INVOLUNTARY UNEMPLOYMENT IN A BARGAINING MODEL WHEN CAPACITY CHOICE IS A BINDING PRECOMMITMENT

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Abstract

A market is studied where prices are formed through a bilateral bargaining process. Players on one side of the market have in an initial stage the option to incur at a cost a binding precommitment which may limit the possible number of trades. It is shown that such a set-up does not necessarily lead to market clearing. To solve the game a refinement of subgame perfection is used and under this extra requirement the sensitivity of bargaining behavior with respect to the specification of outside options is analysed.

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1. Introduction *)

A theory of markets with a finite number of participants must usually take into account the process of price formation. One way to view this issue is to apply non-cooperative bargaining theory and to exploit the characteristics of the market under consideration. This is the approach adopted in the present paper: A market is studied, where the participants can bargain bilaterally on the terms of trade, but where one "side of the market" has the option to enter in an initial stage a binding precommitment, limiting the possible number of trades. In a sense this portrays what may be behind the classical Cournot-oligopoly model with quantity choice, with the variation that the process of price formation, given quantities, is modelled as a (bilateral) bargaining game. As a framework to interpret this set-up industrial labor markets or housing markets are offered: Industrial labor-markets are often characterized by the feature that firms first have to build up capacities, i.e. installing machines and buildings, and can only then enter into negotiations on the wages of its prospective employees; future contracts on wages contingent on (the future installment of) capacity are rarely found on industrial labor-markets. Housing markets are also characterized by the existence of houses or flats before any negotiations on prices or rents can take place. Both kinds of markets typically have first the firms or landlords diciding on capacities and afterwards a process of price-formation during the selling of the existing capacities. In the present paper reference to a labor market will be stressed.

The strategic peculiarity of this kind of markets lies in the fact that capacity choice serves as a binding precommitment for one side of the market. This precommitment may severely limit the number of possible contracts, thereby determining disagreement points. On the other hand the costs of incurring such a precommitment are "strategically sunk", i.e costs of capacities play no role in the ensuing bargaining game. The "tension" between the favourable option to enter a binding precommitment which limits the number of possible contracts and the firms' attempt to profit from as many jobs as possible, will drive the operation of this market.

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Non-cooperative bilateral bargaining games with complete information in the tradition of Rubinstein's seminal paper (1982) can be viewed as exhibiting a surprising efficiency in terms of not wasting resources by delaying agreement. This interpretation is sometimes stretched as far as viewing it as a demonstration of a Coase-theorem with complete information (Farrell, 1987). But immediate agreement in bilateral bargaining under complete information can also be viewed as the consequence of a restriction to pure strategies together with the assumption of acceptance at indifference points. Moreover, immediate agreement may not be the sole measure of efficiency in bargaining models. The present paper offers another view on efficiency by making the outside options in the bargaining endogenous. The simplest bilateral bargaining model of the Rubinstein type shares with its cooperative variant the exogeneity of disagreement points. Making the outside options endogenous by allowing capacity choice in a stage of the game before bargaining takes place raises the questions whether the (non - cooperatively) chosen number of possible contracts will match the maximum number of contracts. If capacity falls short of the maximum number of possible trades, some resources remain unutilized which is clearcut inefficiency. An underutilization of resources amounts to an equilibrium where markets do not clear. Indeed the analysis in the present paper shows that this sort of inefficiency cannot be ruled out.
The argument here is strongly built on Kalecki 's intuition that involuntary unemployment has to serve as a threat against excessive wage claims.

The model presented below focuses on the bargaining which determines the prices at which capacity can be "sold". The market at which firms or landlord "buy" the capacity which they would like to "sell" to workers or prospective tenants and also the market for the product emerging from the successful cooperation of firms with its employees are modelled very traditionally: Capacity can be "bought" on a perfectly competitive market and the product produced jointly by the workers and the firms is again sold by price takers on a perfectly competitive market.

Relations to the literature

The stream of literature to which the present paper is indepted cannot be exhaustively characterized; suffice it to mention here Rubinstein (1982,1985,1986), Rubinstein and Wolinsky (1985), Wolinsky (1986), Shaked and Sutton (1984), Gale (1986a, 1986b). In solving the bargaining game we have to refine Selten's (1965,1975) concept of subgame perfect equilibria in order to obtain unique payoffs from the bargaining stage of the game. The refinement, taylored to the needs of the present game, builds on a broad stream of literature, summarized in van Damme (1987). The name of the refinement "forward induction" is inspired by the use of this term in papers by Kohlberg and Mertens (1986), Hellwig and Leininger (1987) and, in the context of signalling games, by Kreps and Wilson (1982), Cho and Kreps (1987), Grossman and Perry (1986) among many others.

Plan of the paper

Section 2 gives an informal description of the game to be studied and Section 3 presents the correspondig formal definitions and introduces the refinement of subgame perfection. Section 4 studies a variant of the model with costs of mobility. Section 5 presents a version of the game, where the costraints on mobility are relaxed. Section 6 draws some conclusions.

2. Informal description of the model

The game to be studied is an (N+M)−person game. There are M identical players referred to as the "firms" and indexed by $i=1,\dots,M$, $M \geq 2$, and there are N identical players referred to as the "workers" and indexed by $j=1,\dots,N$. The set of "firms", is denoted by $\mathcal{M}$ and the set of "workers" is denoted by $\mathcal{N}$, $\mathcal{M}=\{1,\dots,M\}$, $\mathcal{N}=\{1,\dots,N\}$, $M < N$. The firms will have a limited number of "jobs" which can be thought of as "licences to produce" or as "equipment installed" to make production possible. In case the model is given an interpretation to fit a housing market, the firms can be interpreted as landlords, the jobs as "flats" and the workers as prospective "tenants". In the sequel, however, the reference to firms, workers and jobs will be used.

The firms can buy a capacity of indivisible jobs at a cost $c \in (0,1)$ per job, where the number of indivisible jobs – i.e. the capacity of the firm – is an integer between zero and N and the total capacity cost to the firm equals its number of jobs times c. A job per se yields nothing to the firm, but if a job is combined with a worker, one unit of a universally desired consumption good is produced on this job. This is to say that there is a linear limitational technology such that one worker spending one indivisible unit of labor on one indivisible job produces one unit of, say, apple pie. Hence, if a firm $i \in M$, buys l_i jobs at a cost of cl_i, and then hires x_i workers, $x_i \leq l_i$, to work on these jobs, then the gross product of this combination will equal x_i units of apple pie. The costs will consist of cl_i for the capacity and of the wages for the hired workers. Since jobs and labor units are indivisible, a firm can at most hire a number of workers equal to its capacity.

Payoff functions are also linear. Both firms and workers like apple pie. They can share the gross product of a job in any way agreed upon by both. The marginal utility of apple pie is taken to be unity. As to the workers they only like pie and there is no disutility of labor.

The firm's capacity choice, i.e. their simultaneous decision on the number of jobs they want to buy, constitutes the first stage of the game. Once all firms have chosen their capacities and paid the cost, a matching–technology assigns queues of workers to each firm. A matching technology is a random device which assigns to each $i \in M$ subsets of N of a certain ordering, given the capacity choice of the firms, where the ordered subsets of N will be called "queues". Of course, each $j \in N$ can only be a member of one queue, but in the course of the game workers will have the option of changing their positions, e.g.move from one queue to another or move within the queue they stand in to an other position. The ordering of the queue will determine the succession by which the firm will bilaterally bargain with the workers on their individual wages. It will be assumed that as long as the total number of jobs in the economy does not exceed N each firm will get a queue at least as long as the number of jobs it holds; if the total number of jobs exceeds the number of workers available, then it will be assumed that those firms with the highest number of jobs will be rationed.

Once capacities have been chosen and the matching-technology has assigned queues of workers to the firms, the second stage of the game starts. The second stage is a T-period bargaining game without discounting, where $T > N$ is a known parameter which should be thought of as an astronomous number relative to the number of players. In the first of the T periods each firm enters into a bilateral bargaining game with the first worker in its queue: "Natur" decides whether the firm or the worker will be the proposer; then the proposer makes an offer (which is a number in the interval [0,1]) and the other party - called the responder - may accept the offer or reject it; if an offer is accepted both parties receive payoffs, the worker goes to work until forever (i.e: he leaves the game) and the firm moves to the next worker in its queue to start a new bargaining round; if an offer is rejected both parties have the option either to stay together and resume bargaining or to quit the bargaining process; in case one of the two parties decides to quit the bargaining, the firm will move to the next worker in its queue to start a new bargaining game, and the worker may choose a new position in the present or another queue, where he can only choose among positions not yet taken by other workers. At the end of the current period also all other workers in the economy have the option to choose new positions among those not yet taken, but a worker who has experienced an unsuccsessful bargaining round in the current period has the privileg of choosing his new position first (before the others can decide). In the next period the same structure of the bargaining game is repeated; specifically there is no deterministic alternation of proposers and responders, but rather in each period a random move decides on the proposer.

This bilateral bargaining structure is repeated until either all jobs in the economy are "sold" or until the final period T is reached. In period T the game ends. For most of the paper T will be taken to be finite, but large, because this assumption reduces the number of equilibria drastically. To start the analysis with an infinite T would presumably allow a plethora of equilibria, because "punishments" of arbitrary length become possible (For a discussion on the intuition of this possibility see Fudenberg and Maskin (1986)). To remove the finiteness friction from the solutions of the game, we will, however, frequently study the limits of the solutions of the finite game when T tends to infinity. This will allow us to remove the friction from this market, while at the same time avoiding non-uniqueness and possible non-stationarities. In our view this is an appropriate way to study frictionless markets as limiting cases of a world with frictions.

3. Definition of the Game:

The formal definitions are introduced by first defining the matching technology.

Let $\mathcal{T} = \{1,...,T\}$ and $L = \{0,...,N\}$ the set of possible capacity choices of an individual firm. A matching technology is a mapping p_T: $\mathcal{M} \times \mathcal{N} \times \mathcal{T} \times L^M \rightarrow [0,1]$, the values of which are to be interpreted as the joint conditional probability that firm i, $i \in \mathcal{M}$, will meet worker j, $j \in \mathcal{N}$, at the t-th position, $t \in \mathcal{T}$, of its queue and that neither i nor any other firm has met j before at any position of their queues, given the capacity choice $l \in L^M$.

Now define $J(t,l) = \Sigma_{i \in \mathcal{M}} \min(l_i, t)$ and the set $M(t, l) = \{i \in \mathcal{M} \mid l_i \geq t\}$, $(t, l) \in \mathcal{T} x L^M$.

We then assume on the matching technology:

(A1) (i) for all $i \in M(t,l)$, $t \in \mathcal{T}$, $j \in \mathcal{N}$:

$$p_T((i, j, t) \mid l) = \begin{cases} 1/N, \text{ if } J(t, l) \leq N, \\ 0, \text{ if } J(t, l) \geq N + \mid M(t, l) \mid \\ (N \mid M(t, l) \mid)^{-1}(N - J(t-1, l)), \text{ otherwise;} \end{cases}$$

(ii) for all $i \in M \backslash M(t, l)$, $t \in \mathcal{T}$, $j \in \mathcal{N}$:

$$p_T((i, j, t) \mid l) = \begin{cases} 1/N, \text{ if } (t - l_i)M \leq N - \Sigma_{j \in \mathcal{M}} l_j \\ 0, \text{ if } (t - l_i - 1)M \geq N - \Sigma_{i \in \mathcal{M}} l_i \\ (N\,M)^{-1}(N - \Sigma_{j \in \mathcal{M}} l_j - (t - l_i - 1)M), \text{ otherwise} \end{cases}$$

(where $\mid . \mid$ denotes cardinality).

