DOES CONSUMPTION TAKE A RANDOM WALK?

EVIDENCE FROM MACROECONOMIC FORECASTING DATA*

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Abstract

Professional quarterly forecasts of aggregate U.S. consumption series are used to test Hall's (1978) random walk hypothesis. Results from band spectrum regressions suggest that forecasts predict about 40 percent of the low-frequency variation in growth rates of expenditures on nondurables and services. The forecasts have no explanatory power for high-frequency variations. The rejection of the random walk hypothesis is traced to excess sensitivity of consumption growth to forecasted income growth at the business cycle frequencies.

Zusammenfassung

Das Papier verwendet Wirtschaftsprognosen für den privaten U.S. Konsum, um Hall's (1978) Random Walk Hypothese zu testen. Die Ergebnisse von Band Spektral Regressionen zeigen, daß etwa 40 Prozent der Varianz der Wachstumsraten des privaten Konsums nichtdauerhafter Güter und Dienstleistungen bei niedrigen Frequenzen erfolgreich prognostiziert wird. Schwingungen bei höheren Frequenzen werden nicht prognoziziert. Die Ablehnung der Random Walk Hypothese kann auf eine zu starke Sensitivität der Konsumausgaben auf prognostizierte Einkommensänderungen zurück geführt werden.

I. Introduction

Hall's (1978) interpretation of the life cycle/permanent income hypothesis implies that, to a first approximation, consumption should follow a random walk with drift. The random walk hypothesis (RWH) has been the focus of much research over the last twelve years. In their well-known macroeconomics textbook, Hall and Taylor (1988, p. 210) answer the question posed in the title of this paper as follows,

> "Many economists have tested the random walk theory of consumption for the United States and other countries. Although the theory is approximately correct there are some discrepancies."

The empirical work on the RWH I am aware of employs macroeconomic time series or cross-section data to detect discrepancies between theory and actual aggregate consumption behavior. In this paper, I use quarterly forecasting data issued by Wharton Econometric Forecasting Associates (WEFA) to provide evidence on a central implication of the RWH. A forecast obtained by extrapolating the current level of consumption adjusted for trend can not be improved if consumption evolves as a random walk. In particular, forecasters should not assume that predictable changes in income are of help for predicting changes in consumption.

The use of forecasting data for testing the RWH has two main attractions. First, test power is likely to be increased relative to the usual strategy of regressing consumption changes on lagged variables and testing for the statistical significance of the coefficients. If consumption does not evolve as a random walk, forecasters should have an obvious commercial interest to reveal in their forecasts any information that helps predict consumption changes. Second, forecasting data can be used to distinguish between rejections of the RWH due to predictability of consumption changes at the high-frequency or the low-frequency range of the data. Most of the empirical literature on the RWH appears to proceed under the tacit assumption that tests automatically reveal deviations that occur at business cycle frequencies. Plausibly, deviations from the RWH may also occur because within-year movements in consumption are predictable. The band spectrum regression technique proposed by Engle (1974) provides a tool to evaluate this conjecture.

Section II discusses the forecasting data for real expenditures on nondurables and services issued by WEFA over 1970 III-1987 IV. Section III outlines the test procedure, and section IV contains the empirical results. The RWH is strongly rejected at the low-frequency range of the data, and the results suggest that forecasts account for about 40 percent of the variance at the low frequencies. Further analyses indicate that neither time averaging of consumption data nor the inclusion of goods with durable characteristics in the nondurables and services aggregate are likely to account for the reported rejection of the RWH. Predicted aggregate income growth, however, is found to be strongly correlated with actual consumption growth around the business cycle frequencies. This finding confirms the excess-sensitivity interpretation of rejections of the RWH proposed by Flavin (1981). The conclusions are presented in section V.

