MSc Economics

Banking, Liquidity and Bank Runs: Replication and Analysis of The Solution Method of Gertler/Kiyotaki (2015)

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by

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Vienna, June 6, 2016

MSc Economics

Affidavit

I, Nikola Grga

hereby declare

that I am the sole author of the present Master's Thesis,

Banking, Liquidity and Bank Runs: Replication and Analysis of The Solution Method of Gertler/Kiyotaki (2015)

20 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, June 6, 2016

Signature

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Abstract

The paper of Gertler and Kiyotaki(2015a) is important step forward in the modeling of financial sector in macroeconomic models, because it combines financial accelerator mechanism with the bank runs, two phenomena observed in the last economic crisis. Therefore, aim of the paper is to replicate the results of the Gertler and Kiyotaki(2015a) in order to check their validity. Also the procedure for solving the model is rewritten in order to be precise, conceptually clear and easy to follow. For the replication, the numerical solver based on the quasi-Newton method is used. The results of the replication confirms the results of the Gertler and Kiyotaki(2015a). The model should be further developed in order to incorporate more complex banking sector that includes commercial banks.

1 Introduction

Prior to financial crisis the assumption of market completeness was widely accepted in macroeconomic modeling. According to Leeper and Nason(2014), this assumption implies that any bankruptcy or default that could happen in economy will stay in a set of relevant actors and it won't become systemic. Thus the real and financial sector could be studied separately, i.e. there was no need for financial markets in macroeconomic framework. However, Leeper and Nason(2014) claim that market incompleteness and financial frictions, understood as wide array of departures from complete markets, raise the possibility of financial crisis that will affect entire economy. So, taking in account the example of latest economic crisis, abstraction from complete market assumption and introduction of financial frictions are important for proper modeling of economic reality.

However, the most prominent examples in financial friction literature such as Bernanke et al(1999), Kiyotaki and Moore(1997), Carlstrom and Fuerst(1997) and Curdia and Woodford(2009) miss to introduce banking sector in full sense , which according to Goodhart and Tsomocos(2012) is one of the key issues for analysis of financial stability. Financial stability is defined as: " *A financial system is in a range of stability whenever it is capable of facilitating (rather than impeding) the performance of an economy, and of dissipating financial imbalances that arise endogenously or as a result of significant adverse and unanticipated events.*"(Schinasi 2004:8)

The experience from the latest crisis indicates a great need for theoretical framework to study financial stability issues. Therefore, the preventions or diminution of the effects of some future financial crisis might depend on development of model with financial frictions and sophisticated banking sector. In that sense, interesting step forward in modeling banking sector is Gertler and Kiyotaki(2015a).

This model shows great potential because it incorporates financial accelerator effect with bank runs. Many authors, such as Adrian et al(2013) and Bernanke(2010), found these phenomena in the latest economic crisis. Due to combination of their effects, the possibility of bank run is endogenous and depends on the economic fundamentals, which is novelty in comparison to the other models.

Therefore, the aim of thesis is to replicate the results of Gertler and Kiyotaki (2015a) and to rewrite the procedure for the solving the model.

The thesis is organized in the following manner. First, motivation is given. Then, Gertler and Kiyotaki(2015a) is described. Further, the procedure used for replication is described in the next section while the conclusion is given in the final section.

2 Motivation

Figure 6 in Gertler and Kiyotaki(2015a) show behavior of the model that is in contrast with propositions of the model. Subplot for net worth, n, is negative in the moment of the bank run in period 3. According to Gertler and Kiyotaki(2015a) page 2031 and Gertler and Kiyotaki(2015b) page 1, new worth cannot be negative. Furthermore, the value of net worth in the next period also differs from value Gertler and Kiyotaki(2015a) propose on page 2024. Finally, due to incorrect value of net worth in the period after the run, leverage multiple, ϕ, in period after the run is too high.

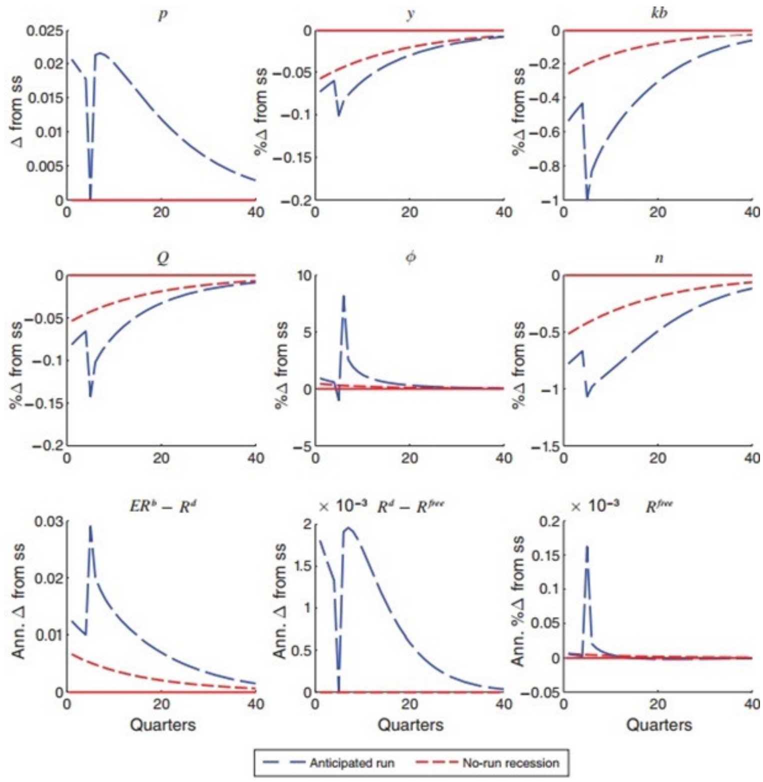

Figure 1: Recession with positive run probability and ex post run

source: Gertler and Kiyotaki (2015a) Figure 6, page 2034

Knowing the importance of Gertler and Kiyotaki(2015a), it is needed to check the result of the paper. Computer code is available with the paper, but it is very complex and not easily tractable, therefore in the thesis it is decided to replicate model on our own. Hence, motivation for the replication of the paper is to check the results. Later, when paper is replicated it turned out that mistake was not conceptual, but computational and benign. On the graphs where the results of the replication are compared with results of Gertler and Kiyotaki(2015a), the correct results of Gertler and Kiyotaki(2015a) are used.

Furthermore, the computational procedure for finding the solution of the model is given in the Gertler and Kiyotaki(2015b). However, the description of the procedure is imprecise and hard to follow. Therefore, it is decided to rewrite the procedure in order to be precise, conceptually clear and easy to follow.

3 Description of Gertler and Kiyotaki(2015a) model

In this section short description of Gertler and Kiyotaki(2015a) is given and all the equation in this section are from Gertler and Kiyotaki(2015a). In the model there are two types of agents, households and bankers, with continuum of the measure unity for each type and two goods, nondurable good and durable asset,i.e. capital. Bankers are specialist in making loans and they intermediate funds between households and productive asset. For K_t^b units of the capital intermediated in period t, a banker receives $Z_{t+1}K_t^b$ units of good in period t+1 and leftover capital K_t^b, where Z_{t+1} is a technology process. Although households can make loans directly, they face management costs that reflect lack of expertise in screening and monitoring of investment projects. The management cost are paid in the moment investment is made while the payoff in the next period is the same as with bankers.

The following description corresponds to the cases when bank run is not anticipated. The changes in the model when bank run is anticipated are given in the subsection 4.3.

3.1 Households

The representative household can save and consume. It can save either through depositing funds to the competitive bank or through holding the capital directly. The household maximizes utility function:

$$U_t = \max_{C_t^h, K_t^h, D_t} E_t\left(\sum_{i=0}^{\infty} \beta^i ln C_{t+i}^h\right) \qquad (2.1)$$

subject to the budget constraint:

$$C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h \qquad (2.2)$$

where:
C_t^h is the household consumption in period t;
K_t^h is the household capital holdings (hereafter household capital) in period t
D_t is the household deposits to the bank in the period t that will mature in period $t+1$;
Q_t is the price of the asset in period t;
R_{t+1} is the household return on deposits, which are made in period t where:
$R_{t+1} = \begin{cases} \bar{R}_{t+1} & if\ no\ bank\ run\ occurs\ in\ period\ t+1 \\ x_{t+1}\bar{R}_{t+1} & if\ run\ occurs\ in\ period\ t+1; \end{cases}$
$\bar{R}_{t+1}$ is the rate that bank promise to pay on the deposits in period $t+1$ in the absence of the bank run;
x_{t+1} is the recovery rate in period t+1;
W^h is the household endowment;
$f(K_t^h)$ is the management cost that is defined as: $f(K_t^h) = \frac{\alpha}{2}(K_t^h)^2$.