What (A1) tries to capture is a sequential pairing of workers and jobs which treats all workers symmetrically and all firms symmetrically:

The assignment of workers to positions in queues starts in an initial round by assigning each worker with a uniform probability to the first place in any of the queues of firms in M(1,l), where M(1,l) denotes the set of firms having a capacity of at least one job. The remaining workers, who did not catch first places in any of these queues, are then distributed in a second round again with uniform probabilities to the second places of the queues of firms in M(2,l). This procedure is repeated until either all workers are distributed or until all positions corresponding to jobs are occupied. In the latter case, i.e. if all firms already have queues exactly as long as their number of jobs and still some unmatched workers remain, the matching technology distributes the remaining workers by an analogous process among all firms: In a first round each of the remaining workers gets assigned to (the first place at the end of) any of the M queues - of course these workers will in equilibrium not have the opportunity to bargain and are, therefore, referred to as the "unemployed". If still unmatched workers remain, the matching technology again distributes these "unemployed" with uniform probability among the (second places at the end of the M) queues, and so on until all workers are matched. The probabilities corresponding to this process are given in (A1).

There are three motivations to adopt this particular matching technology. The first is simply that this allows us to generate simple expressions for expected payoffs in equilibrium. The second is that this matching technology, in case the total number of jobs exceeds the number of workers, ensures that those firms with the highest number of jobs are rationed, viz. the idea of "filling up the bottle from the bottom". The third reason is that, in case umemployment prevails, the unemployed are distributed in a "fair" way, i.e. not all unemployed will cluster in one queue and the unemployed stand as close as possible to the last available job - a property which is neither disliked by workers nor by firms.

Other matching technologies would presumably also do the job, as long as the conventions presented below on the workers' options to reposition themselves during the bargaining game are adopted, but may produce more messy expressions.

Given the matching technology – which is common knowledge – the game can be defined as a (T+1) stage game in extensive form. At the first stage of the game, referred to as period zero, all $i \in M$ choose simultaneously an $l_i \in L$ and pay costs $c\, l_i$, $c \in (0,1)$. The chosen vector $l=(l_1,...,L_M) \in L^M$ becomes known immediately to all players. Given $l \in L^M$ the matching technology assigns queues to the firms and the realizations of the queues become known to all players. This completes the first stage of the game.

At the second stage, i.e. in period one, each firm $i \in M$ enters the game $G_T(i, f_i, l_i)$ with the first worker in its queue.

$G_T(i, f_i, l_i)$ works as follows:

(1) A chance move decides whether the firm (with probability $\alpha \in (0,1)$) is the proposer or the first worker in the queue is the proposer (with probability $(1-\alpha)$).

(2) The proposer makes an offer which is a number in the interval [0,1], e.g. if the worker is the proposer he offers $w \in [0,1]$ which means that he wants the payoff w and the firm is offered a payoff (1−w) from this particular job.

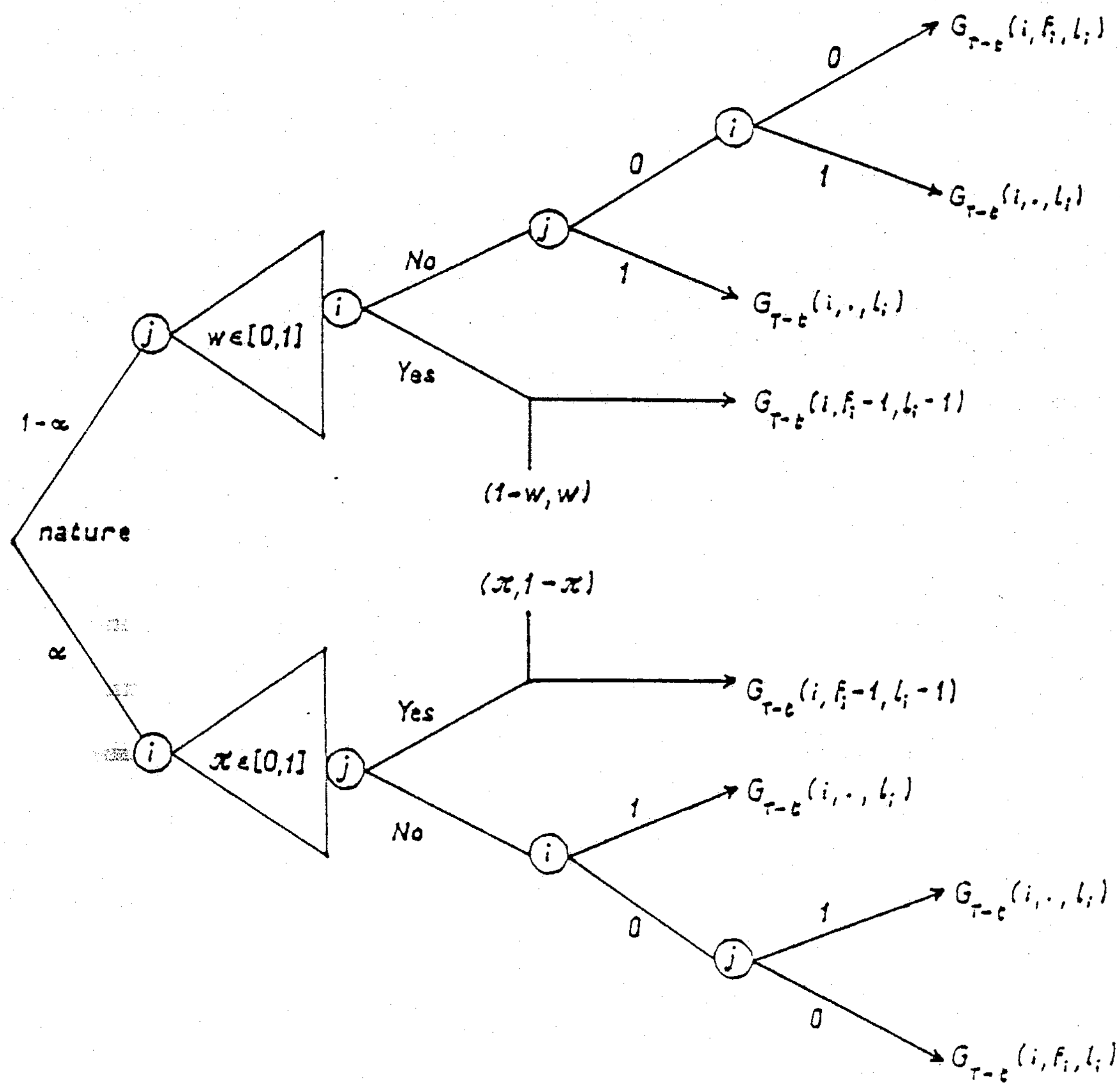
(3) The responder may accept or reject the offer. If an offer is accepted both parties immediately receive payoffs, e.g. if the firm is proposer and offers $\pi \in [0,1]$ and π is accepted, the worker receives $(1-\pi)$ and the firm π. The firm moves to the next worker in its queue, while the worker after receiving his payoff leaves the game. If an offer is rejected, the proposer may decide to quit the bargaining or stay with the present partner, and then the responder may decide to quit or stay, in case the proposer decided to stay.

(4) If both parties decided to stay together for another bargaining round they enter the game $G_{T-1}(i,f_i,l_i)$ which is defined analogously.

(5) If one of the parties decided to quit the bargaining process, the firm moves on to the next worker

FIGURE 1

$G_{T+1-t}(i, f_i, l_i)$:



$G_{T+1-t}(i, f_i, l_i)$ is a subgame–tree, where the first worker in queue f_i is denoted j, T+1–t periods are left until period T and the remaining capacity of firm i is $l_i > 0$. Arrows indicate the next subgames to be entered. The worker's decision on a new position (in any queue) at a node reached after "NO" and (0,1) v (1,0) v (1,1) is not depicted.

in its queue and enters a new G_{T-1}, while the worker may choose a new position in any queue, provided the position is not yet occupied by another worker.

(6) Finally at the end of the period all other workers may choose new positions among those not yet taken at the end of any firm's queue.

Figure 1 depicts the core subgame tree of the bargaining game: Nodes are labelled with the name of the player, who decides at this node, where j is the index of the first worker in i's remaining queue; the triangles indicate choices from the interval [0,1] and 0's and 1's refer to the decision to quit the bargaining process (q=1) or to stay with the present bargaining partner (q = 0). The endpoints (π, $1-\pi$) and (1−w, w) denote payoffs to the firm resp. the worker. The subscript in the name of the game refers to the number of remaining periods until T, the first argument to the index of the firm, the second to the length of its queue and the final argument to the firm's remaining capacity. Arrows indicate which subgame will be reached, after those workers not yet reached have rearranged themselves. The notation $G_{T-t}(i, ., l_i)$ indicates that the next (T−t period) bargaining game to be reached depends on the decision of the workers on their repositioning, specifically on the decision of the unsuccessful worker j whether to position himself at the end of firm i's queue or to the end of another firm's queue.

The structure of G_{T-t+1} is repeated until either

(a) $l_i = 0$ which makes the vertex of G_{T-t+1} a terminal node with payoffs (0,0) or until

(b) T=t which turns all arrows into terminal nodes with payoffs (0,0).

Note that a worker receives a payoff at most once, while a firm receives a payoff as often as it reaches an agreement in the bargaining game and at most as often as its capacity allows.

Let S^k_t be player k's strategy space, $k \in N \cup M$, in period t. Then the choice sets for the players in the periods t=0,...,T can be written as $S^i_0 = L$, for all $i \in M$, $S^j_0 = \phi$ for all $j \in N$, and

S^k_t = [({Yes, No} x [0,1]) x ({0,1} x {0, 1})] U ϕ for all $k \in M \cup N$.

The set of possible histories up to but not including period t is defined recursively:

$H_1 \equiv L^M \times F^*$ where F^* denotes the set of all possible queues, $H_t \equiv H_{t-1} \times (X_{i\in M} S^i_{t-1}) \times (X_{j\in N} S^j_{t-1})$, $t=2,\dots,T$.

A strategy for player $i \in M$ is denoted σ^i and is a (T+1)-tuple $\sigma^i = (\sigma^i_0,\dots,\sigma^i_T)$, where $\sigma^i_0 \in L$ and σ^i_t is a mapping σ^i_t: $H_t \to S^i_t$, $t=1,\dots,T$. A strategy for player $j \in N$ is $\sigma^j = (\sigma^j_1,\dots,\sigma^j_T)$, where σ^j_t: $H_t \to S^j_t$, $t=1,\dots,T$. A strategy profile for a (T+1) period game is defined as $\sigma \equiv ((\sigma^i)_{i\in M},\ (\sigma^j)_{j\in N})$. Given a strategy profile σ the symbol $\sigma \mid h_t$ for $h_t \in H_t$ will denote the (behaviour) strategy profiles induced by σ after h_t for all $t' \geq t$, i.e: the continuation $(((\sigma^i_{t'})_{i\in M})_{t'\geq t},\ ((\sigma^j_{t'})_{j\in N})_{t'\geq t})$ of σ after h_t. From the definition it should be clear that only pure strategies are allowed. Subgames are partial trees emerging from a particular node and the restriction of σ to a particular subgame is the strategy induced by σ on the subgame starting in this node. This completes the formal definition of the game.