II. The Data

The available one-quarter ahead forecasts of real expenditures on nondurables and services were issued by WEFA over the time period 1970 III-1987 IV (70 observations). In the following, I refer to real expenditures on nondurables and services as "consumption" if no misunderstanding can arise. Because I intend to test whether the logarithm of consumption follows a random walk, forecasts and realizations of consumption are expressed in growth rates.[1] National income and product account data undergo regular revisions,

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1 The qualitative results of the paper remain unaffected if levels of consumption instead of logarithms are assumed to follow a random walk.

and there is no obviously correct procedure for calculating predicted and actual growth rates of consumption. The WEFA forecasts of consumption in quarter t are based on information through quarter t-1. The data set used in this paper was compiled by Stephen K. McNees. His data also include the preliminary value of consumption for quarter t-1 as known by the forecasters at the time the forecast was made. I use this preliminary value and the forecasted consumption value to calculate the predicted growth rate of consumption expressed at annual rates in percentage points. The actual growth rates of consumption are based on U.S. Department of Commerce (1986) for 1970 III-1982 IV and on Survey of Current Business (various issues) for 1983 I-1987 IV.[2]

At this point, the question may arise whether consideration of a single forecasting series for consumption can offer a powerful test of the RWH. Intuitively, one would expect that the smaller the forecast errors of a forecast series, the higher the power against the null hypothesis of a random walk in consumption. McNees (1988, Appendix A) compares mean absolute errors (MAE) for one-quarter ahead forecasts of nondurables and services by six forecasting institutions over 1976 II-1987 IV. In this comparison, the

2 A similar procedure for calculating predicted and actual growth rates is employed by Fair and Shiller (1989) in a study of the informational content of ex ante forecasts.

WEFA forecasts exhibit the lowest error statistic. Thus, at least according to the MAE criterion, WEFA forecasts are likely to provide a stringent test of the RWH. But the track record for consumption of nondurables and services of most of the other forecasting institutions is only marginally inferior to the WEFA forecasts. Roughly similar results can therefore be expected for tests of the RWH based on alternative forecast series.[3]

Figure 1 presents the graphs of the actual and predicted growth rates of consumption over 1970 III to 1987 IV. Visual inspection of the graphs points to two noteworthy features of the data. The forecasts appear to track the actual series reasonably well as far as movements in growth rates stretched over several years are concerned. But there appears to be substantial short-run variation in the actual series which is not captured well by the forecasting series. These impressions from figure 1 are vividly confirmed by the sample spectral densities of the series plotted in figure 2.[4] Both

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3 I also analysed forecasts of consumption issued by the Bureau of Economic Analysis (BEA). BEA forecasts give similar results as the WEFA series. The two sets of results are not strictly comparable, however, because the observation for the first quarter in 1981 is missing in the BEA series and the sample period is shorter. Other forecasting series were not available for this study. One major forecasting institution declined permission to use its forecasting series citing misuse of their forecasts by other researchers.

4 The sample spectral densities are based on a rectangular window of width 5 in the frequency domain. The area under the plots is equal to 1/2 of the variance of the series.

power spectra exhibit a peak at the lower frequencies around $\pi/8$ usually associated for quarterly data with business cycle fluctuations. Note that the peak in the spectrum for the actual growth rate at the business cycle frequencies is prima facie evidence against the RWH. There is a second conspicuous peak in the spectrum of the actual growth rate of consumption indicating a cycle in the series that takes less than 1 year to complete. This high-frequency cycle is close to $\pi/2$, and it could represent an artifact of the seasonal adjustment filter Census X-11. The gain of the linearized Census X-11 filter for quarterly data shows that the filter removes frequency bands around the seasonal frequencies $\pi/2$ and π completely and the filter can amplify some of the remaining high-frequency cycles (Laroque (1977, p. 115)). The forecasting series has no noticeable peaks at frequencies smaller than $\pi/4$.