3.2 Banks

Banks in the model corresponds best to the shadow banks. They deposit funds from the households, which they later use for capital investments. Also, the retain profit from the intermediation in one period is used for capital investments in the next period. To prevent banks from accumulating profits to the extent that they can finance capital investments entirely from their own resources and free themselves from the financial frictions, it is assumed that in every period σ percent of the households exit the model. To keep the number of the banks in the model constant it is assumed the same number of the bank enter the model in the that period. In the same period they exit, bankers consume retain profit from the previous period. The expected utility of a banker that continues to operate in period $t+1$ is:

$$V_t = E_t(\sum_{i=1}^{\infty} \beta^i (1-\sigma)\sigma^{i-1} c_{t+i}^b) \qquad (2.3)$$

where:
$(1-\sigma)\sigma^{i-1}$ is the probability of the exiting in the period $t+i$;
c_{t+i}^b is the banker consumption conditional on exit in $t+i$ and it is equal to n_{t+1}.

The retain profit of the bank for the intermediation from the period $t-1$, which is called net worth of the bank in the model, is defined in the following way:

$$n_t = (Z_t + Q_t)k_{t-1}^b - R_t d_{t-1} \qquad (2.4)$$

where:
k_{t-1}^b is the capital holdings of the bank in the period $t-1$;
$d_{tß1}$ is the deposit that one bank took from the households in period t-1.

For entering bank in the period t, net worth is defined in the following way:

$$n_t = w^b \qquad (2.5)$$

where w^b is initial endowment of a bank.
Every bank faces budget constraint of the form:

$$Q_t k_t^b = d_t + n_t \qquad (2.6)$$

Finally, every bank faces incentive constraint due to the moral hazard problem that arises in relationship between depositor and the bank. Since bank can divert assets for the personal use, incentive constraint is proposed to set the incentives right. Also, the incentive constraint limits the bank ability to issue deposits and the size of the bank's portfolio. The incentive constraint is given below:

$$\theta Q_t k_t^b \leq V_t \qquad (2.7)$$

where θ is the percent of the portfolio that banker can divert for personal use without being detected by the depositors. Dishonest behavior of the bank can lead to the default of the bank in the next period.

Bankers optimization problem can be stated as following:

$$\psi_t = \max_{\phi_t} E_t \beta(1-\sigma+\sigma\psi_{t+1})[(R_{t+1}^b - R_{t+1})\phi_t + R_{t+1}] \qquad (2.8)$$

subject to incentive constraint:

$$\theta\phi_t \leq \psi_t \qquad (2.9)$$

where:
ψ is Tobin q ratio that is defined as: $\psi_t = \frac{V_t}{n_t}$;
ϕ_t is the leverage multiple defined as: $\phi_t = \frac{Q_t k_t^b}{n_t}$;
R_{t+1}^b is the return on the capital investment that is defined as: $R_{t+1}^b = \frac{Z_{t+1}+Q_{t+1}}{Q_t}$.

3.3 Aggregate variables

The total holdings of capital is equal to the sum of the total capital holdings of households and banks and it is fixed at unity. The aggregate variables are given in the capitals.

$$1 = K_t^b + K_t^h \qquad (2.10)$$

Having that ϕ_t is independent of individual bank-specific factors, it is possible to aggregate across banks to obtain:

$$Q_t K_t^b = \phi_t N_t \qquad (2.11)$$

where N_t is the total net worth in period t. The evolution of N_t is given below:

$$N_t = \sigma[(Z_t + Q_t)K_{t-1}^b - R_t D_{t-1}] + W^b \qquad (2.12)$$

where the first term is net worth of the bankers that operated at t-1 and survived to period t and W^b is the endowment of all the entering banks in one period that is defined as: $W^b = (1-\sigma)w_b$.

Aggregate consumption of the exiting bankers in period t is given by:

$$C_t^b = (1-\sigma)[(Z_t + Q_t)K_{t-1}^b - R_t D_{t-1}] \qquad (2.13)$$

Finally, resource constraint for the entire economy is given by:

$$Z_t + Z_t W^h + W^b = f(K_t^h) + C_t^h + C_t^b \qquad (2.14)$$

and the net output is given:

$$Y_t = C_t^h + C_t^b \qquad (2.15)$$

4 Replication and the Procedure

Gertler and Kiyotaki (2015a) study four different cases: the run is not anticipated and it does not happen(hereafter no unanticipated run case), run is not anticipated but it does happen(hereafter unanticipated run case), run is anticipated and it does happen (hereafter anticipated run case) and the run is anticipated but it does not happen(hereafter no anticipated run case). In presentation of the replications same logic will be followed.

Shock to technology process Z_t takes place at the beginning of the period 1 and it is the first event in that period. If it is assumed that after the technology shock, household perceives in period t zero probability of the bank run in period t+1 $\forall t : t > 0$, then bank run is not anticipated, i.e. bank run is unanticipated. Otherwise, if after the technology shock, household in period t perceives positive probability of the bank run in t+1 for some $t > 0$, then the bank run is anticipated.

In the thesis, to replicate the results of Gertler and Kiyotaki(2015a), Gertler and Kiyotaki (2015b) is followed closely. However, the computer code written for the purpose of the thesis differs from the original code of Gerlter and Kiyotaki(2015a) in three aspects.

Firstly, for both no unanticipated run case and unanticipated run case, in the thesis the model is solved with numerical solver for solving systems of nonlinear equations based on Broyden method proposed in Press et al(1992), instead of deterministic simulation in Dynare[1] that authors used. The source code for Dynare is written in C++ and hard to follow. In that sense, Dynare is a "black box". Therefore, to have better control over the process of computation, more tractable numerical solver is used in the thesis.

Secondly, for every iteration step in anticipated run case[2], instead of applying numerical solver on the reduced form of the system given in Gertler and Kiyotaki (2015b) and subsequently find the values of the rest of variable on the basis of the solution of the system, in the thesis the entire system of equations provided in Gertler and Kiyotaki (2015b) is solved numerically, including equations with non differentiable functions. In no anticipated run case, the same is done, even though the model is solved in one iteration. We decide to follow mention approach because we were concern with the treatment of non differentiable functions in the original code and the precision of the original code. Having that original code is very complex, the safest approach to check the results is to numerically solve entire system of equations, without any reduction, and then to compare the results with the original ones.

Thirdly, for the anticipated run and no anticipated run case, a numeric solver for solving systems of nonlinear equations based on Broyden method is applied, instead of standard trust region dogleg method[3]. Broyden method is quasi-Newton method that solves the non linear system of equations of the form $f(x) = 0$, where the Jacobian matrix is computed only once at the beginning of the algorithm and then it is updated through the rest of the process[4]. The system of equations solved in the thesis comprises non differentiable functions, which pose a challenge to numerical solvers,i.e. convergence of numerical solver is not guaranteed and if it does converge it might take lot of iterations. In our case, numerical solver based on Broyden method converged, as opposite to the one based on trust region dogleg method that did not converge. Convergence of the numerical solver is a sign that system of equations is solved with

[1]for details see Adjemian et al(2011)
[2]details in anticipated run case subsections
[3]for details see Nocedal and Wright(2006), page 73
[4]for details on Broyden method see Press et al(1992)

certain precision. Numerical solver tries to make the difference between LHS and RHS of every equation in the system equal to zero(equations are provided in appendix). The difference between LHS and RHS of every equation in the system should be less than convergence criterion for numerical solver to converge.

Gertler and Kiyotaki(2015a) propose two equilibria, no bank equilibrium and bank run equilibrium. No bank equilibrium is equilibrium when household decides to roll over the deposit for another period. This is the equilibrium that always exists. Otherwise, if the household decides not to roll over deposit for another period, the economy is in a bank run equilibrium. For the bank run equilibrium, a necessary, but not sufficient condition, is that recovery rate is less than unity. Furthermore, having that a bank run equilibrium coexists with no bank equilibrium in some periods after the technology shock, Gertler and Kiyotaki(2015a) allow for a sunspot which can shift the economy from no bank run to the bank run equilibrium. However, since sunspot shifts economy from one equilibrium to another, it is possible that economy will stay in no bank equilibrium in all periods, i.e. bank run does not happen. Otherwise,if sunspot shifts economy to bank run equilibrium, bank run will happen immediately. All in all, for bank run to happen in period t, the bank run equilibrium must exist in period t, i.e. recovery rate in period t must be less than unity, and sunspot has to move economy from no run to run equilibrium in period t.

In the model there are two potential sources of uncertainty: the exogenous technology process Z_t and the bank run. In all four cases, the Z_t is not a source of uncertainty, because after the initial shock to technology at the beginning of the first period, it follows deterministic process without any further shock until period T, after which it is equal to its steady state.

In no unanticipated run and unanticipated run case, since household does not consider a bank run as an option before the moment bank run really happens, i.e. in every period it assigns a zero probability to a bank run in subsequent period, the bank run is not the source of uncertainty. Although household does not anticipate bank run, the bank run will happen if the conditions for bank run equilibrium are meet. Therefore, the systems of equation that describes model in these two cases are deterministic and household has perfect foresight.