The solution concept for the game will be subgame perfection (refined to meet an extra requirement given later): A particular equilibrium σ of a game is said to be subgame perfect, if for every subgame of the original game the restriction of σ to this subgame constitutes a Nash equilibrium of the subgame (for formal definitions see: Selten (1965),(1975); Rubinstein (1982)).

Games of complete information with perfect recall, involving a high number of repetitions of the same subgame may, however, still exhibit too many subgame perfect equilibria (cf. the Folk-Theorems; Fudenberg and Maskin (1986); van Damme (1987)). Therefore we adopt a set of assumptions basically designed to economize on potential complexity of behaviour.

(A2) Players of the same type ($i \in M$ or $j \in N$) in identical situations (i.e. identical subgames) act identically.

(A3) Inactive workers, who are not currently involved in bargaining, do not change their positions in the queues, if they have no incentive to, i.e. unless they can make themselves strictly better off by changing their positions.

(A4) If a responder is confronted with an offer which makes him indifferent between accepting and rejecting, then he accepts.

(A5) N/M is an integer.

All these assumptions are only informally stated and are now discussed in turn. Assumption (A2) is an "Impersonality assumption" which rules out "personal" arrangements (the terminology is coined by Rubinstein (1986)), i.e. strategies which depend on the players' identities. The way it is stated (A.2) is, however, weaker than the assumption in Rubinstein (1986, p.10) and also weaker than the stationarity assumption used in Rubinstein and Wolinsky (1985) which prescribes the same bargaining tactics against all partners met; because of finiteness of T (A2) is sufficient for our results. Assumption (A3) is vital in the analysis of credible threats: By adopting certain moving strategies from one firm to the next even unemployed workers could substantially change the strategic position of firms – we will have something to say on what happens, if (A3) does not hold, at the very end of the paper. Suffice it here to remark that (A3) may be considered stronger than actually needed: By replacing "inactive workers" by "unemployed workers" the results could still be generated, at the price that, because of indifferences, one can have (even more) multiple, but payoff-equivalent, equilibria. Assumption (A4) is a tie-breaking assumption of the type known from e.g. Rubinstein (1982), where it is used to ensure uniqueness. (A4) will here play a similar role as in the cited paper. Assumption (A5) is simply a matter of conveniance. Extra assumptions will be used in the sequel, but will be stated where they are needed.

To ensure unique equilibrium payoffs in the bargaining part of the present game (the second stage) not even subgame perfection and (A2)–(A4) are sufficiient. An extra requirement which refines subgame perfection is needed. We call this extra requirement "forward induction" (FI), a term which is clearly motivated by its use in the literature (cf. Kohlberg and Mertens (1986); Hellwig and

Leininger (1987); Cho and Kreps (1987)), but we taylor it to the needs of the present game.

Let PE denote the set of subgame perfect equilibria of a game in extensive form. Denote by $D_k(\sigma)$ the set of decision nodes of player k which are reached, when the equilibrium path induced by the strategy (profile) σ is played. Again let $\sigma \mid h_x$ denote the strategy induced by σ in the subgame starting at node x after the history $h_x \in H_x$ (up to, but not including node x). Say that a node y comes after x, denoted $y > x$, if y can be reached from x (but is different from x) and say that y is an immediate successor of x, denoted $y \in S(x)$, if $y > x$ and the number of moves neccessary to reach y from x is exactly one; $S(x) = \min\{y \mid y > x\}$. For a given strategy σ^k of player k let $\sigma^k{}_x$ (by a slight abuse of notation) denote the choice at node $x \in D_k(\sigma)$ of player k induced by σ^k and let $S(x) \mid \sigma^k{}_x$ denote the subset of S(x) reached by the choice $\sigma^k{}_x$ at node x.

Let $(BR_{-k}(\sigma^{*k}),\sigma^{*k}) \mid h_x$, $x \in D_k(\sigma)$, denote the strategy profile which emerges in the subgame starting at node x (reached after history $h_x \in H_x$), when

(i) at node $x \in D_k(\sigma)$ player k substitutes $\sigma^{*k} \mid h_x$ for $\sigma^k \mid h_x$ and

(ii) all other players choose their best response strategies, BR_{-k}, against $\sigma^{*k} \mid h_x$ in the subgame starting at node $x \in D_k$ (x).

Denote by $PE_y(\sigma^{*k} \mid (h_x,\sigma^{*k}{}_x))$, $y \in S(x) \mid \sigma^{*k}{}_x$, the set of subgame perfect equilibria in the subgame starting at node y immediately after h_x and the choice $\sigma^{*k}{}_x$ at node x, where the strategy $\sigma^{*k} \mid h_x$ for player k is held fixed. Clearly $PE_y(\sigma^{*k} \mid (h_x,\sigma^{*k}{}_x))$ may be empty, because $\sigma^{*k} \mid h_x$ is held fixed for the rest of the game. Note that, if only pure strategies are allowed, $S(x) \mid \sigma^{*k}{}_x$ is a singleton. Let $R_k(\sigma \mid h_x)$ denote player k's expected payoff at node x given the strategy σ.

(FI) A $\sigma \in PE$ satisfies <u>forward induction</u>, denoted $\sigma \in FI$, if there exists no $\sigma^{*k} \mid h_x$ such that $R_k((BR_{-k}(\sigma^{*k}),\sigma^{*k}) \mid h_x) > R_k(\sigma \mid h_x)$, for all $(BR_{-k}(\sigma^{*k}),\sigma^{*k}) \mid (h_x,\sigma^{*k}{}_x) \in PE_y(\sigma^{*k} \mid (h_x,\sigma^{*k}{}_x))$, with $y \in S(x) \mid \sigma^{*k}{}_x$, for all $x \in D_k(\sigma)$ and all players k.

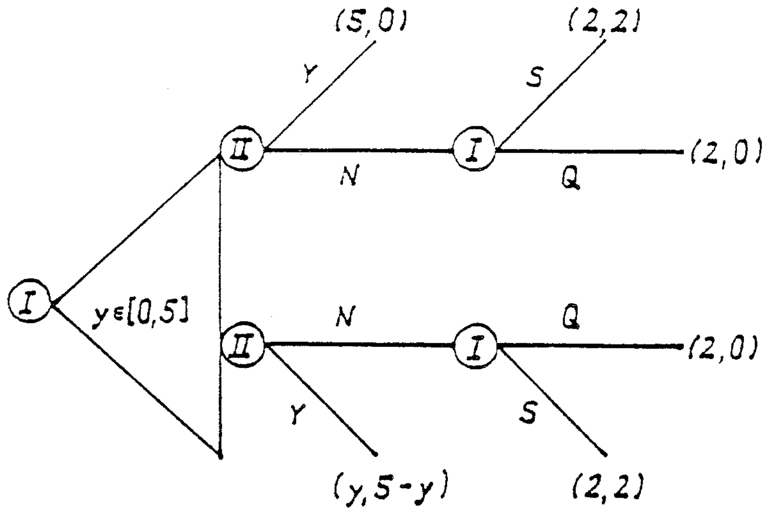
In case $PE_y(\sigma^{*k} \mid (h_x, \sigma^{*k}_x)) = \phi$ the condition in the refinement (FI) is trivially satisfied, implying that in a game with a unique subgame perfect equilibrium (FI) automatically holds. By definition FI is a subset of PE; since for each node $x \in D_k(\sigma)$ there must always be a $\sigma^* \mid h_x$ which is best for player k, there must always be at least one $\sigma \in PE$ satisfying (FI), whenever PE is not empty. (If mixed strategies are allowed, then there is always at least one subgame perfect equilibrium, cf. van Damme (1987), Theorem 6.2.4). [1]

What (FI) does is to rule out "ex ante dominated" choices, i. e. if at some node a player can deviate from his equilibrium strategy in such a way that, whatever subgame perfect equilibrium is played from the successor of this node onwards, he is better off, then the original equilibrium cannot have prescribed to this player an optimal utilization of his future credible threats. Backward induction introduced through a splitting of each player into independent agents at each decision node may fail to capture gains to the players which can be forced by coordination of moves; (FI) allows for a coordination of future credible moves with the present move, such that the present choice can be interpreted as a "signal" on what will be chosen among the future best responses; $\sigma \in FI$ requires that there is no present deviation which can be coordinated with future choices in such a way as to force the other players on paths strictly preferred by the deviator.

The interpretation of (FI) is highlighted by comparing it with the concepts of perfect sequential equilibrium by Grossman and Perry (1986) and the "intuitive criterion" by Cho and Kreps (1987) which, although formulated for signalling games, strongly inspired the present definition: These criteria test for types of players for whom a deviation would be optimal given best responses to the deviation by the other players. (FI) tests for future behavior, which would make a present deviation optimal given the best responses of the other players to the deviation.

What is meant by stating that (FI) requires something like "ex ante admissibility" is illustrated by the example of Figure 2

FIGURE 2:



In the game of Figure 2 player I can choose a number from the interval [0,5]; given this number $y \in [0,5]$ player II can either choose Y, in which case the payoffs are $(y,5-y)$, or he can choose N, in which case I chooses between the payoffs vectors (2,0) and (2,2). Obviously, for any $y > 2$ player I has a strong incentive to induce player II to choose Y; consequently it does not make any sense for I to consider the choice S for the subgame before the game terminates; moreover it never makes sense for I to chose $y<2$. Consider any $2 \leq y < 5$. Clearly there exists $y^* \in (y,5)$ to which II will certainly respond with Y, if y^* is coordinated with the credible threat to play Q, whenever N occurs. Hence any $y \in [2,5)$ is ex ante dominated by some $y^* \in (y,5)$ coupled with the credible threat Q. Therefore the only equilibrium satisfying (FI) is the one where I chooses 5 and II chooses Y, because I makes II indifferent between Y and N by threatening to choose Q later. For I to choose 5 and Q is an "ex ante dominant" move.

The example of Figure 2 allows an interesting comparison of (FI) with stability of equilibria. The concept of a stable set by Kohlberg and Mertens implies that a stable set contains (i) a stable set of any game obtained by deletion of a dominated strategy (Iterated Dominance) and (ii) a stable set of any game obtained by deletion of a strategy which is an inferior response in all equilibria of the set

(Forward Induction). Since a stable set refers to finite games, replace in the game of Figure 2 the interval $[0,5]$ by the set $\{0,1,2,3,5-\delta,5\}$, $\delta \in (0,2)$. For any $y \in \{0,1,2,3\}$ player II's choice Y weakly dominates any other choice and for $y = 5$ player II's choice N weakly dominates. Applying (i), therefore, reduces the game to one where player II has only two pure strategies left. In this reduced game any pure strategy of player I involving an initial choice $y \in \{0,1,2,5\}$ is weakly dominated. Again applying (i) leaves a game in which any pure strategy of player I involving the choice S after $y = 5-\delta$ and II's response N is an inferior response. Reducing the game further by application of (ii) produces a game, where II's choice Y after $y = 5-\delta$ weakly dominates N after $5-\delta$. A final application of (i) then results in a game with the unique equilibrium payoffs $(5-\delta,\delta)$. On the other hand an application of (FI) produces in this example the set of equilibrium payoffs $\{(5-\delta,\delta),(5,0)\}$.