III. The Test Procedure

I take the random walk theory of consumption to imply:

$$\Delta c_t = \epsilon_t \tag{1}$$

where Δc_t is the growth rate of consumption adjusted for constant mean growth, and ϵ_t is a regression disturbance orthogonal to all information available at time t-1.[5] A straightforward implication of (1) is that forecasts of the mean-adjusted consumption growth rate, denoted by $\Delta c_{t,f}$, should not be correlated with actual consumption growth. Thus, in the bivariate regression:

$$\Delta c_t = \beta \Delta c_{t,f} + u_t \tag{2}$$

the coefficient estimate of β should be statistically insignificant and the coefficient of determination R^2 zero.

The informal discussion of the forecasting data in section II suggests that simply running regression (2) may not be an informative procedure to detect deviations from the RWH. Predictability of consumption growth may occur at low and/or high frequencies of the data. Clearly, rejecting the RWH because high-frequency movements in consumption growth are predictable may warrant a different economic interpretation than a rejection due to predictability of low-frequency movements. For this reason, I employ band spectrum regression techniques proposed by Engle (1974) to analyse the predictability of consumption growth at different frequencies. The procedure can be illustrated by considering

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5 To be accurate, equation (1) says that consumption is a martingale. I follow popular terminology by employing the more restrictive but presumably more charming term "random walk".

the spectral densities plotted in figure 2. Basically, I am interested in calculating the amount of variance of actual consumption growth explained by the variance in predicted consumption growth over (A) the whole frequency band, (B) the low frequencies, and (C) the high frequencies. The frequency bands of interest are defined as follows: High-frequency movements include all cycles that take less than one year to complete (frequencies $\geq \pi/2$), whereas low-frequency movements include all cycles taking more than one year to complete (frequencies $< \pi/2$). Band spectrum regressions can be based on data series where the cycles at frequencies not of interest have been deleted by appropriately filtering the data in the frequency domain.

The regression results reported below are based on an elegant technique for performing band spectrum regressions proposed by Harvey (1978). Put briefly, the time series data are first transformed to the frequency domain by using a real finite Fourier transform matrix (Z) with dimension TxT, where T is the number of observations. The elements of the matrix are described in Harvey (1978, p. 509). Equation (2) can be written in the frequency domain as:

$$c^* = \beta c_f^* + u^* \qquad (3)$$

where $c^* = Z\Delta c_t$, $c_f^* = Z\Delta c_{t,f}$, and $u^* = Zu_t$. All transformed series consist of real numbers. Band spectrum regressions for the low-frequency range then simply correspond to running regression (3) using only the first half of the transformed data whereas the regression for the high-frequency range is based on the second half of the transformed data series. All the usual statistical properties of parameter estimates known from time-domain regressions will also hold for the frequency-domain regressions. Moreover, if regression (3) includes all frequencies, it is equivalent to regression (2) in the time domain.[6] Harvey's (1978) regression technique has two main advantages: Transformations using complex Fourier transforms can be avoided, and tests of statistical hypotheses are automatically based on the correct number of degrees of freedom.

IV. Empirical Results

Table 1 reports the results for testing the RWH at different frequencies. Panel A contains the results for regression (3) if all frequencies are included; panels B and

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6 A caveat has to be mentioned at this point. Band spectrum regressions restricted to specific frequency bands require the right-hand side regressor in equation (2) to be orthogonal to the error term at all leads and lags. This strong assumption is unlikely to be met exactly if forecasters adapt their forecasts based on past forecast errors. In fact, there are negative but small cross-correlations between current forecast errors and future WEFA forecasts.

C contain the results if high and the low frequencies are omitted, respectively. For each frequency range, results are reported for the full sample period 1970 III-1987 IV, and for the two subsamples 1970 III-1979 II and 1979 III-1987 IV. The sample split is motivated by the widely held belief that macroeconomic forecasting during the 1970s was unusually difficult due to unprecedented supply shocks.

