THE TERM STRUCTURE OF INTEREST RATES:
A FIRST LOOK AT THE AUSTRIAN CASE*

Albert Jäger

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Die in diesem Forschungsbericht getroffenen Aussagen liegen im Verantwortungsbereich des Autors und sollen daher nicht als Aussagen des Instituts für Höhere Studien wiedergegeben werden.
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ABSTRACT

First, the basic idea of the expectations theory on the term structure of interest rates is sketched. A brief review of empirical evidence on the expectations theory is presented. The paper then derives and estimates a term structure equation for Austrian interest rates. The empirical results for this equation seem not to be at variance with the basic idea of the expectations theory, i.e., representing the long term interest rates as a weighted average of expected short rates. A little bit of evidence for testing bond market efficiency is also presented. The aim of this paper is to see whether the specific version of the expectations theory used in this paper is reasonably in accord with the available data.

ZUSAMMENFASSUNG

I. Introduction

The term structure of interest rates has been the subject of many careful empirical studies during the last 25 years. The abundance and good quality of interest rate data, at least outside Austria, combined with strong testable restrictions implied by term structure theories, have naturally attracted some spirited research efforts. Incidentally, the relationship between long term and short term interest rates is of considerable importance for understanding the effects of debt management and monetary policies.

Curiously enough, as far as the present author is informed, no empirical work based squarely on established term structure theory has been published for Austrian interest rates.\(^1\) The purpose of this paper is to take a first step towards remedying of this unhappy state of affairs.

The basic aim of term structure theories is to explain the relationship between yields pertaining to riskless homogeneous bonds differing only in their maturities. A special case of this general problem is the explanation of the relationship between a representative long term and short term interest rate. The following notation is used: \(R_{Nt}\) is the yield to maturity at time \(t\) of a bond maturing at time \(t+N\). Correspondingly, \(R_{1t}\) denotes the current yield of a bond maturing at the end of time period \(t\). Further, let \(F_{1t}\) be the, hypothetically, one-period forward rate that at time \(t\) pertains to one-period loans to be made at time \(t+m\) and mature at time \(t+m+1\). These rates should be considered as hypothetical, because no actual forward contracts of this kind usually exist.

With this notation at hand, the essential idea of most work on term structures
may be described as assuming that the long rate $R_{nt}$ can be approximated by an average of the current one-period yield $R_{lt}$ and the hypothetical one-period forward yields ranging from time $t+1$ to time $t+N-1$: \(^2\)

\[
R_{nt} = \frac{1}{N} \left[ R_{lt} + t+1F_{lt} + t+2F_{lt} + \cdots + t+N-1F_{lt} \right]
\]

Given corresponding equations like (1) for all $n = 2, 3, \ldots, N$ and data for all $R_{nt}$ the forward rates $t+mF_{lt}$ are uniquely defined and may be explicitly calculated. Thus, equation (1) should, at this point, only be interpreted as a definitional equality devoid of any predictive content.

The expectations theory of the term structure gives predictive power to the basic idea outlined above by postulating that the hypothetical forward rates $t+mF_{lt}$ are forced into equality with the expected forward rates $\hat{R}_{lt+m}$ held by market participants at time $t$:

\[
t+mF_{lt} = \hat{R}_{lt+m} \quad \text{For } m = 1, \ldots, N-1
\]

This "pure" version of the expectations theory has been supplemented by different authors to include liquidity premiums for longer term bonds and inflation expectations. Yet the basic idea of expected short rates determining long rates has always been the fundamental assumption guiding the literature on the term structure phenomenon. \(^3\)

The paper is organised as follows. The next section contains a brief review of empirical evidence presented by various authors. In section III the term structure equation, which is to be estimated, is derived. Also contained in this section is
the rationale for a specific procedure designed to test for market efficiency in the bond market. Section IV reports empirical results and evaluates the evidence. Section V presents the conclusion.

II. A Brief Review of Empirical Evidence

The various tests of the expectations theory reveal quaint cycles of successful verification versus resounding rejection. Interestingly, development and application of new statistical tools coincide to a large extent with these cycles. All studies mentioned below use U. S. data on interest rates.

The first rigorous empirical study of the expectations theory was published by Meiselman (1962). He found strong empirical evidence for the pure version of the theory, assuming an error learning process governing the expectation formation process for future short rates. This unanimous support for one specific version of the expectations theory was slightly qualified by other authors. They found a significant liquidity premium to be present in the data. The liquidity premium augmented version was successfully tested by Roll (1970), who suggested that certain sequences of forward interest rates should follow a martingale. The martingale property follows from assuming expectations of forward rates being formed rationally in the sense of Muth (1961). The rational expectations hypothesis, not to be confused with the expectations theory of the term structure, has since become a standard assumption in this literature.
Sargent's (1972) paper used the framework developed by Roll, but based on sophisticated testing procedures, he rejected Roll's hypothesis. The work by Modigliani and Shiller (1973) was primarily directed towards constructing term structure equations designed to be used in econometric models. They found good statistical fits for their version of the expectations theory, technically known as "Preferred Habitat Model". The authors also presented some positive evidence that expectations in the bond market are formed rationally.