We offer two explanations for the weakness of (FI) in this example: (a) The specific time structure of the game in figure 2, gives player I an advantage which (FI) allows him to exploit, making the elimination of player I's best equilibrium (5, 0) impossible. (b) Procedures (i) and (ii) require player I to protect himself against unfavourable moves of the opponent at indifference points by offering $(5 - \delta, \delta)$. The equilibrium (5, 0) is thereby eliminated. The equilibrium which satisfies both concepts, $(5 - \delta, \delta)$, can be thought of as having y as close as possible to 5. Consequently the boundary equilibrium with $y = 5$ may be interpreted as an approximation of an equilibrium in the stable set, when the finite strategy space is replaced by an interval. Since for the present bargaining game both a determined time structure and the choice of offers from intervals seem essential, (FI) seems to be a reasonable refinement.

4. Mobility costs

The first version of the game to be studied is a variant with mobility costs, i.e. with costs to the workers, if they want to move from one firm to another. These mobility costs may be a taxi bill for the trip to the other firm or the "shoe leather" used as an input for walking around the corner to another firm's office.

(A6) Suppose the matching technology has assigned $j \in N$ to the queue of firm $i \in M$. If j moves in t, t=1,...,T, to the queue of another firm k, $k \neq i$, $k \in M$, then j incurs a cost $m > 0$. (This holds for all $j \in N$ and $i \in M$)

The mobility cost m may be arbitrary small, but positive. It will be seen that this extra friction, even if it is arbitrarily small, changes the strategic situation on the market, i.e. the limit of the solution to the game, when m tends to zero, is not neccessarily the solution of the game for m=0.

The discussion will proceed by presenting a series of lemmas which describe the behavior in the subgame perfect equilibria satisfying (FI) of the bargaining game after paricular classes of histories for a fixed T. The first important class of histories is the one which leaves the firm with a remaining queue exactly as long as the number of remaining jobs and the second important class will be the one which leaves the firm with more workers in its queue than the number of remaining jobs. The first situation may be called "bilateral monopoly", because the firm cannot avoid to deal again with a worker with whom it has unsuccessfully negotiated, due to the privileg of a worker, who has experienced a quitting of the bargaining process, to choose his new position before the other workers in the economy can. This does not hold true after the second class of histories, because positions at the end of the firm's queue are already taken.

To repeat notation, the symbol $\sigma \mid h_t$ will denote the restriction of a strategy profile σ to the subgame after history $h_t \in H_t$, where σ will throughout be taken from the set of subgame perfect equlibrium strategy profiles satisfying (FI). Consistently $\sigma \mid (h_t, \pi)$ will denote the continuation of $\sigma \mid h_t$ after the history h_t extended by the offer π and acceptance of π by the worker and $\sigma \mid (h_t, w)$ is defined analogously; $\sigma \mid (h_t, r(w))$ is the continuation of $\sigma \mid h_t$ after h_t extendended by the offer w, rejection of w and the decision by both bargaining partners to continue bargaining, while $\sigma \mid (h_t, R(\pi))$ is the continuation of $\sigma \mid h_t$ after h_t extended by the offer π, rejection of π and quitting of the bargaining process by at least one of the two partners.

The class of histories $H^i_t(T+1-t-f_i, f_i-l_i)$, $T+1-t \geq f_i \geq 0$, $l_i \geq 0$, will be the class of histories such that after $h_t \in H^i_t(T+1-t-f_i, f_i-l_i)$ firm i is left with a remaining queue of length $f_i = (T+1-t)-(T+1-t-f_i)$ and a number of remaining open jobs $l_i = f_i-(f_i-l_i)$. The major part of the discussion will concern the case where the number of unemployed $U(l) = N - \Sigma_{i \in M} l_i$, is nonnegative.

Lemma 1: Suppose $U(l) \geq 0$ and consider a history $h_t \in H^i_t(x, y)$ with $x + y \leq 0$ and let $j \in N$ be the worker in firm i's queue, $i \in M$, whose turn it is to bargain with i after h_t in period t=1,...,T. Denote by $\pi^i_{T+1-t}(\sigma \mid h_t)$ resp. $v^j_{T+1-t}(\sigma \mid h_t)$ the expected payoffs of the firm resp. the worker in the subgame starting after $h_t \in H^i_t(x, y)$, given $\sigma \in PE \cap FI$. Then, if m is sufficiently small,

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha(T+1-t), \quad v^j_{T+1-t}(\sigma \mid h_t) = (1-\alpha),$$

for all $h_t \in H^i_t(x, y)$, $x+y \leq 0$, t=1,...,T, for all $\sigma \in PE \cap FI$.

Proof: After $h_t \in H^i_t(x, y)$ firm i is left by definition with a number of $l_i = T+1-t-x-y \geq T+1-t$ jobs. For m sufficiently small firm i will have a queue $f_i \geq T+1-t$, because all unemployed have an incentive to change their positions to firm i's queue.

Firm i's offer π will be accepted by worker j only if

$$1 - \pi \geq [1-Q][1-q]\, v^j_{T-t}(\sigma \mid (h_t, r(\pi))), \quad \pi \in [0,1]$$

($(Q, q) \in \{0,1\}x\{0,1\}$ denote the quitting probabilities of the firm resp. the worker). This is true since for Q=1 and/or q=1 the worker will never again have a chance on a job, since the queue is filled up until the end of the game. By assumption (A2) and $l_i \geq T+1-t$

$$\pi^i_{T-t}(\sigma \mid (h_t, r(.)) = \pi^i_{T-t}(\sigma \mid (h_t, R(.))$$

holds. Consequently the worker's offer w will be accepted by the firm only if

$$1 - w + \pi^i_{T-t}(\sigma \mid (h_t, w)) \geq \pi^i_{T-t}(\sigma \mid (h_t, R(w)), \quad w \in [0,1].$$

Consider the subgame starting after nature has decided that the firm is the proposer. Since in a subgame after a rejection the firm would be indifferent between Q=1 and Q=0 it is credible to offer π=1 and this dominates any other offer. Hence by (FI) the expected payoffs to the firm resp. to the worker are by (A4)

$$\pi^{i}_{T+1-t}(\sigma \mid h_t) = \alpha + \alpha\pi^{i}_{T-t}(\sigma \mid (h_t, \pi)) + (1-\alpha)\pi^{i}_{T-t}(\sigma \mid (h_t, R(w))),$$

$$v^{j}_{T+1-t}(\sigma \mid h_t) = (1-\alpha) + (1-\alpha)[\pi^{i}_{T-t}(\sigma \mid (h_t, w)) - \pi^{i}_{T-t}(\sigma \mid (h_t, R(w)))].$$

Now consider t=T; then by (A4) one obtains

$$\pi^{i}_{1}(\sigma \mid (h_T)) = \alpha \quad \text{and} \quad v^{j}_{1}(\sigma \mid h_T) = (1-\alpha).$$

From $l_i \geq T+1-t$ and (A2) it follows that

$$\pi^{i}_{T-t}(\sigma \mid (h_t,w)) = \pi^{i}_{T-t}(\sigma \mid (h_t,R(w))) \quad \text{implying} \quad v^{j}_{T+1-t}(\sigma \mid h_t) = (1-\alpha), \text{ for all } \sigma \in PE \cap FI.$$

Backward induction implies

$$\pi^{i}_{T+1-t}(\sigma \mid h_t) = \alpha + \alpha(T-t) = \alpha(T+1-t), \text{ for all } \sigma \in PE \text{ satisfying FI.}$$

∎

__Lemma 2:__ Suppose $U(l) \geq 0$ and consider a history $h_t \in H^{i}_{t}(x, y)$, with $y \leq 0$ and let j be the worker, whose turn it is to bargain with firm i in t=1,...,T. Denote expected payoffs as in Lemma 1. Then, if m is sufficiently small,

$$\pi^{i}_{T+1-t}(\sigma \mid h_t) = \alpha(T+1-t-x) = \alpha l_i, \quad \text{for all } h_t \in H^{i}_{t}(x, y), y \leq 0,$$

$$v^{j}_{T+1-t}(\sigma \mid h_t) = (1-\alpha), \quad \text{for all } h_t \in H^{i}_{t}(x, y), y \leq 0,$$

t=1,...,T for all $\sigma \in PE \cap FI$ (l_i denotes the number of remaining jobs.).

__Proof:__ (For simplicity the superscripts will be suppressed in the sequel) The hypothesis $y \leq 0$ implies that the remaining queue of firm i is shorter or equal to the number of remaining jobs, $l_i \geq f_i$. Since for $f_i < l_i$, $U(l) \geq 0$ and m sufficiently small all unemployed will try to capture a position in firm i's queue, it suffices to consider the case $f_i = l_i$, i.e. the "bilateral monopoly", $y = 0$.

First we give an argument why the firm can never credibly attract unemployed to the end of its queue; to be precise, this argument should be iterated for each period working backwards; to keep things short, we will only state the argument for $t=T-1$: In a subgame following a rejection of an offer the firm has no way of avoiding to deal again with the unsuccessful worker j, since j has the first option on a new position and can always queue in at the end of firm i's queue – a position which will correspond to an open job after a rejection. The only way credibly to attract unemployed workers, in order to reduce the reservation wages of present workers to zero, is to commit to a positive probability of rejection in the next bargaining game. This can either be done by commiting to ask $\pi > 1$, if the firm becomes the proposer, or by commiting to reject any offer $w \geq 1-\alpha$, if the worker proposes. From Lemma 1 $\pi_1(\sigma \mid h_T) = \alpha$. Thus in the subgame of period $T-1$, where the firm is the proposer, an offer of $\pi > 1$ would yield the firm an expected payoff of α. By asking $\pi = 1$ the firm would have a strictly higher payoff (since this offer would be accepted, if the attraction of unemployed worked.). Consequently the commitment cannot be credible.
Now suppose a commitment to reject any $w \geq 1-\alpha$ by the firm and an offer $w = (1-\alpha)$ by the worker would be a subgame perfect equilibrium. Then there exsists a w', $0 < w' < 1-\alpha$, the acceptance of which would make both the firm and the worker better off than in the conjectured equilibrium; hence a contradiction.

This argument works as long as the future reservation payoff to the worker is positive; if the reservation payoff is zero, then the firm cannot attract unemployed anyway, because their expected payoff from moving to i would be smaller than $m > 0$. The conclusion is that the unsuccessful worker j will always come back.

The offers of the firm resp. the worker will only be accepted, if

$$(1)\quad \begin{aligned} &1-\pi \geq [1-Q][1-q]\, v_{T-t}(\sigma \mid (h_t, r(\pi)) + [Q+q-qQ]\, v_{T-t-l_i}(\sigma \mid (h_t,.)) \text{ resp.} \\ &1-w + \pi_{T-t}(\sigma \mid (h_t, w)) \geq [1-Q][1-q]\, \pi_{T-t}(\sigma \mid (h_t, r(w))) + [Q+q-qQ]\, \pi_{T-t}(\sigma \mid (h_t, R(w))), \end{aligned}$$

where $\pi \in [0,1]$ and $w \in [0,1]$ and (Q, q) denote the quitting probabilities of the firm and the worker. First consider the case $l_i=1$; in this case $(Q,q)=(0,0)$ and $(Q,q)\in \{(0,1),(1,0),(1,1)\}$ leads to the same subgame, because the unsuccessful worker will always come back. Therefore the payoffs, given <u>any</u> σ, are

$$\pi_{T+1-t}(\sigma \mid h_t) = \alpha - \alpha v_{T-t}(\sigma \mid (h_t, r(\pi))) + (1-\alpha)\pi_{T-t}(\sigma \mid (h_t, r(w))),$$
$$v_{T+1-t}(\sigma \mid h_t) = \alpha v_{T-t}(\sigma \mid (h_t, r(\pi))) + (1-\alpha)[1-\pi_{T-t}(\sigma \mid (h_t, r(w)))].$$

Now $\pi_1(\sigma \mid h_T) = \alpha$ and $v_1(\sigma \mid h_T) = (1-\alpha)$ from Lemma 1; hence assume for $l_i = 1$ that $\pi_{T-t}(\sigma \mid h_{t+1}) = \alpha$ and $v_{T-t}(\sigma \mid h_{t+1}) = (1-\alpha)$ for all $h_{t+1} \in H^i_{t+1}(x, 0)$; Since $(h_t, r(.)) \in H^i_{t+1}(x, 0)$, one has $\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha$ and $v_{T+1-t}(\sigma \mid h_t) = (1-\alpha)$ for all $h_t \in H^i_t(x, 0)$, all $\sigma \in PE \cap FI$.

