, it is by drawing capital in that the borrowing country becomes more dependent upon trade, and better able to exploit its monopoly position in trade. In both cases the criterion, which furnishes the question for empirical research, is the same: whether, in a regime of protectionism with unimpeded capital movements, capital inflows or outflows make a country more dependent upon, and therefore better able to exploit, international trade.

FOOTNOTES

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1. Removal of the assumption of passive reaction would be a natural next step in the study of this model. For the case in which trade is unrestricted and only capital flows are subject to interference, a way has been shown by Hamada [8].

2. Ethier and Ross [5] have considered this problem in a comparative dynamics setting in which capital is assumed to be produced and ownership patterns may change as a result of saving. They have characterized types of saving behavior that allow for the possibility of efficient production with diversification in the long run.

3. An important difference should be noted between the model studied here and that of Mundell [20]. In Mundell's, production functions are assumed to be identical as between countries, and it follows from this that if a single instrument is introduced (a tariff or a tax on income from invested capital), and reversal of factor intensity is ruled out, equilibrium with diversification is no longer possible. This is no longer the case if we allow (as we must - cf. Chipman [3]) production functions to be different as between countries, in both industries. This can be seen from the following simple argument: Let $g_i(w, r)$, $g_i^*(w^*, r^*)$ be the minimum unit-cost

functions for commodity i in the home and foreign countries respectively, where w, r and w^*, r^* denote the wage rate and rental of capital in the home and foreign country. Let the home country be a borrower and suppose a tax of 100t' % is imposed on the income of foreign capital, so that $r^* = (1 - t')r$ (see formula (5.14') of section 5); let commodity 1 be exported by the home country and a tax of 100τ % be imposed on exports, so that $p_1^* = (1 + \tau)p_1$, where p_i, p_i^* are the prices of commodity i at home and abroad, and $p_2 = p_2^* = 1$ (see formula (5.12) of section 5). Then trade and capital mobility together with diversification of production in both countries imply the existence of a solution to the equations

$$g_1(w, r) = p_1 \; ; \qquad g_1^*(w^*, (1 - t')\, r) = (1 + \tau)p_1 \; ;$$

$$g_2(w, r) = 1 \; ; \qquad g_2^*(w^*, (1 - t')\, r) = 1 \; .$$

If $g_i = g_i^*$ for $i = 1, 2$, and if $t' = 0$ and $\tau > 0$, the bottom two equations imply $w = w^*$, contradicting the top two; this is essentially Mundell's result. Likewise, if $g_i = g_i^*$ for $i = 1, 2$, and if $t' > 0$ and $\tau = 0$, then the top pair of equations state that the points (w, r) and (w^*, r^*) both lie in the curve $g_1(w, r) = p_1$, and the bottom pair states that these same points both lie on the curve $g_2(w, r) = 1$. Then these curves must cross at two different points, implying factor intensity reversal.

If $g_i \neq g_i^*$ for $i = 1, 2$, and an isolated solution to the above equations exists for $\tau = 0$ and $t' = 0$, then a slight increase in one or both of the parameters τ and t' is equivalent to a slight perturbation of the functions g_1^* and g_2^*, hence an isolated solution will continue to exist. Compare the argument in Inada and Kemp [10].

4. That is, given any $(L_i^o, K_i) \geq (0, 0)$, there exist $w^o > 0$ and $r^o > 0$ such that

$$w^o L_i + r^o K_i > w^o L_i^o + r^o K_i^o$$

for all $(L_i, K_i) \geq (0, 0)$ such that $f_i(L_i, K_i) = f_i(L_i^o, K_i^o)$ and $(L_i, K_i) \neq (L_i^o, K_i^o)$ (cf. Hurwicz and Uzawa [9, p. 131]). Here the symbol $\geq$ has the usual meaning that $(L_i, K_i) \geq (0, 0)$ if and only if $L_i \geqq 0$, $K_i \geqq 0$, and not both $L_i = K_i = 0$.

5. This function was first defined by Samuelson [24, p. 10]. Mathematically it is the support function of the set $\mathfrak{Y}(L, K)$; see Fenchel [6].