In no anticipated run and anticipated run case, the bank run is a sole source of the uncertainty. The household considers the bank run before it really happens in these two cases, i.e. it assigns a probability to the bank run in the subsequent period according to the equation (4.5), while the bank run itself depends on a sunspot, which is a random event. Therefore, the household considers a random event, when it makes intertemporal decisions, which implies it faces the uncertainty. Contrary to the previous two cases,the systems of equation that describes model in these two cases are stochastic and household has rational expectations.

The bank run will happen every time conditions are meet and for the numerical examples that are replicated in the thesis, Gertler and Kiyotaki(2015a) propose that bank run conditions are meet at the beginning of the third period in unanticipated run and anticipated run cases, while in the other two cases they are never meet. Also, in unanticipated run and anticipated run cases, the conditions are meet exactly ones.

The deterministic technology process Z_t that was used in the thesis, equation (1), has the same form as technology process Z_t used in the original code for Gertler and Kiyotaki(2015a) that differs from the form Z_t is given in the Gertler and Kiyotaki(2015a) paper[5].

[5] see Gertler and Kiyotaki(2015a) page 2038

$$Z_t = exp(-0.05 * 0.95^{t-1})Z_{ss} \quad for\, t = 1...T \qquad (4.1)$$

For periods $t > T$, $Z_t = Z_{ss}$. In Gertler and Kiyotaki(2015a) $T = 120$ in all four cases. In the thesis, for no unanticipated run and unanticipated run cases $T = 120$ as well, while for no anticipated run and anticipated run cases $T = 150$. In latter cases, T had to be chosen big enough to prevent sudden oscillation of the variables in the periods slightly before period T.

In the thesis, for the resource constraint equation when run happens in period t (4.2) and when it does not happen in period t (4.3), forms given in the original code for Gertler and Kiyotaki (2015a), which differs from the forms they are given in Gertler and Kiyotaki (2015b), are used. More to the point, instead of being a simple multiple of a technology process, household endowment is a multiple of a technology process normalized by its steady state, i.e. $\frac{Z_t W^h}{Z^{ss}}$ instead of $Z_t W^h$. Since $Z^{ss} = 0.0126$, the household endowment and output when household endowment is a multiple of Z_t are always below the household endowment and output when household endowment is multiple of normalized Z_t. Having that results are produced with the code, to exactly replicate results of Gertler and Kiyotaki (2015a), it is necessary to use the equations (4.1),(4.2) and (4.3), instead of the their counterparts given in the paper.

$$C_t^h + \frac{\alpha}{2} = Z_t + \frac{Z_t}{Z_{ss}} W^h \qquad for\, t = 1...T \qquad (4.2)$$

$$C_t^h + \frac{(1-\sigma)}{\sigma}(N_t - W) + \frac{\alpha}{2}(K_t^h)^2 = Z_t + \frac{Z_t W^h}{Z^{ss}} + W \qquad for\, t = 1...T \qquad (4.3)$$

After the shock, no matter if bank run happens or not, the economy turns back to the same steady state it was before the shock. Gertler and Kiyotaki(2015a) parametrize model to prevent bank runs in steady state. In the replications, original calibration from Gertler and Kiyotaki (2015a) for all 4 cases are used.

Because the systems of equations comprise non differentiable functions, in the thesis the following settings for numerical solver are used:

1.) convergence criterion is set to 1e-5,
2.) step length for forward difference Jacobian is also set to 1e-5,
3.) maximal number of iterations is set to 2000.

In every case, numerical solver converged, i.e. $max(|f(x^*)|) < 1e-5$, where x* is the solution of the system found by numerical solver based on Broyden method and f is the function, $f : R^M \rightarrow R^M$ where M is number of variables, that finds the residuals between LHS and RHS of every equation in the system. The systems solved for every case are provided in the appendix.

The results, as well as the original results of Gertler and Kiyotaki(2015a), are given in the form of impulse response function. In all 4 cases, replications exactly match the original results of the Gertler and Kiyotaki(2015a)

4.1 Run Is Not Anticipated And It Does Not Happen

First case describes the situation when households do not anticipate bank run and bank run does not take place. To find the paths for variables after the unexpected shock at the beginning of the first period, deterministic problem that consists of equal number of equation as variable should be solved numerically, with solver based on Broyden method. Having that prior to technology shock system is in steady state and after the shock it tends to the same steady state, the vector of steady state values is used as initial guess for numerical solver.

The system which solution are the paths for variables from period 1 to period T is given in the appendix. The equation (A1.1) is the resource constraint of the economy and it corresponds to equation (2.14). In every period the expenditures, i.e. sum of the households consumption, bankers consumption and the management cost have to be equal to the income, i.e. the sum of capital output and household and bank endowments. The equation (A1.2) is Euler equation for capital and the equation (A1.3) is Euler equation for deposits. These two equations are the first order conditions from household optimization problem. The equations (A1.4)-(A1.7) are related to the banks. The equation (A1.4) is a definition of the leverage multiple and it corresponds to (2.11).The equation (A1.5) is a incentive constraint for banks when incentive is binding[6] and it limits the leverage level and the size of banks' portfolios.The equation (A1.6) is a budget constraint that banks face, i.e. investment in the assets must equal net worth of the banks and the deposits bankers took from the household. The equation (2.6) gives the budget constraint of the individual bank and it will correspond to (A1.6) when individual constraints are summarized across the banks. The equation (A1.7) is a definition of the net worth of entire banking system and it corresponds to the equation (2.12). The initial conditions for household capital, deposits and the interest rate are given by the equations (A1.8)-(A1.10), while the terminal conditions for asset price, household consumption, net worth and leverage are given by the equations (A1.11)-(A1.14).

The results of the replication match the original results of Gertler and Kiyotaki (2015a). The comparison between the results is presented in figure 1 and figure 2.

[6]for conditions see Gertler and Kiyotaki (2015a) page 2020

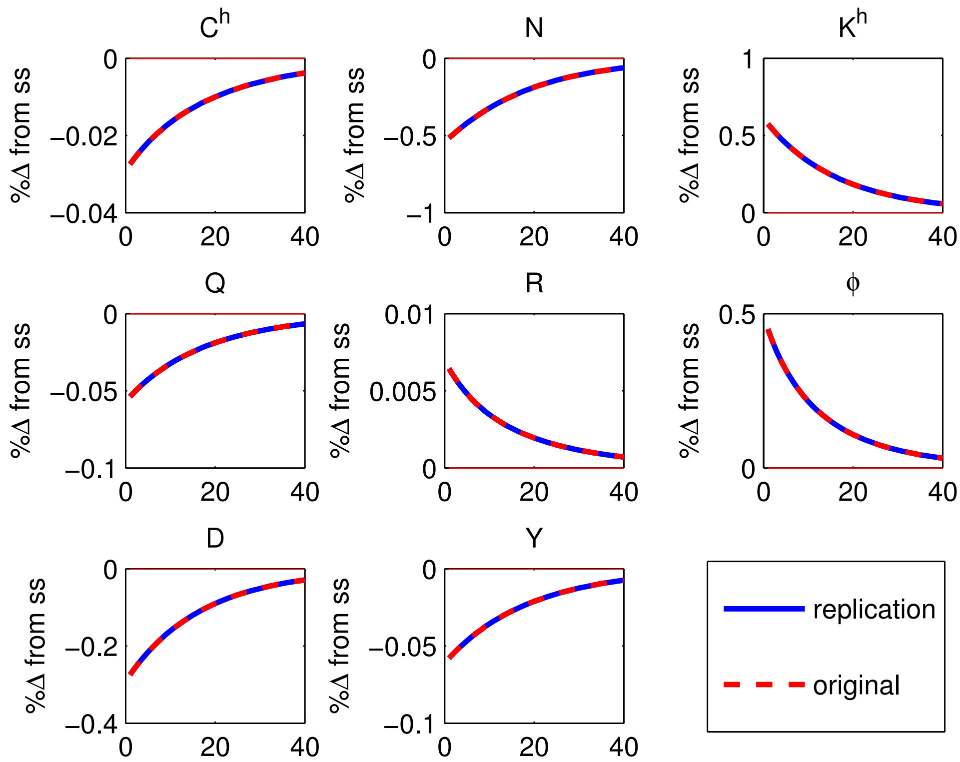

Figure 2: Comparison between replication and original results for no unanticipated run case

source: my computations

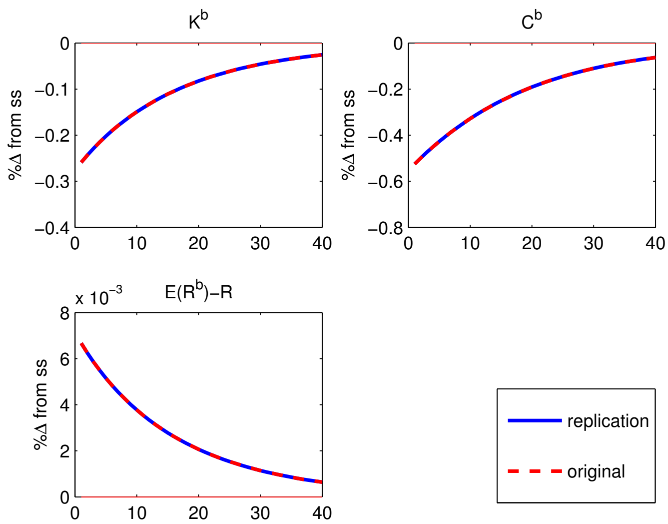

Figure 3: Comparison between replication and original results for no unanticipated run case: continuation

source: my computations

4.2 Run Is Not Anticipated but It Happens in 3rd Period

The second case describes the situation when bank run takes place at the beginning of the third period, although households do not anticipate the bank run. Prior to the bank run economy behaves as in the no unanticipated case. In the moment of the bank run, the system is reset. The new levels of the variables in moment of bank run are given by the equations (A2.8-A2.14) in appendix and they follow Gertler and Kiyotaki(2015b). Therefore, a path for every variable consists of two parts. The first part, for first two periods, is identical to the case when no unanticipated run happens, and the second part, for the periods 3 onward, is the solution of the system of equations given in the appendix. This is deterministic system with equal number of equation as variable that is solved numerically, with solver based on Broyden method.