Let $l_i > 1$. Again the acceptability constraints (1) apply and a commitment to a rejection in order to attract unemployed from other firms is impossible . If $l_i = T+1-t$, then Lemma 1 applies and confirms Lemma 2. For $l_i < T+1-t$ assume $(Q, q) =(0,0)$; we will show that this strategy has the same payoffs as $(Q, q) \in \{(0,1),(1,0),(1,1)\}$.

Expected payoffs are

$$\pi_{T+1-t}(\sigma \mid h_t) = \alpha - \alpha v_{T-t}(\sigma \mid (h_t, r(\pi))) + \alpha\pi_{T-t}(\sigma \mid (h_t, \pi) + (1-\alpha)\pi_{T-t}(\sigma \mid (h_t, r(w))) ,$$
$$v_{T+1-t}(\sigma \mid h_t) = (1-\alpha) + (1-\alpha)[\pi_{T-t}(\sigma \mid (h_t, w)) - \pi_{T-t}(\sigma \mid (h_t, r(w)))] + \alpha v_{T-t}(\sigma \mid (h_t, r(\pi))).$$

To apply backward induction assume

$$\pi_{T-t}(\sigma \mid h_{t+1}) = \alpha(T-t-x') \quad \text{for all } h_{t+1} \in H^i_{t+1}(x', 0),\ x' \geq 0 ,$$
$$v_{T-t}(\sigma \mid h_{t+1}) = (1-\alpha) \quad \text{for all } h_{t+1} \in H^i_{t+1}(x', 0),\ x' \geq 0.$$

Since $h_t \in H^i_t(x,0)$ implies $(h_t,r(.)) \in H^i_{t+1}(x-1,0)$ and $(h_t,a) \in H^i_{t+1}(x,0)$, for $a \in [0,1]$, one obtains $\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha - \alpha(1-\alpha) + \alpha^2(l_i-1) + \alpha(1-\alpha)l_i = \alpha l_i$, where $l_i=T+1-t-x$, the number of open jobs, and $v^j_{T+1-t}(\sigma \mid h_t) = (1-\alpha) - \alpha(1-\alpha) + \alpha(1-\alpha) = (1-\alpha)$ as required. If $(Q,q) \in \{(0,1),(1,0),(1,1)\}$ then the workers reservation payoff will be calculated from the case $l_i = 1$ and will equal $(1-\alpha)$ again. Applying the induction hypothesis then again generates the payoffs above, as required by the Lemma.

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Remark: The first two Lemmas are formulated to apply to all $\sigma \in PE \cap FI$. The reason for this can be seen from the proof of Lemma 2: There are several payoff equivalent equilibria, all in the same component and differing only with respect to the choice of quitting strategies. Payoff equivalence is the important feature for the solution of the "Cournot–Nash"-game in capacities at the first stage.

The first two lemmas concern classes of histories after which the firm is in a weak strategic position, because it cannot threaten workers with substituting them by a competitor. The firm will, therefore, only have an expected payoff per job which is a fraction α of the pie. This changes entirely once the firm can credibly threaten with employing a substitute worker from its queue.

Lemma 3: Suppose $U(l) \geq 0$ and consider $h_t \in H^i_t(x, y)$, with $y > 0$ and let j be the worker, whose turn it is to bargain with i in t, t=1,...,T. Denote expected payoffs as before. Then

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha(T+1-t-x-y) + \alpha(1-\alpha) \sum_{k=0}^{x+y-1} \sum_{s=0}^{T-t-x-y} \binom{s+k}{k} \alpha^s (1-\alpha)^k$$

for all $h_t \in H^i_t(x, y)$, $y>0$, $t = 1,\dots,T$, for all $\sigma \in PE \cap FI$.

Proof: (Again superscripts for the players' identities are suppressed.) If $y > 0$, then $f_i = T+1-t-x > l_i = T+1-t-x-y$ and, therefore, an unsuccsessfully negotiating worker has no chance ever again to catch a job after a bargaining process is terminated by quitting. In the latter situation the worker's expected payoff is non–positive whatever he does. (He cannot catch a job in any queue, because all positions corresponding to jobs are taken and in firm i's queue also the last position is taken.) The worker's optimal choice is, therefore, to move to the end of firm i's queue, where, although he, thereby, cannot catch a job, he avoids the mobility cost $m > 0$. Given this behaviour the firm can credibly threaten to play Q=1 in any subgame following a rejection and by applying (FI) all $\sigma \in PE \cap FI$ have Q=1 in all subgames, where the firm has been appointed the proposer. Indeed this threat dominates any other behaviour in a subgame where the firm has been chosen the proposer. In subgames where the worker is the proposer again multiple, but payoff-equivalent, equilibria differing by the choice of (Q,q) are possible.

The acceptability constraint implies the following expected payoff to the firm

$\pi_{T+1-t}(\sigma \mid h_t) = \alpha + \alpha\pi_{T-t}(\sigma \mid (h_t,\pi)) + (1-\alpha)\pi_{T-t}(\sigma \mid (h_t,R(w)))$.

Now $h_t \in H^i_t(x, y)$, $y > 0$ implies $(h_t, \pi) \in H^i_{t+1}(x, y)$ and $(h_t, R(w)) \in H^i_{t+1}(x-1, y)$. First consider $l_i=1$; then

$\pi_{T+1-t}(\sigma \mid h_t) = \alpha + (1-\alpha)\pi_{T-t}(\sigma \mid (h_t, R(w)))$;

$t=T-1$ implies $\pi_2(\sigma \mid h_{T-1}) = \alpha + \alpha(1-\alpha) = \alpha + \alpha(1-\alpha) \sum_{k=0}^{0}(1-\alpha)^k$;

$$\pi_{T-t}(\sigma \mid h_{t+1}) = \alpha + \alpha(1-\alpha) \sum_{k=0}^{x+y-2} (1-\alpha)^k$$

for all $h_{t+1} \in H^i_{t+1}(x-1,y)$ with $T-t-x+1-y=1$ (i.e. $x+y=T-t$) implies

$$\pi_{T+1-t}(\sigma \mid h_t) = \alpha + \alpha(1-\alpha) + \alpha(1-\alpha) \sum_{k=1}^{x+y-1} (1-\alpha)^k = \alpha + \alpha(1-\alpha) \sum_{k=0}^{x+y-1} (1-\alpha)^k \text{ , as required.}$$

Next for $l_i = T+1-t-x-y > 1$ assume

$$\pi_{T-t}(\sigma \mid h_{t+1}) = \alpha(T-t-x'-y) + \alpha(1-\alpha) \sum_{k=0}^{x'+y-1} \sum_{s=0}^{T-t-x'-y-1} \binom{s+k}{k} \alpha^s(1-\alpha)^k$$

for all $h_{t+1} \in H^i_{t+1}(x', y)$, $y > 0$. Then this implies

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha + \alpha^2(T-t-x-y) + \alpha(1-\alpha)(T+1-t-x-y) +$$

$$+ \alpha(1-\alpha) \sum_{k=0}^{x+y-1} \sum_{s=1}^{T-t-x-y} \binom{s+k-1}{s-1} \alpha^s(1-\alpha)^k + \alpha(1-\alpha) \sum_{k=1}^{x+y-1} \sum_{s=0}^{T-t-x-y} \binom{s+k-1}{s} \alpha^s(1-\alpha)^k =$$

$$= \alpha(T+1-t-x-y) + \alpha(1-\alpha)[1 + \sum_{s=1}^{T-t-x-y} \alpha^s + \sum_{k=1}^{x+y-1} (1-\alpha)^k + \sum_{k=1}^{x+y-1} \sum_{s=1}^{T-t-x-y} \binom{s+k}{s} \alpha^s(1-\alpha)^k] =$$

$$= \alpha(T+1-t-x-y) + \alpha(1-\alpha) \sum_{k=0}^{x+y-1} \sum_{s=0}^{T-t-x-y} \binom{s+k}{k} \alpha^s (1-\alpha)^k$$

as reqired .

I

Corollary 1: If $T \to \infty$, $U(l) \geq 0$ and $h_t \in H^i_t(x, y)$, $y > 0$, $1 \leq t < \infty$ and $l_i = T+1-t-x-y$ the number of jobs remaining in firm i after h_t,

$\lim_{T\to\infty} \pi^i_{T+1-t}(\sigma \mid h_t) = l_i$, for all $h_t \in H^i_t(x, y)$, $y > 0$, for all $\sigma \in PE \cap FI$.

Proof: see Appendix.

From the Corollary it follows that when T tends to infinity a firm with a queue longer than the number of jobs will capture all the surplus. That this does not depend on the number by which the queue exceeds the number of jobs is a consequence of (A6): Since mobility costs deter the worker from reducing the excess in the firm's queue, one extra worker is sufficient to threaten all with a substitute. Hence no firm gains from a queue $f_i > l_i+1$. Rather this would be a waste of a potential profitable job.

Now suppose there is excess unemployment, $U(l) < 0$. By (A.6) no unsuccessfully negotiating worker will have an incentive to change his position to another firm, because there will always be a free job in the present firm. Hence the situation is equivalent to Lemma 2. This argument demonstrates:

Lemma 4: Suppose $U(l) < 0$ and consider any history $h_t \in H_t$ and let j be the worker, whose turn it is to bargain with firm i in t, t=1,...,T. Denote expected payoffs as before. Then

$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha f_i$

$v^j_{T+1-t}(\sigma \mid h_t) = (1-\alpha)$ for all $h_t \in H_t$, t=1,...,T,

for all $\sigma \in PE \cap FI$, where f_i denotes the number of workers assigned to firm i.

Remark: Lemmas 1–3 are stated for m sufficiently small. The situation for large m would, however, not differ. A large $m > 0$ would reduce the game to one between a single firm and f_i workers, yielding the same results as above.

We summarize the analysis of the bargaining game by a Theorem on expected payoffs to the firm in period one, i.e. after completion of the first stage of the overall game.

Theorem 1: Assume (A1) – (A6). Then there exists a unique set of payoff equivalent subgame perfect equilibria satisfying (FI) of the bargaining game (stage 2), given capacity choices $l \in L^M$ and the realization of the matching technology. The expected payoffs to the firms $i \in M$ are given by

$$\pi^i_T(\sigma \mid (l,f)) = \begin{cases} \alpha s_i, \text{ where } s_i = f_i, \text{ if } U(l) < 0, \text{ and } s_i = l_i, \text{ if } U(l) \geq 0 \text{ and } f_i \leq l_i \\ \alpha l_i + \alpha(1-\alpha) \sum_{k=0}^{T-l_i-1} \sum_{s=0}^{l_i-1} \binom{s+k}{k} \alpha^s (1-\alpha)^k, \text{ if } f_i > l_i, \ U(l) \geq 0, \end{cases}$$

for all $\sigma \in PE \cap FI$, where l_i denotes the capacity of firm i and f_i denotes the length of firm i's queue and $f = (f_1,...,f_M)$.