6. The sufficiency follows from the fact that the constraints in (2.4) are concave; see Theorem 3 of Kuhn and Tucker [16].

7. The vector (p_1, p_2) is said to be semi-positive, written $(p_1, p_2) \geq (0, 0)$, if $p_1 \geqq 0$, $p_2 \geqq 0$ but not both $p_1 = p_2 = 0$.

8. Compare the definition in footnote 4.

9. A formal proof goes as follows. Let $y^o = (y_1^o, y_2^o)$ and $y^1 = (y_1^1, y_2^1) \in \mathfrak{Y}(L, K)$ and let $f_i(L_i^h, K_i^h) = y_i^h$ and $L_1^h + L_2^h \leqq L$, $K_1^h + K_2^h \leqq K$ for $h = 0, 1$. Denote $y^\theta = (1 - \theta)y^o + \theta y^1$, $L_i^\theta = (1 - \theta)L_i^o + \theta L_i^1$, $K_i^\theta = (1 - \theta)K_i^o + \theta K_i^1$, where $0 \leqq \theta \leqq 1$. We are to show that $y^\theta \in \mathfrak{Y}(L, K)$. By the concavity of f_i we have

$$f_i(L_i^\theta, K_i^\theta) \geqq (1 - \theta) f_i(L_i^o, K_i^o) + \theta f_i(L_i^1, K_i^1) = y_i^\theta .$$

Define $\lambda_i = y_i^\theta / f_i(L_i^\theta, K_i^\theta)$ if $y_i^\theta > 0$ and $\lambda_i = 0$ if $y_i^\theta = 0$; then $f_i(\lambda_i L_i^\theta, \lambda_i K_i^\theta) = y_i^\theta$ by the homogeneity of f_i, and moreover $\lambda_1 L_1^\theta + \lambda_2 L_2^\theta \leqq L_1^\theta + L_2^\theta \leqq L$ and $\lambda_1 K_1^\theta + \lambda_2 K_2^\theta \leqq K_1^\theta + K_2^\theta \leqq K$.

10. This may be generalized to the case of n commodities and m factors, in which case the condition corresponding to (2.7) is that any set of k rows of the Jacobian matrix $\partial g/\partial w$ be linearly independent. In particular, this requires $n \leqq m$ (see footnote 12).

11. Cf. Shephard [26].

12. Cf. Samuelson [24, p. 10]. Samuelson stated the proposition for n commodities and m factors, omitting mention of the requirement of non-switching from specialization to diversification as well as of the condition $n \leqq m$ needed to assure the single-valuedness of the functions Y_i.

13. The fact that it is immaterial whether the distortion is introduced in the domestic price of the import or export good is, of course, the substance of Lerner's symmetry theorem; cf. [17].

14. This is only to "fix ideas"; the subsequent analysis in no wise depends, and indeed should not depend, on this interpretation, since we cannot know in advance which commodities will be imported or exported.

15. See the lucid discussion by Connolly and Ross [4], who emphasize that conditions (4.10a) and (4.10b) simply characterize efficiency, and do not depend on the particular choice of a social welfare function (which enters only via the function $\tilde{X}_1$ in (4.10c)). It should also be stressed, however, that this possibility of splitting up the optimization problem into an efficiency problem and a distributive problem <u>does</u> depend on the hypothesis that there is <u>some</u> social welfare function which forms the basis for the subsequent redistribution (by a system of taxes and subsidies supplementary to those to be discussed in the following section).

16. This assumption is implicit in the treatments of both Kemp [15] and Jones [12], that is, they both tacitly assume that the foreign offer function is derived by maximization of an aggregate utility function.

17. The amount $-(K - C)(\partial^2\Pi/\partial K^2)dK$ corresponds to the area of the rectangle EDJI in MacDougall's Diagram I [18, p. 15].