The regression results including all frequencies clearly reject the RWH. According to the R^2-statistics reported in panel A, about 20 percent of the variance of actual consumption growth is successfully predicted by the forecasts. The results appear to be stable across the two subsamples although the Ljung-Box Q-statistic for serial correlation in the regression errors indicates some autocorrelation for the subperiod 1979 III-1987 IV. The regression results in panels B and C suggest that the rejection of the RWH is a low-frequency phenomenon. About 40 percent of the variance of consumption growth is explained at the low frequencies whereas the R^2-statistics for the regressions including only high frequencies are close to zero. It is conceivable that the latter finding can be explained by assuming that forecasters deliberately ignore within-year cycles on the ground that their forecasts would otherwise exhibit too much short-run variation.

The rejection of the restrictions imposed by the RWH raises the question whether forecasting data contain clues on specific reasons for the findings. I first consider the argument that time averaging of consumption data is the reason for rejecting the RWH. If the logarithm of consumption follows a random walk at a time interval smaller than a quarter, quarterly consumption will be a time average. Working's (1960) famous result implies that the growth rate of time-averaged quarterly consumption follows:

$$\Delta c_t = \epsilon_t + \alpha\epsilon_{t-1}, \tag{4}$$

where α is 0.221 if monthly consumption is a random walk. By exploiting information contained in ϵ_{t-1}, the maximum R^2 a forecaster can achieve is $\alpha^2/(1+\alpha^2)$ or about 0.0046 for α equal to 0.221. According to this calculation, time averaging can account only for about 20 percent of the amount of predicted variance of 0.227 reported for nondurables and services in panel A of table 1. Furthermore, the sample spectral density for actual consumption growth in figure 2 is not similar to the negatively sloped curve typically expected for a first-order moving average process with small positive MA-coefficient.

Another possible explanation is that durability characteristics of some of the goods included among the nondurables and services aggregate are responsible for the failure of the RWH. Obviously, this argument presumes that growth rates of expenditures on durables are predictable. I therefore used the same method as for expenditures on nondurables and services to investigate whether WEFA forecasts of the growth rate of expenditures on durables are correlated with the realized growth rates. The results are contained in table 2. The R^2-statistics show that durables are about equally well predicted at all frequencies. The forecasts explain close to 60 percent of the variance of the actual growth rates.[7] If durability characteristics are responsible for the rejection of the RWH, the results in table 2 suggest that high-frequency movements in nondurables and services should also be predictable to some extent. But this implication is not confirmed by the results in table 1.

The presumably most prominent explanation for rejections of the RWH holds that consumption growth is predictable because actual consumption growth is related to predicted income growth. Because forecasting data on disposable household income are not available, I employ WEFA forecasts of growth in real gross national product (GNP) to investigate whether predictable income growth is related to consumption. The relevant regression in the time domain is:

$$\Delta c_t = \theta \Delta y_{t,f} + u_t \tag{5}$$

7 This finding is at odds with the claim by Mankiw (1982) and Startz (1989) that U.S. expenditures on durable consumption goods are well approximated by a random walk with drift.

where $\Delta y_{t,f}$ is the mean-adjusted WEFA prediction of real GNP growth. Table 3 contains the results for band spectrum regressions for both expenditures on nondurables and services and expenditures on durables. The results in panel A for nondurables and services show that predicted GNP growth explains about the same amount of variance at the different frequencies as predicted consumption growth. Moreover, similar to predicted consumption growth in figure 2, the sample spectral density of predicted income growth has most of its power concentrated at the business cycle frequencies. The evidence from predicted income growth therefore indicates that the ability of forecasters "to beat the random walk" rests on excess sensitivity of actual consumption growth to predicted income growth around the business cycle frequencies. The results in panel (B) for expenditures on durables show that a relatively small part of the variance of durables is explained by predicted income growth. Other types of information, for example from consumer surveys, must be of substantial help in predicting growth in expenditures on durables.

V. Conclusion

In this paper I examined professional forecasting data in order to provide new evidence on discrepancies between the

RWH proposed by Hall (1978) and actual aggregate consumption behavior. Forecasters should have a strong profit-motivated incentive to exploit any information that is useful for predicting future consumption growth. I find that the testable restrictions imposed by the RWH are soundly rejected by the forecasting data. A systematic relationship between predicted income growth and actual consumption growth around the business cycle frequencies is identified as the most likely cause for the rejection.