Mishkin (1978a, 1978b) tested for bond market efficiency by using implications of the expectations theory cum rational expectations. He interpreted his positive evidence as supporting the argument of the Lucas critique (Lucas, 1976) on econometric policy simulation. Term structure equations with built in rational expectations will not remain invariant to a discretionary change in monetary policy. Thus these equations should not be used to calculate the presumable effects of a short term interest rate reduction, say, on long term interest rates and GNP.

An enduring wave of negative evidence for the pure version of the expectations hypothesis was initialized by Shiller (1979). He presented evidence that long term interest rates, compared to short term interest rates, are "too volatile" to be determined by equations like (1) and (2). These "variance bound" tests were criticized by Flavin (1984) as being statistically biased. Flavin's approach, based on calculating the response of long rates to innovations in short rates and inflation, generally confirmed the implications of the pure expectations theory.
III. The Model

The specification of the term structure equation in this paper is similar to that developed by Modigliani and Shiller (1973). The maturity spectrum is reduced to the relationship between one long rate and one short rate, both being considered as representative for the "short" and the "long" maturity region. From a theoretical point of view, the model to be developed consists of the pure version of the expectations theory supplemented with a constant liquidity premium and the assumption of rational expectations.

According to the expectations theory, the long term interest rate $R_t$ is determined by a weighted average of expected forward short term rates. Assuming risk averse agents in the bond market, a time independent liquidity premium $K'$ is added, reflecting the reward necessary to compensate agents for binding them long.\(^{5}\)

\begin{equation}
R_t = a_0 R_{lt} + \sum_{m=1}^{N-1} a_m A_{lt+m} + K'
\end{equation}

The expectations held by market participants of forward rates are assumed to be formed rationally. This amounts to setting the expected forward rates equal to their conditional expectation at time $t$:

\begin{equation}
\hat{R}_{lt+m} = E_t[R_{lt+m}/\Omega_t] \quad \text{For } m = 1, \ldots, N-1
\end{equation}

Here $E_t$ denotes the expectations operator applied to the probability distribution of $R_{lt+m}$ conditional on the information set $\Omega_t$. To derive operational expressions for the $\hat{R}_{lt+m}$, one has to be specific about the probability
distribution which generates the $R_{1t+m}$ as well as on the content of the information set $\mathcal{I}_t$. The hypothesis of rational expectations essentially postulates that agents know the "true" stochastic mechanism generating the variable of interest. Furthermore, agents use all "relevant" information known to them at time $t$ to predict future realisations.

The two specification problems thus posed (concerning stochastic mechanism and relevant information set) are solved in this paper somewhat boldly. First, following Modigliani and Shiller (1973), the expected forward rate $\hat{R}_{1t+m}$ is decomposed into an "inflation component" $\hat{P}_{1t+m}$ and a "real component" $\hat{I}_{1t+m}$:

$$(5) \quad \hat{R}_{1t+m} = \hat{P}_{1t+m} + \hat{I}_{1t+m} \quad \text{For all } m = 1, \ldots, N-1$$

This decomposition may be justified on purely intuitive grounds. If prices have been subject to considerable variability in the past, it seems reasonable to suppose that expected nominal interest rates will include an inflation component not adequately accounted for in past nominal rates. Whether this conjecture really applies is, of course, a matter of empirical testing.

Expectations for both components of the nominal interest rate are modeled separately, assuming the two time series to be generated by stationary autoregressive processes of infinite order with coefficients constant over time.

$$(6) \quad \begin{align*}
P_{1t} &= \alpha_0 + \alpha_1 P_{1t-1} + \alpha_2 P_{1t-2} + \cdots + \varepsilon_{1t} \\
I_{1t} &= \beta_0 + \beta_1 I_{1t-1} + \beta_2 I_{1t-2} + \cdots + \varepsilon_{2t}
\end{align*}$$

where the $\varepsilon_{it}$ are assumed to be i.i.d. random errors obeying:
\[ E(\varepsilon_{it}) = 0 \]

and \( E(\varepsilon_{it}, \varepsilon_{jt}) = \begin{cases} \sigma_i^2 & i = j \\
0 & i \neq j \end{cases} \)

The expected value of \( R_{lt+1} \) is given by:

\[ R_{lt+1} = E_t (P_{lt+1} + I_{lt+1}) = \left( \alpha_0^{(n)} + \beta_0^{(n)} \right) + \sum_{j=1}^{\infty} \left( \alpha_0^{(n)} P_{lt+1-j} + \beta_0^{(n)} I_{lt+1-j} \right) \]

The upper symbols of the coefficients indicate that (7) is the forecasting equation for \( t+1 \).