Proof: Follows from Lemmas 1–4.

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The rest of this section is devoted to the investigation of the first stage of the game, the "Cournot-Nash" (CN) game in capacities.

Suppose there is a "Cournot - Nash" equilibrium with $U(l) < 0$. This implies that $l_i \geq N/M$, for all firms, because if there exists an i such that $l_i < N/M$ this firm would be better off with l_i+1. This is the case because the matching technology ensures that i gets a queue $f_i \geq l_i + 1$, if it choses $l_i' = l_i + 1$; also $\alpha - c > 0$ must hold in this case since otherwise at least one firm would make a loss, contradicting the assumption of an equilibrium. Hence by $(l_i + 1)(\alpha - c) > l_i(\alpha - c)$, the firm would be better off with $l_i' = l_i + 1$. The assumption $U(l) < 0$ and the implication $l_i \geq N/M$ for all firms then implies that there exists an $i \in M$, which is rationed with probability one (i.e. $f_i < l_i$). This firm i would certainly be better off by reducing its capacity and saving the cost c. We have shown:

Lemma 5: In any CN–Equilibrium of the first stage of the game $U(l) = N - \Sigma_{i \in M} l_i \geq 0$.

To study more closely the CN–equilibria of the first stage of the game, the limit of the solution of the bargaining game as T tends to infinity is exclusively taken into consideration. This makes both things easier and is a proper notion of an "almost" frictionless bargaining at the second stage of the game.

Lemma 5 restricts the search for CN–equilibria to the set of $l \in L^M$ satisfying $U(l) \geq 0$, i.e. there can never be excess employment in any equilibrium of the game. Moreover it was already noted that $U(l) \leq M$ in any CN–equilibrium (see Corollary 1).

For $0 \leq U(l) \leq M$ the net expected profit of firm i can be written as

$P_i(l) = l_i[1 - c - (1-\alpha)(M - U(l))/M]$, for all $i \in M$,

since the matching technology implies that firm i will get a worker in its queue in excess of the number of jobs with the probability $U(l)/M$ and a queue exactly as long as the number of jobs with probability $1 - U(l)/M = (M-U(l))/M$.

Theorem 2: Assume (A1) – (A6) and let $T \to \infty$. There exists a set of CN–equilibria in pure strategies all of which have

$\min[M, \max\{0, (M/(M + 1))(N/M - M(\alpha - c)(1 - \alpha)^{-1} + 1)\}] \geq U(l) \geq$

$\geq \min[M, \max\{0, (M/(M + 1))(N/M - M(\alpha - c)(1 - \alpha)^{-1} - 1)\}]$.

Proof: see Appendix.

We conclude this section by some comparative static interpretations of the results:

1. From Lemmas 1–4 and Theorem 1 it is clear that the probability of the firm being the proposer, $\alpha \in (0,1)$, plays a crucial role. As α tends to unity the firm will always be able to capture the whole surplus. As a consequence it never pays the firm, for $\alpha \to 1$, to generate any unemployment and full employment will be the only equilibrium. The other extreme, $\alpha \to 0$ shows a dicontinuity at $\alpha=0$: No firm will choose any capacity at $\alpha=0$ and unemployment will leave the interval defined in Theorem 2, since the workers would capture all the surplus in the bargaining game, irrespective of the size of unemployment. As long as $\alpha > 0$, the firms can offset the bargaining power of the workers by sufficiently high unemployment.

2. The driving force of the game is the twofold role which capacity choice plays. Capacity choice gives the firms the option of a binding precommitment, thereby determining their threat points. On the other hand capacity choice determines the total product that workers and firms can divide among each other. Since the costs of incurring this precommitment are "strategically sunk" in the bargaining game, i.e. no worker will care about capacity costs, the firms are forced to generate credible threats against excessive wage–claims; the only available threat here is unemployment. This can be seen most clearly from Theorem 2: A sufficient condition for $U(l) > 0$ in equilibrium is that $\alpha - c \leq 0$.The difference $(\alpha-c)$ contains the two crucial parameters of the game, namely the probability that the firm will make the offer and the capacity cost. High capacity costs generate unemployment!

3. Let the integer N/M be fixed and let M tend to infinity. The influence of an individual firm then becomes small. From Theorem 2 it can be seen that for $\alpha-c > 0$ unemployment will tend to zero. For $\alpha - c < 0$ unemployment will still be positive. Hence also in a "large market" inefficiency will not vanish for all parameter values.

4. Assumption (A6) is responsible for the absence of inter–firm mobility of the workers which could threaten the firm with a shift from the situation of Lemma 3 to situations like those in Lemmas 1 and 2. Note that this holds for any arbitrary small but positive m.

5. The restriction to pure strategies does help in the present model. If mixed strategies are allowed, then presumably positive probabilities of delay in bargaining could emerge.

6. Discounting and a proper definition of the length of periods which can be made arbitrarily small, when T approaches infinity, would not change the results.

5. Increasing Mobility

The main feature of the second version of the game is increased mobility of workers. There are two ways to think about this enhanced mobility: The first is to drop assumtion (A6), such that workers can freely move between firms; the second is to retain (A6) but amending it by:

(A7) At the end of each period, every worker may move to a position which is costlessly accessible for workers and which is outside of any queue. (The expected payoff from being in this "neutralized" position is zero.)

Viennese would probably prefer to think of the position described in (A7) as coffee-houses, where workers can virtually costlessly retreat from any contact with firms.

In view of Lemmas 1 and 2 this version of the game exhibits the feature that unsuccessful workers, having experienced a quitting of their bargaining process, are not forced anymore to queue in at the end of the present firm's queue and thereby improve the firm's strategic position. Quite on the contrary workers can now threaten the firm with reducing the excess (if there is any) in the firm's queue over the number of jobs still "unsold". It is, therefore, sensible to expect in the present version a lot of continuation of bargaining in subgames after a rejection. This is in contrast to Section 4, where frequent quitting of the bargaining process after rejections was found.

Before exploring this difference we argue that for all histories generating a "bilateral monopoly" situation the solutions to the present version are the same as in Section 4.

Whenever the firm finds itself in a position reached after a history $h_t \in H^i_t(x,y)$ with $x + y \leq 0$, implying that $l_i = T+1-t-x-y$, the number of open jobs, exceeds the number of remaining periods, then clearly the strategic situation is equivalent to the one of Lemma 1, because the firm will have no incentive to continue bargaining after a rejection. Consequently Lemma 1 also applies to the version with increased mobility. Under the hypothesis $U(l) \geq 0$ there is, as in Lemma 2, no need to seperately consider the situation after $h_t \in H^i_t(x, y)$ with $y < 0$ (i.e. $f_i < l_i$), but it suffices to consider the situation after $h_t \in H^i_t(x,0)$. By (A3) it is again impossible for the firm to attract unemployed from other firms to the end of its queue, because the firm can neither credibly give up its chance to propose by commiting to an unacceptable proposal, nor can the firm commit to reject an offer which makes the firm indifferent, because the latter would contradict the equilibrium assumption. Hence Lemma 2 also applies to the present version, i.e. the unique equilibrium payoffs to "bilateral monopoly" situations are unaltered by the new assumptions . Also we have:

Lemma 6: Suppose $U(l) \geq 0$ and consider a history $h_t \in H^i_t(x,y)$ with $x \leq 0$, i.e. $f_i \geq T+1-t$. Denote expected payoffs as before. Then , for $i \in \mathcal{M}$, $y \leq T+1-t$,

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha(T+1-t-y) + \alpha(1-\alpha) \sum_{k=0}^{T-t-y} \sum_{s=0}^{y-1} \binom{s+k}{k} \alpha^k (1-\alpha)^s$$

for all $h_t \in H^i_t(x,y)$, $x \leq 0$, $t = 1,\dots,T$ for all $\sigma \in PE \cap FI$.

Proof: Since the game terminates in period T the situation $x \leq 0$ is equivalent to $x = 0$, i.e. $f_i = T+1-t$. It suffices to consider $h_t \in H^i_t(0,y)$, where the number of jobs still open is given by $l_i = T+1-t-y \geq 0$. After a rejection of an offer the firm will always be indifferent between quitting the bargaining process and continuation of bargaining with the present worker - even if the worker moves in response to quitting (Q=1) to another firm or to the "neutralized" position of (A7). There is still a queue sufficiently long to fill the remaining time. By the fact that the remaining queue exceeds the number of remaining periods, (FI) requires the firm to play Q = 1 in response to a rejection, if the firm was the proposer offering $\pi = 1$. Hence Lemma 6 is merely a special case of Lemma 3, specialized by setting $x = 0$.

∎

Corollary 2: Let $h_t \in H^i_t(0,z)$ and $h'_t \in H^i_t(z,0)$ for some z satisfying $T+1-t > z > 0$. Then for all $i \in M$,

$\pi^i_{T+1-t}(\sigma \mid h_t) > \pi^i_{T+1-t}(\sigma \mid h'_t)$, for all $\sigma \in PE \cap FI$, $t = 1,...,T$.

Proof: Compare Lemma 2 and Lemma 6.

∎

What Lemma 6 shows is that the analogy between Section 4 and the present section even extends to the case, where the lenght of the firm's queue exceeds the number of remaining periods. Beyond this, however, the analogy ends: Apart from the "boundary" histories just studied which will for sufficiently high T not be reached in equilibrium the two versions behave entirely different. The flavor of the reason for this divergence is contained in Corollary 2: The worker's threat to move to another firm or to the "neutralized" position of (A7) is now both credible and effectiv.

Lemma 7: Suppose $U(l) \geq 0$ and consider a history $h_t \in H^i_t(x,y)$ with $x > 0$, $y > 0$, $T+1-t > x+y$. Denote expected payoffs as before. Then, for all $i \in M$,

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha(T+1-t-x-y) + \alpha(1-\alpha) \sum_{k=0}^{T-t-x-y} \sum_{s=0}^{y-1} \binom{s+k}{k} \alpha^k (1-\alpha)^s$$

for all $h_t \in H^i_t(x,y)$, $x > 0$, $y > 0$, $T+1-t > x+y$, for all $\sigma \in PE \cap FI$.