In the moment of a bank run the state variables, K_t^h and D_t are reset to the new levels. $D_t = 0$ because entire banking system vanishes in the moment of the bank run and $K_t^h = 1$ because banks sell their entire capital to the households in the moment of the bank run. Consequently, after the bank run model starts from new values, completely unrelated to the values in the previous periods. Therefore, the paths after bank run can be computed independently of the the path before the run. Also, no matter whether household anticipates bank run or not nor in which period bank run happens, the model is always reset to the same values.

In the moment of the bank run, bankers' consumption is equal to zero, the entire capital is in hands of household and bank endowment is equal to zero by definition. Therefore, resource constraint equation in case of bank run (A2.12) is used to pin down the household consumption in that moment. Similarly, the equation (A2.8) is Euler equation for capital that is used to pin down the asset price in the moment of the run.

The equation (4.4) gives the level of the banks' net worth one period after the run. Although it is explained in Gertler and Kiyotaki(2015a), it needs further clarification. The bankers who should enter in the moment of run to substitute bankers who had to exit by the assumptions of the model, wait the next period to enter the model. For that reason banks' net worth in period 4 is equal the net worth of bankers that enters in period 4 and the net worth of the banker that should have entet in period 3, but had to wait by the assumption of the model until the period 4, reduced by the number of bankers exited at beginning of period 4. The equation (4.4) is taken from Gertler and Kiyotaki(2015a), page 2024

$$N_4 = (1 + \sigma)W \tag{4.4}$$

Having that after the bank run system starts from new values and then gradually goes back to steady state, the initial guess for numerical solver is the vector that comprises the linear paths from the bank run values to steady state.

Once the paths for period 3 are found, they are joined with the paths for no unanticipated run case in periods 1 and 2. In that way, the paths for all T periods are created. The results of the replication match the original results of Gertler and Kiyotaki (2015a). The comparison between the results is presented in figure 3 and figure 4.

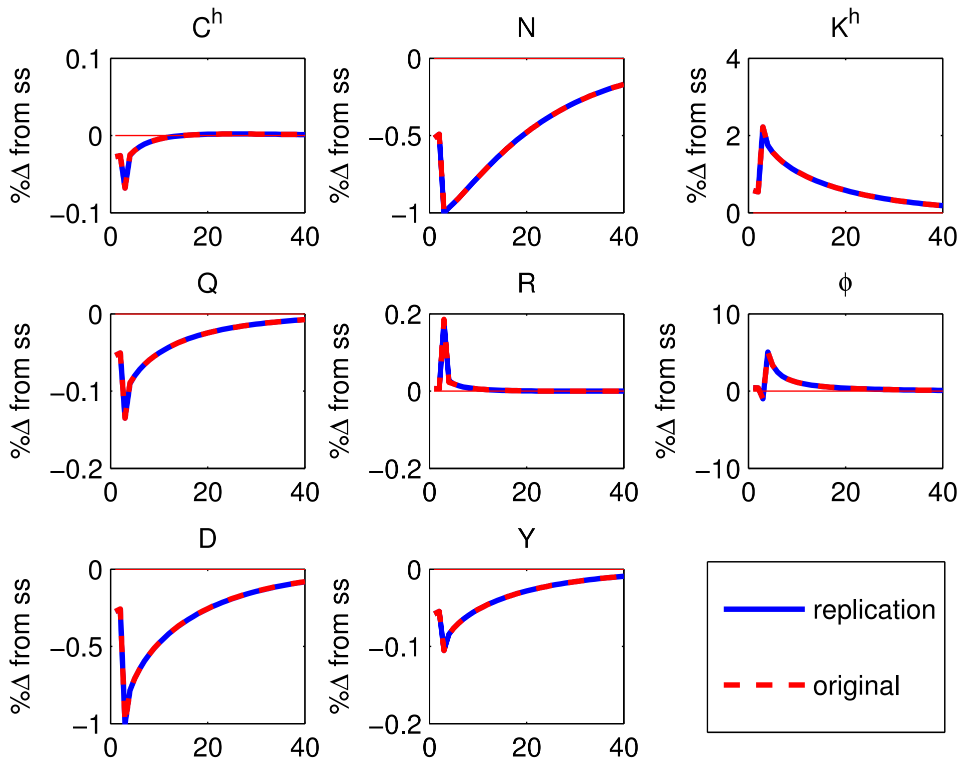

Figure 4: Comparison between replication and original results for unanticipated run case

source: my computations

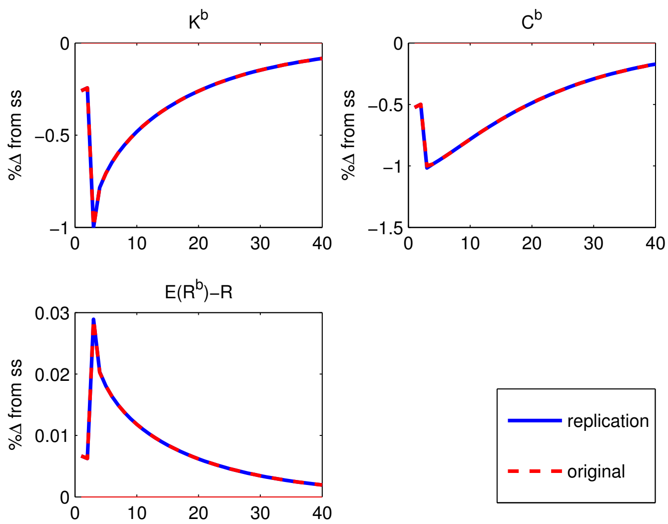

Figure 5: Comparison between replication and original results for unanticipated run case: continuation

source: my computations

4.3 Run Is Anticipated and It Does Happen in the 3rd period

In this section, household anticipates the bank run and bank run happens in period 3. As well as with unanticipated run case, the paths for variables comprise two parts. The first part, periods 1 and 2, corresponds to no anticipated run case(discussed in the next subsection), and the second part, from the 3rd period onwards, describes the behavior of the system after the bank run.

The introduction of the bank run probability changes the system of equations that describes the model. Household's Euler equations for assets (4.5) and deposits (4.6) reflect that household is aware of the potential bank run when it makes inter-temporal choices. Furthermore, the equation that limits bank portfolio size (4.8) is also altered and the equation (4.7), which defines bank run probability as a function of expected recovery rate [7] is introduced. In every period, the household uses equation (4.5) to find the probability of the bank run in the subsequent period. Equations (4.5)-(4.8) are taken from Gertler and Kiyotaki(2015b), page 5:

$$p_t = 1 - min[\frac{(Z_{t+1} + Q^*_{t+1})(1 - K^h_t)}{\bar{R}_{t+1} D_t}, 1] \tag{4.5}$$

$$Q_t + \alpha K^h_t = \beta[(1 - p_t)\frac{C^h_t}{C^h_{t+1}}(Z_{t+1} + Q_{t+1}) + p_t \frac{C^h_t}{C^{h*}_{t+1}}(Z_{t+1} + Q^*_{t+1})] \tag{4.6}$$

$$1 = \beta \bar{R}_{t+1}[(1 - p_t)\frac{C^h_t}{C^h_{t+1}} + p_t \frac{C^h_t}{C^{h*}_{t+1}} min(\frac{(Z_{t+1} + Q^*_{t+1})(1 - K^h_t)}{\bar{R}_{t+1} D_t}, 1)] \tag{4.7}$$

$$\theta\phi_t = \beta(1 - \sigma + \sigma\theta\phi_{t+1})[\phi_t(\frac{(Z_{t+1} + Q_{t+1})}{Q_t} - \bar{R}_{t+1}) + \bar{R}_{t+1}] \tag{4.8}$$

In order to facilitate convergence of the numerical solver, in the thesis equations (4.5) and (4.7) are slightly modified. For equation (4.5), probability of bank run is given in terms of max instead of min function (see equation A3.28 in appendix). In that way, probability still can not be negative, but min function in equation (4.7) can be dropped because any time $\frac{(Z_{t+1}+Q^*_{t+1})(1-K^h_t)}{\bar{R}_{t+1}+D_t} > 1$, probability of bank run is equal to 0 and corresponding part of equation (4.7) is equal to 0 (see equation A3.37 in appendix).