18. Alternatively, keeping in mind the convention (4.9), and the fact that the essential relations corresponding to (5.12) and (5.12') are

$$\text{(5.12a)} \qquad \frac{p_1^*}{p_2^*} = (1 + \tau)\frac{p_1}{p_2} \; ; \qquad \frac{p_1}{p_2} = (1 + \tau')\frac{p_1^*}{p_2^*} \; ,$$

if we instead of (4.9) adopt the normalization $p_1 = p_1^* = 1$ then we have

$$\text{(5.12b)} \qquad p_2 = (1 + \tau)p_2^* \; ; \qquad p_2^* = (1 + \tau')p_2 \, .$$

Then if $\tau < 0$ this can be interpreted as meaning either that imports of commodity 2 are subsidized at a rate of -100τ %, or that exports of commodity 2 are taxed at a rate of $100\tau'$ %.

19. The notation adopted here for γ^*, δ^*, ε^*, μ^* is the same as that of Kemp [15] and Jones [12]; however, the sign of their η^* is the reverse of that of the Marshallian definition (5.17) adopted here.

20. In terms of formula (5.21a) below, condition (5.20) is equivalent to the requirement that $\tau > -1$.

21. For a discussion of the "paradoxes" which result from erroneous interpretation of formulas such as (5.21), see the excellent discussion by Graaff [7, p. 136].

22. The term "possible" is subject to the reservation that the existence of solutions with the sign patterns indicated in Table 1 must still be established; thus "not proved impossible" would be more accurate. Nevertheless, the discussion following (5.26) below will provide a basis for an existence proof, though a formal proof will not be attempted here.

23. Inada and Kemp [10] have shown that if an initial free-trade equilibrium with diversification exists, then a tariff-cum-tax equilibrium with diversification will exist if the tariff and tax are sufficiently small. A similar continuity argument could extend this to optimal tariff and tax rates, if conditions could be found under which the benefits from further extension of these rates come to an end before specialization has been reached.

24. The general formula (when the foreign country diversifies) is

$$\frac{\partial V}{\partial K^*} = R^* - R - (p_1^* - p_1)\frac{\partial Y_1^*}{\partial K^*} - (p_2^* - p_2)\frac{\partial Y_2^*}{\partial K^*}$$

where $\partial Y_1^*/\partial K^* = - \partial Z_1^*/\partial K^*$ on account of (3.15). If instead of (4.9) we assume $p_1 = p_1^*$ and interpret τ as a tariff on imports (see footnote 18), then in a situation of unimpeded capital flows we have $\partial V/\partial K^* =$ $(p_2 - p_2^*)\,\partial Y_2^*/\partial K^* = \tau p_2^*\,\partial Y_2^*/\partial K^*$. Under (4.9) we have $\partial V/\partial K^* = -\tau p_1\,\partial Y_1^*/\partial K^*$ (see (5.12)).

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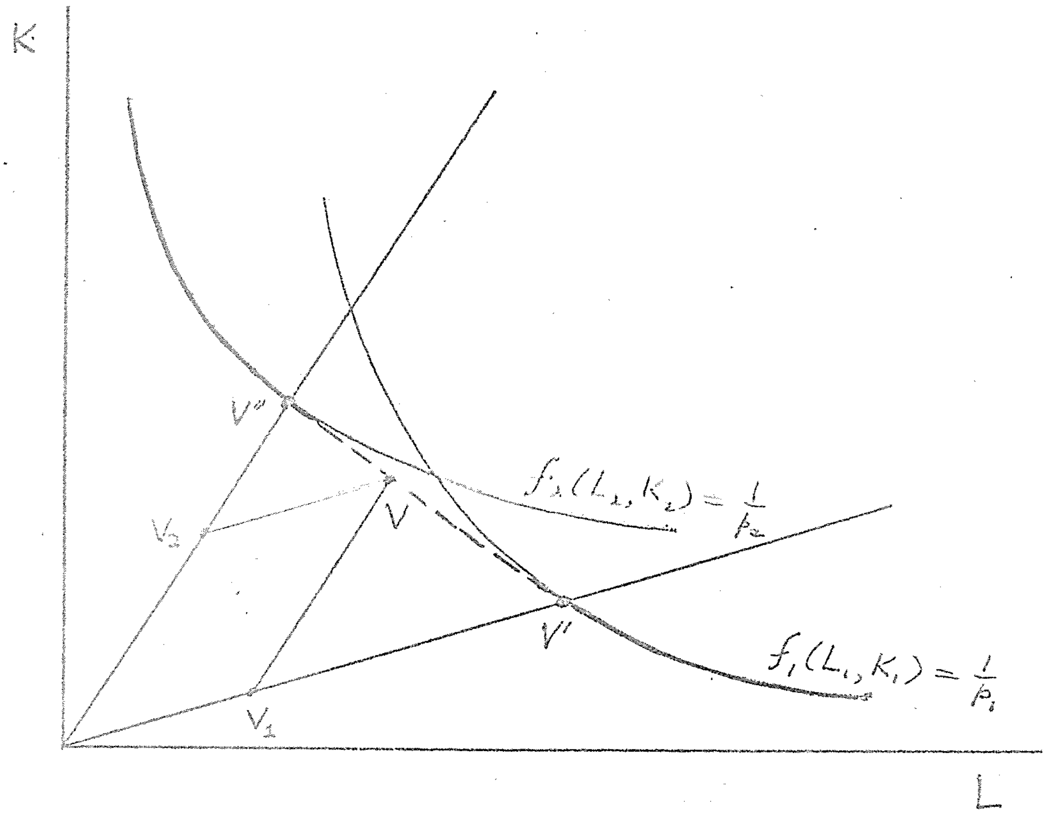