Proof: First consider $h_t \in H^i_t(1,1)$, such that $f_i = T-t$ and $l_i = T-t-1$. Consider the subgame, where the firm has been chosen as the proposer and assume the firm offers $\pi = 1$ (as in Section 4). If the worker would indeed expect the firm to play $Q = 1$ in response to a rejection, it would be optimal for him to accept $\pi = 1$. If, however, the worker deviates to a rejection of $\pi = 1$ coordinated with his decision not to quit the bargaining process, and to move to another firm (or the "neutralized" position of (A7)), if the firm plays $Q = 1$, then given the rejection the firm is confronted with the problem of choosing beween $R(\pi)$, where $(h_t,R(\pi)) \in H^i_{t+1}(1,0)$ and $r(\pi)$, where $(h_t,r(\pi)) \in H^i_{t+1}(0,1)$. According to Corollary 2 the best response is to choose $r(\pi)$ i.e. to stay and play $Q = 0$, which, however, makes the worker strictly better off than with an acceptance of $\pi = 1$. By (FI) we, therefore, must have that the firm's offer π satisfies

$$1 - \pi \geq v^j_{T-t}(\sigma \mid (h_t,r(\pi)), \quad \pi \in [0,1].$$

Now consider the subgame, where the worker proposes. He might propose $w = 1 + \pi^i_{T-t}(\sigma \mid (h_t,w)) - - \pi^i_{T-t}(\sigma \mid (h_t,R(w)))$ and threaten to quit the bargaining process, $q = 1$, and move to another firm (or to the "neutralized" position), if the firm does not accept, such that $(h_t,w) \in H^i_{t+1}(1,1)$ and $(h_t,R(w)) \in H^i_{t+1}(1,0)$. If the firm indeed believes that the worker will play $q = 1$ and move to another firm, if it does not accept, then acceptance is optimal. Confronted with this offer w the firm may, however, deviate to a rejection coordinated with its decision to stay, i.e. to play $Q = 0$, if the worker stays, i.e. plays $q = 0$. Since from $U(l) \geq 0$ the worker can by playing $q = 1$ only get zero payoff, his best response to the firm's deviation is to stay, $q = 0$, which gives the firm a payoff of $\pi^i_{T-t}(\sigma \mid (h_t,r(w))) > \pi^i_{T-t}(\sigma \mid (h_t,R(w)))$, because $(h_t,r(w)) \in H^i_{t+1}(0,1)$. Hence (FI) implies that the worker's offer has to satisfy

$$1 - w + \pi^i_{T-t}(\sigma \mid (h_t,w)) \;\geq\; \pi^i_{T-t}(\sigma \mid (h_t,r(w))), \quad w \in [0,1]$$

with $(h_t,w) \in H^i_{t+1}(1,1)$ and $(h_t,r(w)) \in H^i_{t+1}(0,1)$, if $h_t \in H^i_t(1,1)$. The resultant payoff to the firm is

$$\pi^i_{T+1-t}(\sigma \mid h_t) \;=\; \alpha - \alpha v^j_{T-t}(\sigma \mid (h_t,r(\pi))) + \alpha\pi^i_{T-t}(\sigma \mid (h_t,\pi)) + (1-\alpha)\pi^i_{T-t}(\sigma \mid (h_t,r(w))),$$

i.e. after a rejection bargaining is continued (instead of quitting, as in Section 4). From the preceding Lemma we have, for $(h_t, r(.)) \in H^i_{t+1}(0,1)$,

$$\pi^i_{T-t}(\sigma \mid (h_t,r(.))) + v^j_{T-t}(\sigma \mid (h_t,r(.)) = 1 + \pi^i_{T-t-1}(\sigma \mid (h_t,r(.),\pi))$$

with $(h_t,r(.),\pi) \in H^i_{t+2}(0,1)$, implying

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha\pi^i_{T-t}(\sigma \mid (h_t,\pi)) + \pi^i_{T-t}(\sigma \mid (h_t,r(w))) - \alpha\pi^i_{T-t-1}(\sigma \mid (h_t,r(.),\pi)),$$

for all $h_t \in H^i_t(1,1)$.

Solving the latter recursive equation yields for all $\sigma \in PE \cap FI$

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha(T-t-1) + \alpha(1-\alpha) \sum_{k=0}^{T-t-2} \alpha^k \qquad \text{for all } h_t \in H^i_t(1,1),$$

in accordance with the statement of the Lemma. This result together with Lemmas 6 and 2 yields , for $h_t^0 \in H^i_t(0,2)$, $h_t^1 \in H^i_t(1,1)$ and $h_t^2 \in H^i_t(2,0)$, the inequalities

$$\pi^i_{T+1-t}(\sigma \mid h_t^0) > \pi^i_{T+1-t}(\sigma \mid h_t^1) > \pi^i_{T+1-t}(\sigma \mid h_t^2), \quad \text{for all } t=1,\dots,T.$$

Using the above as a starting point, we will now apply a backward induction argument.
Assume for some z satisfying $T+1-t > z > 0$ and the histories $h_t^k \in H^i_t(k,z-k)$, $k = 0,\dots,z$, that the following series of inequalities holds:

$$(3) \qquad \pi^i_{T+1-t}(\sigma \mid h_t^k) > \pi^i_{T+1-t}(\sigma \mid h_t^{k+1}), \qquad \text{for all } k = 0,1\dots,z-1, \text{ for all } t;$$

assume also for the moment, that the worker will move to another firm (or the "neutralized" position), in case the bargaining process is quitted. Then $h_t \in H^i_t(k,z+1-k)$, $k=1,\dots,z$, implies that

$(h_t,R(.)) \in H^i_{t+1}(k,z-k)$ and $(h_t,r(.)) \in H^i_{t+1}(k-1,z+1-k)$. By the same reasoning as above the refinement (FI) requires that from the series of inequalities (3) the second assumption follows directly, because both the firm and the worker have coordinated moves to make the equilibrium behavior under (FI) $(Q,q)=(0,0)$, i.e. continuation of bargaining after a rejection. This also generates the acceptibility constraints, for $h_t \in H^i_t(k,z+1-k)$,

$$1 - w + \pi^i_{T-t}(\sigma \mid (h_t,w)) \geq \pi^i_{T-t}(\sigma \mid (h_t,r(w))) , \quad w \in [0,1] ,$$
$$1 - \pi \geq v^j_{T-t}(\sigma \mid (h_t,r(\pi))) , \quad \pi \in [0,1]$$

with $(h_t,r(.)) \in H^i_{t+1}(k-1, z+1-k)$, and the expected payoff

$$\begin{aligned}\pi^i_{T+1-t}(\sigma \mid h_t) &= \alpha - \alpha v^j_{T-t}(\sigma \mid (h_t,r(\pi))) + \alpha\pi^i_{T-t}(\sigma \mid (h_t,\pi)) + (1-\alpha)\pi^i_{T-t}(\sigma \mid (h_t,r(w))) = \\ &= \alpha\pi^i_{T-t}(\sigma \mid (h_t,\pi)) + \pi^i_{T-t}(\sigma \mid (h_t,r(.))) - \alpha\pi^i_{T-t-1}(\sigma \mid (h_t,r(.),\pi)) ,\end{aligned}$$

with $h_t \in H^i_t(k,z+1-k)$, $(h_t,r(.)) \in H^i_{t+1}(k-1,z+1-k)$ and $(h_t,r(.),\pi) \in H^i_{t+2}(k-1,z+1-k)$, $k = 1,...,z$. Solving the latter recursive equation yields

$$\pi^i_{T+1-t}(\sigma \mid h_t) = \alpha(T-t-z) + \alpha(1-\alpha) \sum_{j=0}^{T-t-z-1} \sum_{s=0}^{z-k} \binom{s+j}{j} \alpha^j(1-\alpha)^s$$

which is strictly decreasing in $k = 1,...,z$. Combining this result with Lemmas 6 and 2 yields the series of inequalities

$$\pi^i_{T+1-t}(\sigma \mid h_t^k) > \pi^i_{T+1-t}(\sigma \mid h_t^{k+1}) , \qquad \text{for all } k = 0,1...,z, \quad \text{for all } t,$$

with $h_t^k \in H^i_t(k,z+1-k)$, $k = 0,1,...,z+1$, for all $\sigma \in PE \cap FI$. Hence backward induction yields the desired result.

∎

Remark: From the proof of Lemma 7 it should be noted again that equilibrium behavior is not unique; since in the subgame where the worker proposes followed by rejection either both partners stay together or the firm quits and the worker queues in at the end of the firm's queue. Still equilibrium payoffs are unique, because (FI) rules out any equilibrium behavior which would generate payoffs different from those in the statement of Lemma 7.

Remark: Although this is not explicitely calculated in Lemma 7 it is of some interest to note that

$$v^j_{T+1-t}(\sigma \mid h_t) = (1-\alpha) \sum_{k=y}^{T-t-x} \binom{T-t-x}{k} (1-\alpha)^k \alpha^{T-t-x-k} \qquad \text{for all } h_t \in H^i_t(x,y)$$

for all $\sigma \in PE \cap FI$. Since this expression is strictly decreasing in t, no worker assigned to an "early" place in firm i's queue by the matching technology has an incentive to leave his position in order to position himself on a "later" place in the same queue, because this would make him worse off.

We summarize the bargaining stage in the following:

Theorem 3: Assume (A1) - (A5). (Alternatively assume (A1)-(A7).) There exists a unique set of payoff equivalent subgame perfect equilibria satisfying (FI) of the bargaining game (stage 2), given the capacity choices $l \in L^M$ and the realization of the matching technology. Provided $U(l) \geq 0$, the expected payoff to the firms $i \in M$ are, irrespective of T, given by

$$\pi^i_T(\sigma \mid (l,f)) = \alpha l_i + \alpha(1-\alpha) \sum_{k=0}^{l_i-1} \sum_{s=0}^{f_i-l_i-1} \binom{s+k}{k} \alpha^k (1-\alpha)^s$$

for all $\sigma \in PE \cap FI$, where l_i denotes capacity of firms and f_i denotes the length of firm i's queue; $f = (f_1,...,f_M)$.

Proof: Follows from Lemmas 1–2 and 6–7.

∎

Until this point there is no difference between dropping (A6) and assuming (A6)and (A7), because only capacity choices satisfying U(l) ≥ 0 have been considered. Taking into account the possibility of excess capacity, U(l) < 0, changes things. If (A6) is dropped, behavior under excess capacity can be entirely different from the case where (A6) and (A7) holds. Since this is not at the focus of the paper, we did not solve the game without assuming (A6) under excess capacity. Retaining (A6) and assuming (A7) makes the case of excess capacity, U(l) < 0, equivalent to the "bilateral monopoly" described in Lemma 2. Then, by the same argument as in Lemma 4, it is easy to show that excess capacity cannot be a CN -equilibrium of the first stage of the game. With this result at hand, the payoffs given in Theorem 3 can be used to define an M person normal-form game with the space of capacities , L, as strategy spaces for the first stage of the game. The Nash-Theorem then ensures existence of an equilibrium in this normal-form game at least in mixed strategies (Nash, 1951) and this equilibrium will always have U(l) ≥ 0.

The point we wish to emphasize here is, however, that the increased mobility of workers in the present section does not neccessarily lead to lower unemployment. On the contrary, it is possible to show that the sufficient condition for positive unemployment in Section 4, namely $\alpha - c \leq 0$, is also sufficient for positive unemployment under increased mobility; but moreover, it is possible to show that in the present version unemployment may exceed M, the upper bound for U(l) in Section 4. This claim is independent of whether (A6) is dropped or retained and amended by (A7).

Proposition 1: For any CN-equilibrium in pure strategies,

$\alpha - c \leq 0$ implies U(l) > 0.

Proof: Assume to the contrary for $\alpha \leq c$ there exists a CN−equilibrium in pure strategies satisfying $U(l) \leq 0$. There are two possible cases:

(i) $U(l) < 0$: In the present variant of the game the workers always have the option of imitating their behavior from Section 4 (although they may now do better than that). But then the firms' payoffs cannot exceed the "bilateral monopoly" payoffs from Lemma 2 and are, consequently, non−positive. By the hypothesis $U(l) < 0$ there must be at least one firm which is rationed with (strictly) positive probability by the matching technology, (A1). This firm will then certainly make a loss (in terms of expected payoff) and can do better by reducing capacity and saving the cost $c > 0$ − a contradiction.