The introduction of the bank run probability makes the procedure of finding the solution more complicated. Since household in this case considers the bank run as an option when it makes inter-temporal decisions, in every period t household needs to know the asset price in case of bank run in the subsequent period, Q^*_{t+1}, and its consumption in case of bank run in the subsequent period, C^{h*}_{t+1}(see equations 4.5-4.8). To find Q^*_{t+1} and C^{h*}_{t+1}, the simulations when bank run happens just once in period t+1, for $t \geq 1$ have to be done. Also simulations should be done backwards, i.e. the first simulation that should be done is when bank run happens in the last period (the discussion on the last period is given bellow) and the last simulation when bank run happens in the second period.

The equation (4.9) defines the value of the asset price in period t if bank run happens in period t, for $t > T$. Also, the equation (4.10) defines the household consumption in period t when bank run happens in that period, for $t > T$. Having that $Z_t = Z_{ss}$ for $t > T$, Q^*_t=Q^* and C^{h*}_t=C^{h*} for all $t > T$. In addition, the other variables always reset to same value no matter in which period $t > T$ bank run happens. Hence, the paths after the bank run in period $t > T$ will be the same for all $t > T$. Therefore, the

[7] the recovery rate for anticipated run cases in Gertler and Kiyotaki(2015a) is defined as $x_{t+1} = min[\frac{(Z_{t+1}+Q^*_{t+1})(1-K^h_t)}{\bar{R}_{t+1}+D_t}, 1]$

first simulation can be done when bank run happens in any period $t > T$. However, to make the procedure more tractable the first simulation is done when run happens in T+1. Solving the model when bank run happens in period T+1 is a "basic step" in Gertler and Kiyotaki(2015b) and the equations (4.9) and (4.10) are taken from Gertler and Kiyotaki(2015b), page 5.

$$Q_t^* + \alpha = \beta(\frac{C_t^{h*}}{C_{t+1}^h}(Z_{ss} + Q_{t+1}))\, for\, t > T \qquad (4.9)$$

$$C_t^{h*} + \frac{\alpha}{2} = Z_{ss} + W^h\, for\, t > T \qquad (4.10)$$

To find the solution of the model in the basic step the system with equal number of equations and variables is solved numerically, with solver based on the Broyden method. The system of equations solved in "basic step" is provided in appendix.

To find the good initial guess for numerical solver, first the steady state is computed as if $Q^* = 0.9018$.Then, having that system starts and ends at the same steady state, the vector of these guesses for steady state is used as the initial guess for the solver in the "basic step".

The importance of Q_t^* and C_t^{h*} goes much further than just finding the paths when bank run happens in period t-1. The information on the Q_t^* and C_t^{h*} is important when the household makes inter-temporal decisions in period t-1. If $p_{t-1} = 0$, i.e. bank run is not possible in the moment t, then the information on Q_t^* and C_t^{h*} is irrelevant for household inter-temporal decision made in the period t-1, because they are neutralized with $p_{t-1} = 0$(see equations 4.6 and 4.7). However, if bank run happens in some period m, such that $1 < m < t$, where $p_m \neq 0$, the p_{t-1} might not be zero any more, because probability is a function of the other variables, besides Z_t, which value might change due to the bank run in the period m(see equation 4.5). Therefore, information on Q_t^* and C_t^{h*} is relevant anytime $p_t \neq 0$.

More to the point, in anticipated run case p_T, where $T = 150$, will be equal to 0, but the basic step and its main products, Q^* and C^{h*}, are not irrelevant for the procedure of finding the paths in this case. First, Q^* is needed to compute p_T. Further, Q^* and C^{h*} are needed for the simulation when bank run happens in period t,where $t < T$, for which $p_T \neq 0$. In that case, the household considers bank run possible in period T+1 and its decisions in period T are affected by Q^* and C^{h*}.

Once Q^* and C^{h*} are known from the basic step, it is possible to go one step back and compute the paths when bank run happens in period T, i.e. Q_T^* and C_T^{h*} can be found.The equation (4.11) defines asset price when bank run happens in period $t = 1...T$ and equation (4.12) defines household when bank run happens in period $t = 1...T$. Having that periods $t = 1...T$ $Z_t \neq Z_{ss}$, $Q_i^* \neq Q_{t+1}^*$ and $C_t^{h*} \neq C_{t+1}^{h*}$ for all $t = 1...T$.Therefore, in periods $t = 1...T$ asset price and household consumption in case of bank run depends on the timing of the bank run. The equations (4.11) and (4.12) are taken from Gertler and Kiyotaki(2015b), page 5.

$$Q_t^* + \alpha = \beta(\frac{C_t^{h*}}{C_{t+1}^h}(Z_{t+1} + Q_{t+1}))\, for\, t = 1...T \qquad (4.11)$$

$$C_t^{h*} + \frac{\alpha}{2} = Z_{t+1} + W^h\, for\, t = 1...T \qquad (4.12)$$

Once Q_T^* and C_T^{h*} are known, it is possible to compute the paths when bank run happens in period T-1. Therefore,by iterating backwards it is possible to find the paths when bank run happens in T-k,$k = 1...T-2$. Solving the model when bank runs happens in period t, where $t = 1..T$, is called "inductive step" in Gertler and Kiyotaki(2015b).

For every iteration step in "inductive step", the system with equal number of equations and variables is solved numerically, with solver based on the Broyden method. The system of equations solved in one iteration step is provided in appendix. Furthermore, the solutions of the model if bank run happen in period t, are used as initial guesses for numerical solver when bank run happen in period $t-1$, for $t = 2...T$. Although bank run happens in period 3, the system should be solved backwards until the first period is reached, because the information on Q_2^* and Q_3^* will be needed for no anticipated run case.

Finally, the paths when bank run happen in period 3 are joined with paths in no anticipated run case for first 2 periods. The results of the replication match the results of Gertler and Kiyotaki (2015a). The comparison between the results is presented in figure 5 and figure 6. The slight difference in path for spread between deposit rate and risk free rate in figure 6 is due to the precision of the computation.

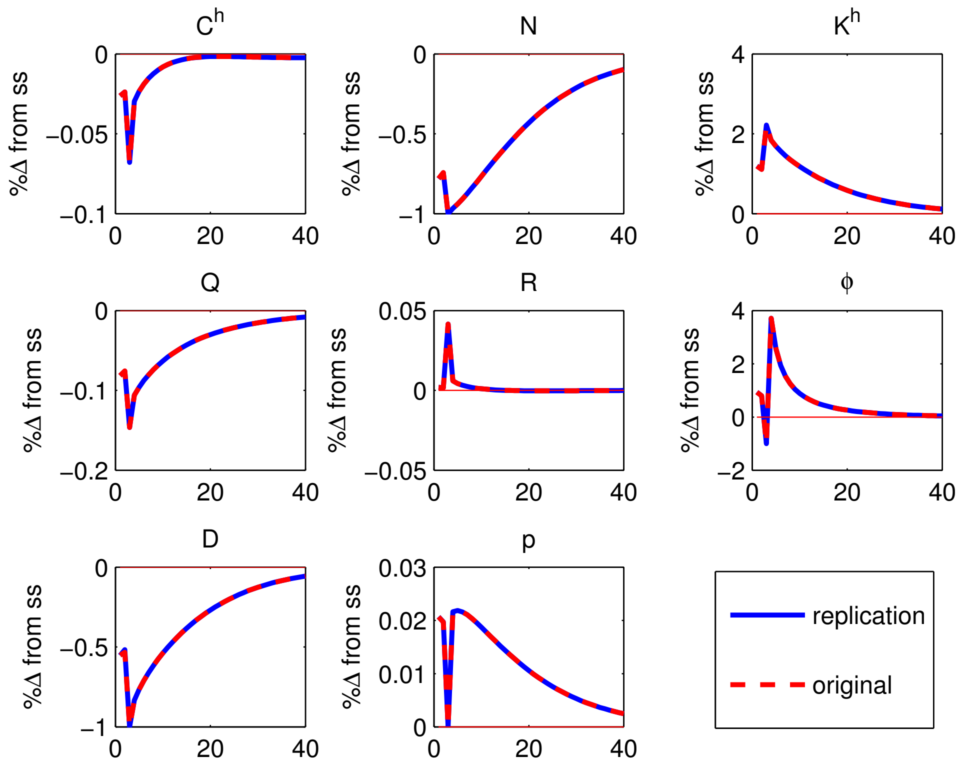

Figure 6: Comparison between replication and original results for anticipated run case