The results presented in this paper support the conclusions of work on aggregate consumption behavior which traces rejections of the RWH to excess sensitivity of consumption to predictable income fluctuations (Flavin (1981)). A sizeable fraction of households may be unable to set consumption as suggested by the RWH because they are subject to liquidity constraints. Following the work by Campbell and Mankiw (1989), the estimate of the coefficient θ in panel A can be interpreted as the portion of income that accrues to "rule of thumb" households which simply consume their current income in each time period. This interpretation suggests that capital market imperfections are important for understanding aggregate consumption behavior (Jappelli and Pagano (1989)).

Table 1.-Predictability of Growth Rate of Expenditures on Nondurables and Services

A. All Frequencies

Time Period	β	R^2	Q	DF
(1) 1970 III-1987 IV	0.668 (4.541)	0.227	28.37 (0.25)	69
(2) 1970 III-1979 II	0.611 (2.944)	0.189	20.51 (0.31)	35
(3) 1979 III-1987 IV	0.766 (3.141)	0.236	22.95 (0.09)	33

B. Low Frequencies

Time Period	β	R^2	Q	DF
(4) 1970 III-1987 IV	0.747 (5.060)	0.427	9.71 (0.84)	34
(5) 1970 III-1979 II	0.648 (2.894)	0.319	7.58 (0.58)	17
(6) 1979 III-1987 IV	0.433 (4.356)	0.575	6.66 (0.57)	16

C. High Frequencies

Time Period	β	R^2	Q	DF
(7) 1970 III-1987 IV	0.137 (0.320)	0.001	22.25 (0.10)	34
(8) 1970 III-1979 II	0.376 (0.658)	0.018	7.66 (0.57)	17
(9) 1979 III-1987 IV	-0.370 (0.509)	0.016	12.89 (0.17)	16

Notes: Numbers below coefficient β are t-statistics. Q is the Ljung-Box test statistic for serial autocorrelation. Numbers below Q are marginal significance levels. DF denotes the number of degrees of freedom in the regression.

Table 2.-**Predictability of Growth Rate of Expenditures on Durables**

Frequencies	β	R^2	Q	DF
(1) All	1.180 (9.708)	0.569	23.59 (0.49)	69
(2) Low	1.077 (7.205)	0.580	6.63 (0.97)	34
(3) High	1.372 (6.648)	0.565	10.60 (0.78)	34

Notes: Numbers below coefficient β are t-statistics. Q is the Ljung-Box test statistic for serial autocorrelation. Numbers below Q are marginal significance levels. DF denotes degrees of freedom.

Table 3.-Actual Consumption Growth and Predicted GNP Growth

A. Growth of Expenditures on Nondurables and Services

Frequencies	θ	R^2	Q	DF
(1) All	0.271 (4.073)	0.191	28.14 (0.25)	69
(2) Low	0.326 (4.849)	0.407	11.75 (0.70)	34
(3) High	-0.040 (0.229)	0.002	21.303 (0.78)	34

B. Growth of Expenditures on Durables

Frequencies	θ	R^2	Q	DF
(4) All	2.019 (4.067)	0.178	24.30 (0.44)	69
(5) Low	2.185 (4.466)	0.331	4.64 (0.99)	34
(6) High	1.082 (0.775)	0.017	13.323 (0.58)	34

Notes: Numbers below coefficient θ are t-statistics. Q is the Ljung-Box test statistic for serial autocorrelation. Numbers below Q are marginal significance levels. DF denotes the number of degrees of freedom in the regression.