FIGURE 1

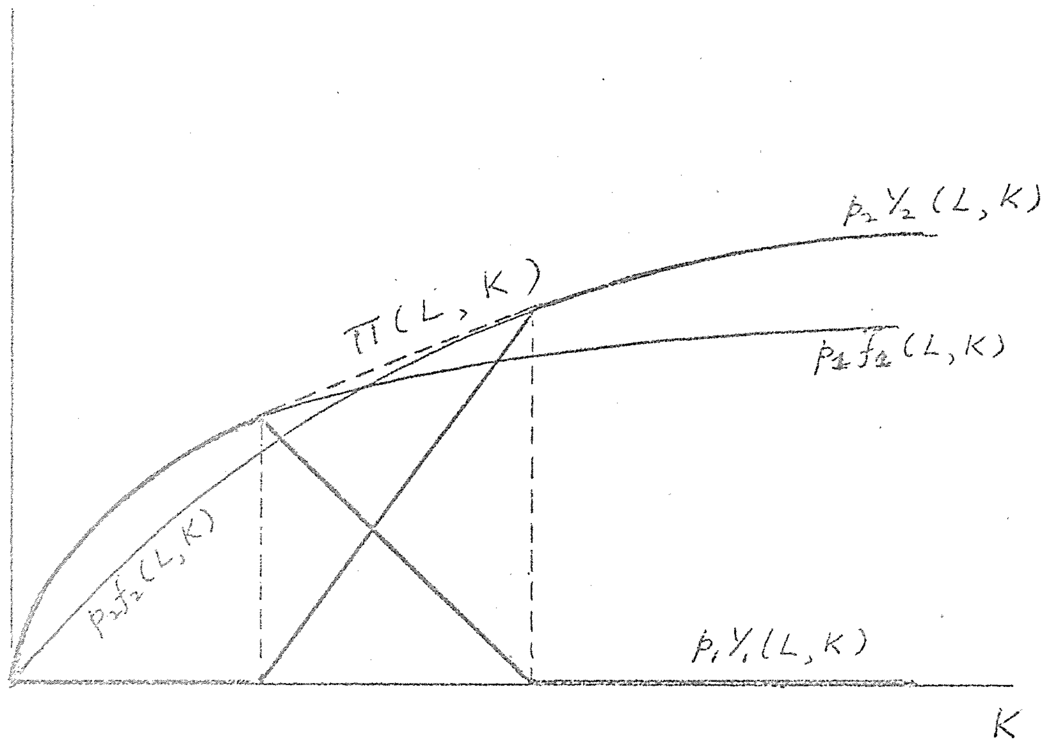

FIGURE 2