source: my computations

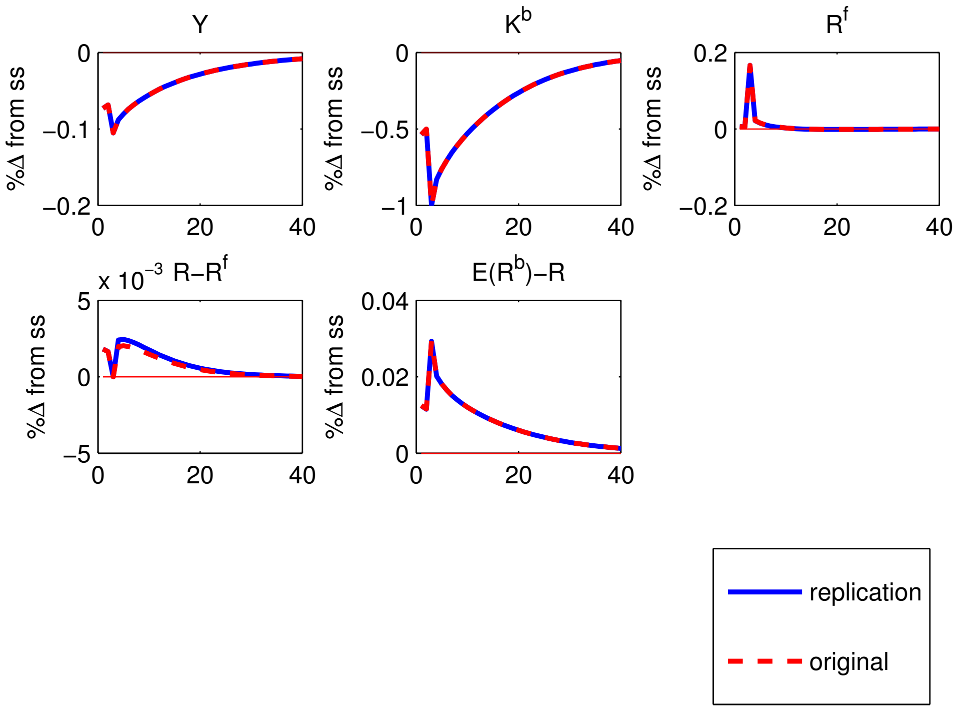

Figure 7: Comparison between replication and original results for anticipated run case: continuation

source: my computations

4.4 Run Is Anticipated but It Does Not Happen

In the last case, households anticipate bank run, but bank run does not happen. The paths for no anticipated run case are described by the system with equal number of equations and variables given in the appendix. Once the asset prices and household consumption in case of bank run in periods 2 to T+1 are computed in the anticipated run case, the system that describes no anticipated run case can be solved numerically, with solver based on the Broyden method. Furthermore, the paths when bank run happens in period 1 are used as initial guesses for numerical solver in no anticipated run case.

The results of the replication match the original results of Gertler and Kiyotaki (2015a). The comparison between the results is presented in figure 7 and figure 8.

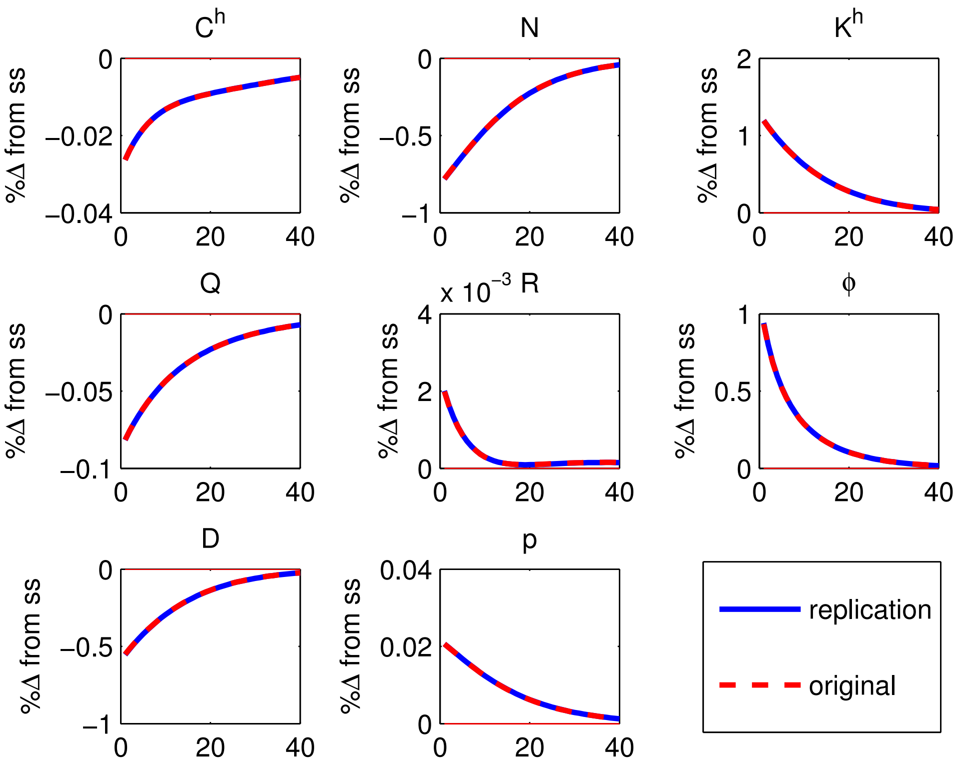

Figure 8: Comparison between replication and original results for no anticipated run case

source: my computations

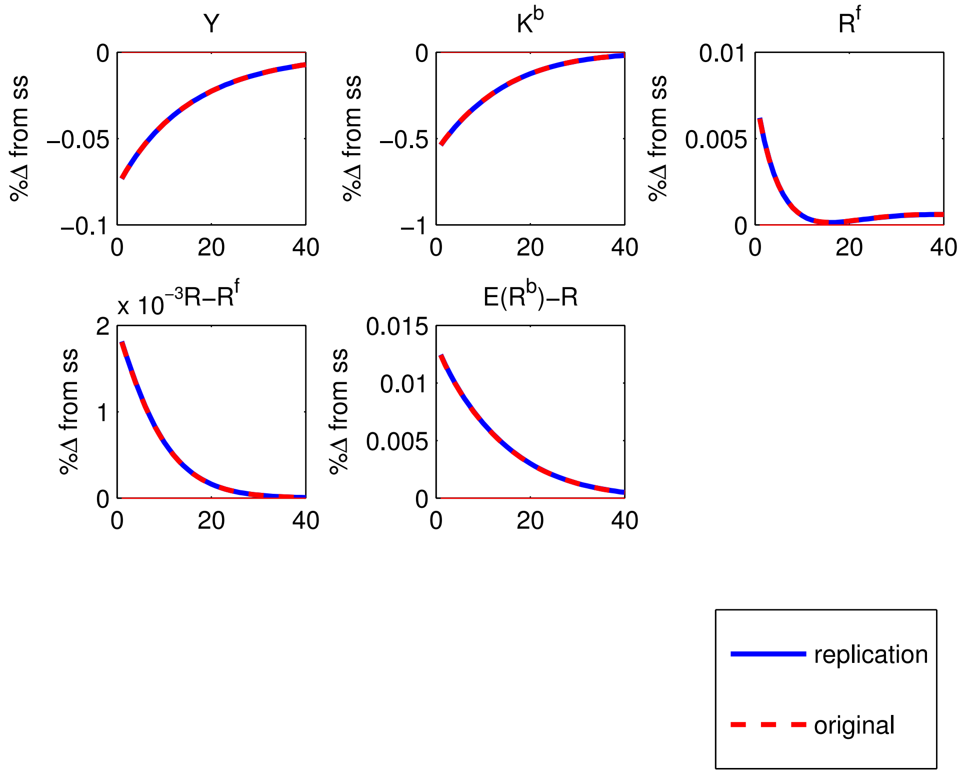

Figure 9: Comparison between replication and original results for no anticipated run case: continuation

source: my computations

5 Conclusion

In the thesis the results of the paper Gertler and Kiyotaki(2015a) are replicated. The results of the replication confirms the original results of the paper. Also, the procedure for finding the solution of the model is now written in more precise and conceptually clear manner. Also, in the thesis it is pointed out that there is a mistake in the figure 6 in Gertler and Kiyotaki(2015a). The mistake is corrected on the figures presented in the thesis.

Model of Gertler and Kiyotaki(2015a), because of the innovative way banking sector is modeled, shows great potential for future use in financial stability analysis. However, to be capable for financial stability analysis, the banking sector should be more complex and it must include commercial banks. Commercial banks are very important players, because they have huge funds under their management. Introduction of the commercial banks into the framework of Gertler and Kiyotaki(2015a) requires additional work. Therefore, the introduction of the commercial banks in this framework could be a topic for a PhD thesis.