FIGURE 1. - ACTUAL VS PREDICTED GROWTH OF EXPENDITURES ON NONDURABLES AND SERVICES (1970 III-1987 IV)

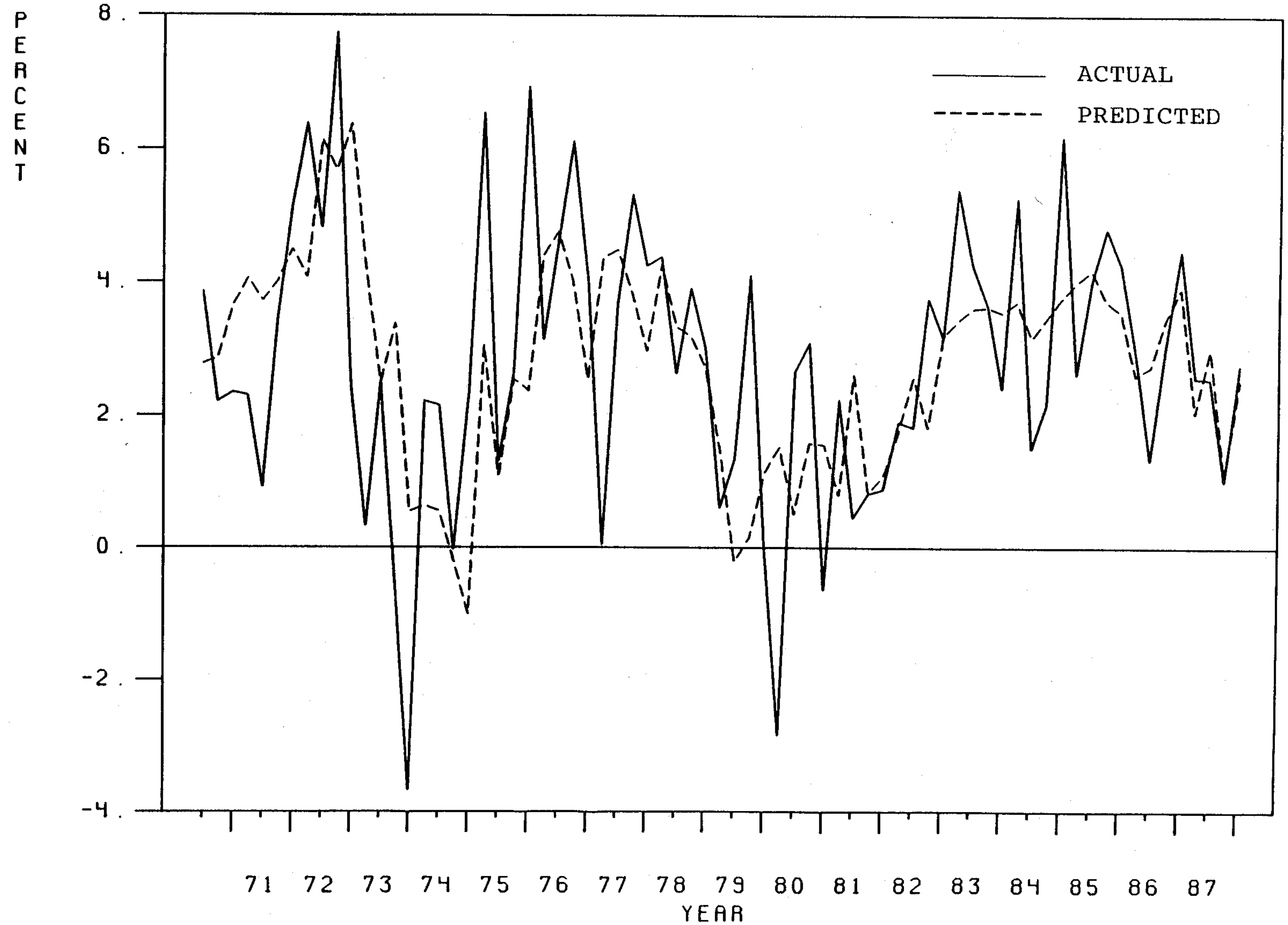