(ii) $U(l) = 0$: Let $F(l) = \max\{n \in N^* \cup \{0\} \mid n \leq U(l)/M\}$, where N^* is the set of integers, and denote by $P(l_i, l_{-i})$, $l_{-i} = (l_1, \ldots, l_{i-1}, l_{i+1}, \ldots, l_M) \in L^{M-1}$, the net (of capacity cost) expected payoff to firm $i \in M$ which is from Theorem 3 and the matching technology

$$P(l_i, l_{-i}) = (\alpha - c)l_i + \alpha(1-\alpha) \sum_{j=0}^{l_i-1} \sum_{s=0}^{F-1} \binom{s+j}{j} \alpha^j (1-\alpha)^s + \alpha[U/M - F] \sum_{j=0}^{l_i-1} \binom{F+l_i}{j} \alpha^j (1-\alpha)^{F+li-j}$$

where $F = F(l)$ and $U = U(l)$. In the present case $U = F = 0$ implies

$$P(l_i, l_{-i}) = (\alpha - c)l_i \leq 0.$$

Using the definition of expected payoffs, there exists a z, $z \in \{1, \ldots, \min(l_i.M-1)\}$ such that

$$(\alpha - c)(l_i - z) + M^{-1}\alpha z(1 - \alpha^{li-z}) > (\alpha - c)l_i\,,$$

making a downward deviation in capacities (increasing unemployment) profitable − again a contradiction. The conclusion is the statement of the Proposition.

∎

In Section 4 unemployment remained bounded from above by M for all parameter values. In the present version the increased mobility of workers gives them a more favorable strategic position and, thereby, forces the firms to develop more severe threats against excessive wage claims – an intuition which is at variance with the standard intuition that higher mobility leads to a more efficient allocation of factors of production. An example demonstrates this claim:

Let $M = 2$, $N = 4$ and $\alpha = 1/2$, $c = 3/4$. The claim is that $l_1 = 0$ and $l_2 = 1$, implying $U(l_1,l_2) = 3$, is an equilibrium. Since $\alpha < c$ Proposition 1 ensures that $U(l_1,l_2) \geq 1$ in any equilibrium. From the expected payoffs in Proposition 1 we have

$$P_1(0,1) = 0,\ P_1(1,1) = 0,\quad P_1(2,1) = -5/16.$$

$$P_2(0,0) = 0,\ P_2(1,0) = 1/16,\ P_2(2,0) = -1/8\quad P_2(3,0) = -\ 17/32\ ,$$

verifying that $(l_1,l_2) = (0,1)$ is an equilibrium with $U(0,1) = 3 > 2 = M$. In this examople high capacity costs jointly with the higher mobility of workers lead to unemployment exceeding M. Clearly this conclusion holds irrespective of wether (A6) is dropped or adopted jointly with (A7).

To conclude some remarks are presented:

1. In the version of the game of the present section discounting together with a proper definition of the length of periods may indeed change our results. Although all results would go through for a sufficiently large discount factor ("very little" discounting), for low discount factors Lemma 7 may not go through anymore. The reason is that the series of inequalities between expected payoffs to the firm from certain classes of histories may not hold for some sufficiently long time horizon: At some point in the game it may become favorable to the firm to quit the bargaining process after a rejection (instead of continuing to bargain with the present worker), thereby introducing a non – stationarity in the firm's behavior in equilibrium. An extra discounting friction in addition to the finiteness friction may change results in Section 5, if the discounting friction is sufficiently strong.

2. Now suppose (A6) <u>and</u> (A3) are dropped . This is again a case which sheds doubt on a familiar line of reasoning: One may argue that workers could unionize and agree to lower their wage claims

sufficiently in order to induce the firms to generate full employment. But in the present model strongly unionized workers (where "unionizations" also encompasses potential unemployed) would not have to lower their wage claims below the full employment wage $(1-\alpha)$. Unionized workers could force full employment for all parameter values $\alpha > c$ by playing the following strategy: If there is full employment everything works as described above; whenever a firm deviates and produces unemployment, any unemployed assigned to this firm will immediately move away from it, thereby bringing the firm into a "bilateral monopoly" situation, where the firm can again only get α per job and has wasted jobs. Against all other firms workers would ask $w = (1-\alpha)$ and accept no offer $1 - \pi < (1-\alpha)$; if a firm refuses to accept $w = (1-\alpha)$ or asks $\pi > \alpha$, all unemployed in the firm's queue would run away from this firm, thereby again bringing the firm into the "bilateral monopoly" situation. This is credible behavior and also satisfies (FI). Hence strong unionization of workers can force efficiency in terms of unemployment, without requiring the workers to "give in " in terms of wages.

6. Conclusions

The present paper studies a market, where the suppliers of indivisible "jobs" can incur a binding commitment, limiting the number of possible trades, at a certain cost. While the possibility of this commitment by capacity choice works in favor of the "firms", the costs of capacity are sunk during the negotiations on "wages". The latter effect forces the "firms" to deter excessive wage claims by reducing capacity in order to generate credible threats. This may result in a non-market clearing capacity choice in equilibrium. It is shown that such an inefficiency cannot be ruled out and that, moreover, it does not necessarily decrease with higher mobility.

In the course of the derivation a refinement of subgame perfection is used which emphasizes the possibility of coordinating the various agents of one single player. Relying on this refinement it is shown that the bargaining behavior is rather sensitive to the specification of outside options, even if the payoff to the player, who uses his outside option, is always zero: The effect of playing the outside option on the opponent can make a difference. Only a proper specification of the outside options, derived from considering the environment in which bargaining takes place, can lead to broader economic conclusions on efficiency. How outside options are specified also determines whether the "Walrasian" disequilibrium-argument, that the short side of the market aquires all the surplus, holds: Without the extra friction of mobility costs there is no such "jump" to the boundary, but rather the division of the surplus depends smoothly on excess supply. The reason for this inertia, when there are no costs of mobility, is that excess supply cannot completely eliminate competition between players on the short side of the market.

In one sense the game presented here is merely another example of a bargaining model. In another sense it is a step towards refining our view on how markets operate by analyzing simple models which display, in a stylized fashion, the main characteristics of certain types of markets in a world with frictions.

Appendix

Proof of Corollary 1: (For simplicity again we will drop the index i resp. j in the sequel.)

Substituting l=T+1−t−x−y in the formula of Lemma 3 yields for finite T

$$\pi_{T+1-t}(\sigma \mid h_t) = \alpha l + \alpha(1-\alpha) \sum_{k=0}^{T-t-l} \sum_{s=0}^{l-1} \binom{s+k}{k} \alpha^s (1-\alpha)^k$$

Now

$$\sum_{k=0}^{T-t-l} \sum_{s=0}^{l-1} \binom{s+k}{s} \alpha^s (1-\alpha)^k = \sum_{s=0}^{l-1} \alpha^s \sum_{k=0}^{T-t-l} \binom{s+k}{s} (1-\alpha)^k =$$

$$= \sum_{s=0}^{l-1} \alpha^s \sum_{k=s}^{T-t-l+s} \binom{k}{s} (1-\alpha)^{k-s} = \sum_{s=0}^{l-1} \alpha^s \sum_{j=0}^{T-t-l} \binom{T+1-t-l+s}{j} (1-\alpha)^j \alpha^{T-t-l-j} =$$

$$= \sum_{s=0}^{l-1} \alpha^{-1} \sum_{j=0}^{T-t-l} \binom{T+1-t-l+s}{j} (1-\alpha)^j \alpha^{T+1-t-l+s-j} \longrightarrow$$

$$\xrightarrow[T \to \infty]{} l / \alpha$$

using the following manipulation:

$$\sum_{k=n}^{z} \binom{k}{n} x^{k-n} = \sum_{k=0}^{z-n} \binom{z+1}{k} x^k (1-x)^{z-n-k} , \qquad |x| < 1 .$$

for all n,z = 0,1,2,..., $z \geq n$, which can be proved by induction over n.

∎

<u>Proof of Theorem 2</u>: Restrict U(l) to $0 \leq U(l) \leq M$. The choice l_i of firm i has to satisfy

$$l_i[\alpha - c + M^{-1}(N - \Sigma_{j\in M} l_j)(1-\alpha)] \;\geq\; (l_i+s)[\alpha - c + M^{-1}(N - \Sigma_{j\in M} l_j - s)(1-\alpha)]$$

for all $s \in \{-l_i,...,-1,1,...U(l)\}$, all $i \in M$. Manipulating these inequalities yields

$$(2) \quad ½[N + M(\alpha-c)(1-\alpha)^{-1} + 1 - \Sigma_{j\neq i} l_j] \geq l_i \geq ½[N + M(\alpha-c)(1-\alpha)^{-1} - 1 - \Sigma_{j\neq i} l_j]$$

which is equivalent to

$$(2') \quad M(\alpha-c)(1-\alpha)^{-1} + U(l) + 1 \geq l_i \geq M(\alpha-c)(1-\alpha)^{-1} + U(l) - 1.$$

In (2) and (2') the left hand inequalities ensure that downward deviations from l_i are unprofitable, while the right hand inequalities ensure that upward deviations from l_i are unprofitable. Note that a manipulation of (2) yields the main characterization of U(l) in the Theorem. It remains to show existence.

Parameter constellations may fall into three cases:

<u>1. case:</u>

$M < (M/(M+1))(N/M - M(\alpha-c)(1-\alpha)^{-1} + 1)$ which is equivalent to $N/M - 1 > M(\alpha-c)(1-\alpha)^{-1} + M - 1$ such that (2') implies that no upward deviation from $l_i = N/M - 1$, for all $i \in M$, can be profitable. This implies that $U(l) = M$. Because of (A5) $l_i = N/M - 1$ is therefore an equilibrium on the boundary with $U(l) = M$.

<u>2. case:</u>

$0 > (M/(M+1))\,(N/M - M(\alpha-c)(1-\alpha)^{-1} - 1)$ which is equivalent to $N/M < M(\alpha-c)(1-\alpha)^{-1} + 1$ in this case (2') implies that no downward deviation from $l_i = N/M$ for all $i \in M$ can be profitable, yielding by (A5) the second boundary equilibrium with $U(l) = 0$.

<u>3. case:</u>

The first and the second case are simultaneously violated which is equivalent to

$1 \leq N/M - M(\alpha-c)(1-\alpha)^{-1} \leq M$. In this case pick an integer U such that

$N/M - M(\alpha-c)(1-\alpha)^{-1} \geq U \geq N/M - M(\alpha-c)(1-\alpha)^{-1} - 1;$

clearly such an integer always exists and always satisfies $M \geq U \geq 0$.

Now set $l_i = N/M-1$, for all $i = 1,...,U$ and $l_i = N/M$, for all $i = U+1,...,M$. This choice yields

$M(\alpha-c)(1-\alpha)^{-1} + U + 1 > N/M - 1 = l_i \geq M(\alpha-c)(1-\alpha)^{-1} + U - 1$, for all $i = 1,...,U$. and

$M(\alpha-c)(1-\alpha)^{-1} + U + 1 \geq N/M = l_i > M(\alpha-c)(1-\alpha)^{-1} + U - 1$, for all $i = U+1,...,M$ and

U(l) = U, satisfying (2') which confirms that an equilibrium has been constructed. (A5) makes matters simple, because it makes it unneccessary to search for close integers in the intervals defined by (2'); without (A5) the argument would, however, not be altered.

Footnotes:

1 After completion of the present paper we have learned about the concept of a strategic equilibrium by Wolfgang Leininger (1988) which is slightly stronger and more general than our definition of (FI), but captures essentially the same logic as (FI) does. The present definition of (FI) can, in this light, be seen as an adaption of strategic equilibrium to the present game.

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