6 Bibliography

Adjemian Stephane , Bastani Houtan, Juillard Michel, Karame Frederic, Mihoubi Ferhat, Perendia George, Pfeifer Johannes, Ratto Marco and Villemot Sebastien (2011): Dynare: Reference Manual Version 4, Dynare Working Papers, CEPREMAP, Paris

Adrian Tobias, Colla Paolo and Shin Hyun Song (2013): Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007 to 2009, In: Acemoglu, Parker and Woodford(Ed.), NBER Macroeconomics Annual 2012, Vol.27, University of Chicago Press, Chicago, pages 159–214

Bernanke Ben S. (2010): Causes of the Recent Financial and Economic Crisis, Before the Financial Crisis Inquiry Commission. September 2, 2010. Washington, DC: Board of Governors of the Federal Reserve System

Bernanke, Ben S., Gertler Mark and Gilchrist Simon (1999): The financial accelerator in a quantitative business cycle framework, Handbook of Macroeconomics, Vol. 1, Part C, pages 1341-1393, Elsevier, Amsterdam

Carlstrom Charles T. and Fuerst Timothy S. (1997): Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis, The American Economic Review, Vol.87, No.5, pages 893-910

Curdia Vasco and Woodford Michael (2009): Credit Frictions and Optimal Monetary Policy, BIS Working Paper No. 278.
`http://ssrn.com/abstract=1440289` - accessed on: 5.6.2015

Gertler Mark and Kiyotaki Nobuhiro (2015a): Banking, Liquidity and Bank Runs in an Infinite Horizon Economy, American Economic Review, Vol. 105(7), pages 2011-43

Gertler Mark and Kiyotaki Nobuhiro (2015b): Online Appendix of Banking, Liquidity and Bank Runs in an Infinite Horizon Economy,
`https://assets.aeaweb.org/assets/production/articles-attachments/aer/app/10507/20130665_app.pdf` - accessed on date:5.6.2015

Goodhart Charles and Tsomocos Dimitrios(2012): The Challenge of Financial Stability: a New Model and Its Applications, Edward Elgar Publishing, Cheltenham, UK

Kiyotaki Nobuhiro and Moore John (1997): Credit Cycles, Journal of Political Economy, Vol.105, No.2, pages: 211-248

Leeper Eric M. and Nason James M.(2014): Bringing Financial Stability into Monetary Policy, `http://ssrn.com/abstract=2523466` - accessed on date:5.6.2015

Nocedal Jorge and Wright Stephen J.(2006): Numerical Optimization, 2^{nd} ed., Springer Series in Operations Research, Springer Verlag, New York

Press William H., Teukolsky Saul A.,Vetterling William T. and Flannery Brian P.(1992): Numerical Recipes in C: The Art of Scientific Computing, 2^{nd}ed., Cambridge

University Press, Cambridge

Schinasi Garry J. (2004): Defining Financial Stability, IMF Working Paper WP/04/187,October

A Appendices

Having that the model of Gertler and Kiyotaki(2015a) is replicated, the equations presented in the appendix will follow closely Gertler and Kiyotaki(2015b).

A.1 Equations for No Unanticipated Run Case

The system of equations which solution gives the paths of variables from period 1 to period T is presented below. The subscripts present time indices and $t = 1, ...T - 1$.

$$C_t^h + \frac{(1-\sigma)}{\sigma}(N_t - W^b) + \frac{\alpha}{2}(K_t^h)^2 = Z_t + \frac{Z_t W^h}{Z^{ss}} + W^b \qquad (A1.1)$$

$$Q_t + \alpha K_t^h = \beta(\frac{C_t^h}{C_{t+1}^h}(Z_{t+1} + Q_{t+1})) \qquad (A1.2)$$

$$1 = \beta(\frac{C_t^h}{C_{t+1}^h} R_{t+1}) \qquad (A1.3)$$

$$Q_t(1 - K_t^h) = \phi_t N_t \qquad (A1.4)$$

$$\theta\phi_t = \beta(1 - \sigma + \sigma\theta\phi_{t+1})[\phi_t(\frac{(Z_{t+1} + Q_{t+1})}{Q_t} - R_{t+1}) + R_{t+1}] \qquad (A1.5)$$

$$Q_t(1 - K_t^h) = N_t + D_t \qquad (A1.6)$$

$$N_t = \sigma[(Z_t + Q_t(1 - K_{t-1}^h) - D_{t-1}R_t] + W^b \qquad (A1.7)$$

$$K_0^h = K_{ss}^h \qquad (A1.8)$$

$$D_0 = D_{ss} \qquad (A1.9)$$

$$R_1 = R_{ss} \qquad (A1.10)$$

$$Q_T = Q_{ss} \qquad (A1.11)$$

$$C_T^h = C_{ss}^h \qquad (A1.12)$$

$$\phi_T = \phi_{ss} \qquad (A1.13)$$

$$N_T = N_{ss} \qquad (A1.14)$$

$$Z_t = exp(-0.05 * 0.95^{t-1})Z_{ss} \qquad (A1.15)$$

$$Z_T = exp(-0.05 * 0.95^{T-1})Z_{ss} \qquad (A1.16)$$

A.2 Equations for Unanticipated Run Case

The system of equations which solution gives the paths of variables from period 3 to period T is presented below. The subscripts present time indices and $t = 4, ...T - 1$.

$$C_t^h + \frac{(1-\sigma)}{\sigma}(N_t - W^b) + \frac{\alpha}{2}(K_t^h)^2 = Z_t + \frac{Z_t W^h}{Z^{ss}} + W^b \quad (A2.1)$$

$$Q_t + \alpha K_t^h = \beta(\frac{C_t^h}{C_{t+1}^h}(Z_{t+1} + Q_{t+1})) \quad (A2.2)$$

$$1 = \beta(\frac{C_t^h}{C_{t+1}^h} R_{t+1}) \qquad (A2.3)$$

$$Q_t(1 - K_t^h) = \phi_t N_t \qquad (A2.4)$$

$$\theta\phi_t = \beta(1 - \sigma + \sigma\theta\phi_{t+1})[\phi_t(\frac{(Z_{t+1} + Q_{t+1})}{Q_t} - R_{t+1}) + R_{t+1}] \qquad (A2.5)$$

$$Q_t(1 - K_t^h) = N_t + D_t \qquad (A2.6)$$

$$N_{t+1} = \sigma[(Z_{t+1} + Q_{t+1}(1 - K_t^h) - D_t R_{t+1}] + W^b \qquad (A2.7)$$

$$Q_3 + \alpha = \beta(\frac{C_3^h}{C_4^h}(Z_4 + Q_4)) \qquad (A2.8)$$

$$K_3^h = 1 \qquad (A2.9)$$

$$D_3 = 0 \qquad (A2.10)$$

$$N_3 = 0 \qquad (A2.11)$$

$$C_3^h + \frac{\alpha}{2} = Z_3 + \frac{Z_3}{Z_{ss}} W^h \qquad (A2.12)$$

$$\phi_3 = 0 \qquad (A2.13)$$

$$1 = \beta(\frac{C_3^h}{C_4^h} R_4) \qquad (A2.14)$$

$$N_4 = (1 + \sigma)W^b \qquad (A2.15)$$

$$C_T^h = C_{ss}^h \qquad (A2.16)$$

$$Q_T = Q_{ss} \qquad (A2.17)$$

$$\phi_T = \phi_{ss} \qquad (A2.18)$$

$$R_{T+1} = R_{ss} \qquad (A2.19)$$

$$K_T = K_{ss} \qquad (A2.20)$$

$$D_T = D_{ss} \qquad (A2.21)$$

$$Z_{t-1} = exp(-0.05 * 0.95^{t-2}) Z_{ss} \qquad (A2.22)$$

$$Z_T = exp(-0.05 * 0.95^{T-1}) Z_{ss} \qquad (A2.23)$$

A.3 Equations for Anticipated Run Case

The system of equations which solution gives the paths of variables for basic step is presented below. Bank run happens in period $j > T$ and $J = j+T-1$. The subscripts present time indices and $t = j+1, ...J-1$.