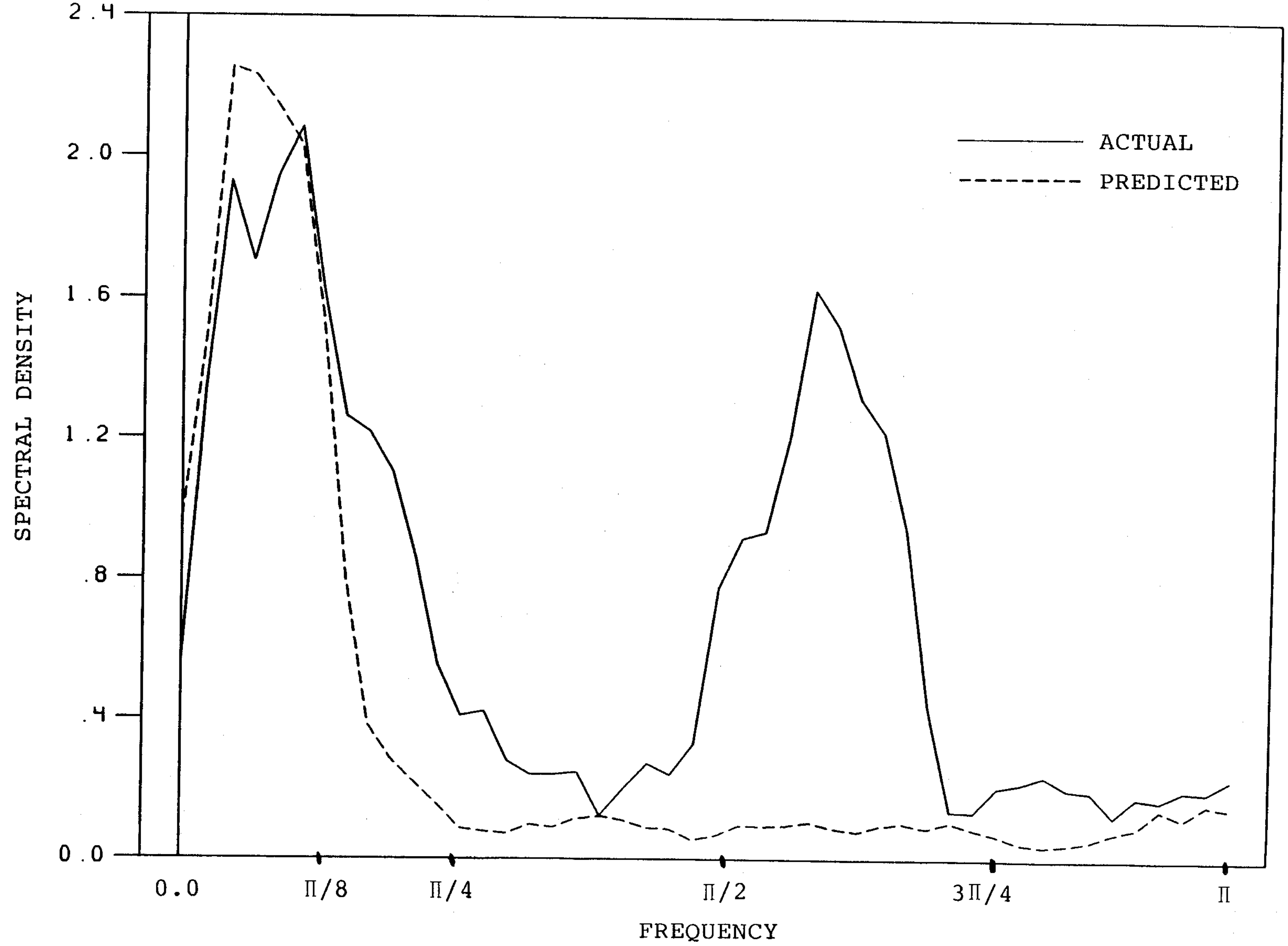


FIGURE 2. - SPECTRAL DENSITIES OF ACTUAL AND PREDICTED CONSUMPTION GROWTH



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