$$p_j = 0 \qquad (A3.1)$$

$$p_t = max[0, 1 - \frac{(Z_{ss} + Q^*)(1 - K_t^h)}{R_{t+1}D_t}] \quad (A3.2)$$

$$p_J = max[0, 1 - \frac{(Z_{ss} + Q^*)(1 - K_J^h)}{R_{J+1}D_J}] \quad (A3.3)$$

$$C_j = C^{h*} \quad (A3.4)$$

$$C^{h*} + \frac{\alpha}{2} = Z_{ss} + W^h \qquad (A3.5)$$

$$C_t^h + \frac{(1-\sigma)}{\sigma}(N_t - W) + \frac{\alpha}{2}(K_t^h)^2 = Z_{ss} + W^h + W^b \quad (A3.6)$$

$$C_J^h + \frac{(1-\sigma)}{\sigma}(N_J - W) + \frac{\alpha}{2}(K_J^h)^2 = Z_{ss} + W^h + W^b \quad (A3.7)$$

$$Q_j = Q^* \quad (A3.8)$$

$$Q^* + \alpha = \beta(\frac{C^{h*}}{C_{j+1}^h}(Z_{ss} + Q_{j+1})) \qquad (A3.9)$$

$$Q_t + \alpha K_t^h = \beta[(1 - p_t)\frac{C_t^h}{C_{t+1}^h}(Z_{ss} + Q_{t+1}) + p_t\frac{C_t^h}{C^{h*}}(Z_{ss} + Q^*)] \quad (A3.10)$$

$$Q_J + \alpha K_J^h = \beta[(1 - p_J)(Z_{ss} + Q_J) + p_J\frac{C_J^h}{C^{h*}}(Z_{ss} + Q^*)] \quad (A3.11)$$

$$1 = \beta R_{j+1}[(1 - p_j)\frac{C_j^h}{C_{j+1}^h} + p_j\frac{C_j^h}{C^*}\frac{(Z_{ss} + Q^*)(1 - K_j^h)}{R_{j+1}D_j}] \qquad (A3.12)$$

$$1 = \beta R_{t+1}[(1 - p_t)\frac{C_t^h}{C_{t+1}^h} + p_t\frac{C_t^h}{C^*}\frac{(Z_{ss} + Q^*)(1 - K_t^h)}{R_{t+1}D_t}] \qquad (A3.13)$$

$$1 = \beta R_{J+1}[(1 - p_J) + p_J\frac{C_J^h}{C^*}\frac{(Z_{ss} + Q^*)(1 - K_J^h)}{R_{J+1}D_J}] \qquad (A3.14)$$

$$K_1^h = 1 \qquad (A3.15)$$

$$Q_t(1 - K_t^h) = \phi_t N_t \qquad (A3.16)$$

$$Q_J(1 - K_J^h) = \phi_J N_J \qquad (A3.17)$$

$$\phi_j = 0 \qquad (A3.18)$$

$$\theta\phi_t = \beta(1 - \sigma + \sigma\theta\phi_{t+1})[\phi_t(\frac{(Z_{ss} + Q_{t+1})}{Q_t} - R_{t+1}) + R_{t+1}] \qquad (A3.19)$$

$$\theta\phi_J = \beta(1 - \sigma + \sigma\theta\phi_J)[\phi_J(\frac{(Z_{ss} + Q_J)}{Q_J} - R_{J+1}) + R_{J+1}] \qquad (A3.20)$$

$$D_j = 0 \qquad (A3.21)$$

$$Q_t(1-K_t^h) = N_t + D_t \qquad (A3.22)$$
$$Q_J(1-K_J^h) = N_J + D_J \qquad (A3.23)$$
$$N_j = 0 \qquad (A3.24)$$
$$N_{j+1} = (1+\sigma)W^b \qquad (A3.25)$$
$$N_{t+1} = \sigma[(Z_{ss} + Q_{t+1}(1-K_t^h) - D_t R_{t+1}] + W^b \qquad (A3.26)$$

The paths for one iteration step in "inductive step",i.e. paths when bank run happens in period $i : 1 < i < T+1$, where $I = i+T-1$,are described by the equations given bellow. Variables with subscript ss have the same value as corresponding variables in basic step in period J. The subscripts present time indices and $t = i+1, ...I-1$.

$$p_i = 0 \qquad (A3.27)$$
$$p_t = max[0, 1 - \frac{(Z_{t+1} + Q^*_{t+1})(1-K_t^h)}{R_{t+1}D_t}] \quad (A3.28)$$
$$p_I = p_{ss} \quad (A3.29)$$
$$C_i^h + \frac{\alpha}{2} = Z_i + \frac{Z_i}{Z_{ss}}W^h \quad (A3.30)$$
$$C_t^h + \frac{(1-\sigma)}{\sigma}(N_t - W) + \frac{\alpha}{2}(K_t^h)^2 = Z_t + \frac{Z_t}{Z_{ss}}W^h + W^b \quad (A3.31)$$
$$C_I^h = C_{ss}^h \quad (A3.32)$$
$$Q_i + \alpha = \beta(\frac{C_i^h}{C_{i+1}^h}(Z_{i+1} + Q_{i+1})) \qquad (A3.33)$$
$$Q_t + \alpha K_t^h = \beta[(1-p_t)\frac{C_t^h}{C_{t+1}^h}(Z_{t+1} + Q_{t+1}) + p_t\frac{C_t^h}{C_{t+1}^{h*}}(Z_{_{t+1}} + Q_t^*)] \qquad (A3.34)$$
$$Q_I = Q_{ss} \qquad (A3.35)$$
$$1 = \beta R_{i+1}[(1-p_i)\frac{C_i^h}{C_{i+1}^h} + p_i\frac{C_i^h}{C_{i+1}^*}\frac{(Z_{i+1} + Q^*_{i+1})(1-K_i^h)}{R_{i+1}D_i}] \qquad (A3.36)$$
$$1 = \beta R_{t+1}[(1-p_t)\frac{C_t^h}{C_{t+1}^h} + p_t\frac{C_t^h}{C_{t+1}^*}\frac{(Z_{t+1} + Q^*_{t+1})(1-K_t^h)}{R_{t+1}D_t}] \qquad (A3.37)$$
$$R_{I+1} = R_{ss} \qquad (A3.38)$$
$$K_i^h = 1 \qquad (A3.39)$$
$$Q_t(1-K_t^h) = \phi_t N_t \qquad (A3.40)$$
$$K_I^h = K_{ss}^h \qquad (A3.41)$$
$$\phi_i = 0 \qquad (A3.42)$$
$$\theta\phi_t = \beta(1-\sigma+\sigma\theta\phi_{t+1})[\phi_t(\frac{(Z_{t+1}+Q_{t+1})}{Q_t} - R_{t+1}) + R_{t+1}] \qquad (A3.43)$$
$$\phi_I = \phi_{ss}(A3.44)$$
$$D_i = 0 \qquad (A3.45)$$

$$Q_t(1-K_t^h) = N_t + D_t \qquad (A3.46)$$

$$D_I = D_{ss} \qquad (A3.47)$$

$$N_i = 0 \qquad (A3.48)$$

$$N_{i+1} = (1+\sigma)W \qquad (A3.49)$$

$$N_{t+1} = \sigma[(Z_{t+1} + Q_{t+1}(1-K_t^h) - D_t R_{t+1}] + W^b \qquad (A3.50)$$

$$Z_t = \begin{cases} Z_t & t \leq T \\ Z_{ss} & t > T \end{cases} \qquad (A3.51)$$

A.4 Equations for No Anticipated Run Case

The system of equations which solution gives the paths of variables for no anticipated run case is presented below. Variables with subscript ss have the same value as corresponding variables in basic step in period J. The subscripts present time indices and $t = 1, ...T-1$.

$$p_t = max[0, 1 - \frac{(Z_{t+1} + Q^*_{t+1})(1-K_t^h)}{R_{t+1}D_t}] \quad (A4.1)$$

$$p_T = p_{ss} \qquad (A4.2)$$

$$C_t^h + \frac{(1-\sigma)}{\sigma}(N_t - W) + \frac{\alpha}{2}(K_t^h)^2 = Z_t + \frac{Z_t W^h}{Z_{ss}} + W^b \qquad (A4.3)$$

$$C_T^h = C_{ss}^h \qquad (A4.4)$$

$$Q_t + \alpha K_t^h = \beta[(1-p_t)\frac{C_t^h}{C_{t+1}^h}(Z_{t+1} + Q_{t+1}) + p_t \frac{C_t^h}{C_{t+1}^{h*}}(Z_{t+1} + Q^*_{t+1})] \quad (A4.5)$$

$$Q_T = Q_{ss} \qquad (A4.6)$$

$$R_1 = R_{ss} \qquad (A4.7)$$

$$1 = \beta R_{t+1}[(1-p_t)\frac{C_t^h}{C_{t+1}^h} + p_t \frac{C_t^h}{C_{t+1}^{h*}} \frac{(Z_{t+1} + Q^*_{t+1})(1-K_t^h)}{R_{t+1}D_t}] \qquad (A4.8)$$

$$K_0^h = K_{ss}^h \qquad (A4.9)$$

$$Q_t(1-K_t^h) = \phi_t N_t \qquad (A4.10)$$

$$\theta\phi_t = \beta(1-\sigma+\sigma\theta\phi_{t+1})[\phi_t(\frac{(Z_{t+1}+Q_{t+1})}{Q_t} - R_{t+1}) + R_{t+1}] \qquad (A4.11)$$

$$\phi_T = \phi_{ss} \qquad (A4.12)$$

$$D_0 = 0 \qquad (A4.13)$$

$$Q_t(1-K_t^h) = N_t + D_t \qquad (A4.14)$$

$$N_t = \sigma[(Z_t + Q_t(1-K_{t-1}^h) - D_{t-1}R_t] + W^b \qquad (A4.15)$$

$$N_T = N_{ss} \qquad (A4.16)$$

$$Z_t = exp(-0.05 * 0.95^{t-1})Z_{ss} \qquad (A1.17)$$

$$Z_T = exp(-0.05 * 0.95^{T-1})Z_{ss} \qquad (A1.18)$$