The rational expectations hypothesis implies that expectations for the other future rates (i.e. for \( m > 2 \)) should be built up recursively according to Wold's (1963) "chain rule of forecasting". This implication of rational expectations is known as the "consistency property". The expected value for \( R_{lt+m} \), where \( 1 \leq m \leq N-1 \), is:

\[ R_{lt+m} = \left( \alpha_0^{(n)} + \beta_0^{(n)} \right) + \sum_{j=1}^{\infty} \left( \alpha_0^{(n)} P_{lt+1-j} + \beta_0^{(n)} I_{lt+1-j} \right) + \sum_{j=m}^{\infty} \left( \alpha_0^{(n)} P_{lt+1-j} + \beta_0^{(n)} I_{lt+1-j} \right) \]

Inserting recursively for the expected \( P_{lt+m} \) and \( I_{lt+m} \) from their respective forecasting formulae, the expectation equation (8) has an equivalent representation in terms of current and past \( P \)'s and \( I \)'s only of:
(9) \[ \hat{R}_{lt+m} = \alpha_{l}^{(m)} + \sum_{j=1}^{m} \alpha_{3j}^{(m)} R_{l-t+j} + \sum_{j=1}^{m} \alpha_{3j}^{(m)} P_{l-t+j} \]

The weights \( \alpha_{3j}^{(m)} \) (and \( \beta_{3j}^{(m)} \) analogously) are recursively built up according to (a fact easily proved by the method of undetermined coefficients):

(10) \[ \alpha_{3j}^{(m)} = \alpha_{m-1+j}^{(m)} + \sum_{i=0}^{m-3} \alpha_{3i}^{(m)} \alpha_{3j}^{(m-1-i)} \]

Having derived suitable expressions for the \( \hat{R}_{lt+m} \) before inserting them in equation (3), the non-observable real rate \( I_{lt} \) is everywhere replaced by the definitional equality:

(11) \[ R_{lt+m} - P_{lt+m} = I_{lt+m} \]

Replacing the so defined \( I_{lt+m} \) in all forecasting equations and plugging these equations into (3) we get the sought for term structure equation:

(12) \[ R_{t} = \sum_{j=0}^{m} b_{j} R_{l-t+j} + \sum_{j=0}^{m} c_{j} P_{l-t+j} + K \]

where the coefficients \( b_{j}, c_{j} \) and \( K \) are defined as:

\[ b_{0} = a_{0} + \sum_{i=1}^{N} a_{i} \beta_{i}^{(m)} \quad K = K' + \sum_{i=1}^{N} \alpha_{i} [\alpha_{0}^{(i)} + \beta_{0}^{(i)}] \]

(13) \[ b_{j} = \sum_{i=1}^{N} a_{i} \beta_{i}^{(m)} \quad j = 1, \ldots \]

\[ c_{j} = \sum_{i=1}^{N} a_{i} (\alpha_{i}^{(m)} - \beta_{i}^{(m)}) \quad j = 0, 1, \ldots \]

Section IV reports the estimation results for equation (12).
The empirical evidence found by estimating equations like (12) is useful for judging the adequacy of the specific term structure theory assumed in terms of statistical fit.\textsuperscript{9}) The so-called "efficient market approach" provides a framework for testing the twin assumption of long rates being an average of expected short rates and of expectations being formed rationally. Rewriting equation (1) and substituting the rationally expected values for the forward rates, we get:

\begin{equation}
R_t = \frac{1}{N} \left[ R_{1,t} + E_t R_{1,t+1} + E_t R_{1,t+2} + \ldots + E_t R_{1,t+N-1} \right]
\end{equation}

where the conditioning information sets \( \mathcal{F}_t \) have been omitted for notational convenience. We also left out the constant liquidity premium, because it will not affect the argument. Lagging (14) by one time period and subtracting the resulting equation from (14) gives:

\begin{equation}
R_t - R_{t-1} = \frac{1}{N} \left[ R_{1,t} - E_{t-1} R_{1t} + (E_t R_{1t+1} - E_{t-1} R_{1t+1}) + \ldots + (E_t R_{1t+N-2} - E_{t-1} R_{1t+N-2}) + \frac{1}{N} (E_{t-1} R_{1t+N-1} - R_{1t-1}) \right]
\end{equation}

The revision terms in brackets are "innovations" representing news, and as an implication of rationality their expectation at time \( t-1 \) is zero. Equation (15) may therefore be written as:

\begin{equation}
R_t - R_{t-1} = \epsilon_t + \frac{1}{N} \left[ E_{t-1} R_{1,t+n-1} - R_{1,t-1} \right]
\end{equation}

If \( N \) is sufficiently large, the second term on the right side of (16) may safely be set to zero, if \( E_{t-1} R_{1,t+N-1} - R_{1,t-1} \) does not become infinite (normally...
ruled out by a terminal condition). Thus the testable result emerges that the difference between successive long rates should be uncorrelated with any information known by agents at time t-1. The reason for this implication is that \( R_{t-1} \) has built into it all the information known at t-1, because of the rational expectations assumption. Correlation between \( (R_t - R_{t-1}) \) and information known at t-1 would imply unexploited arbitrage opportunities if the costs of arbitrage are negligible. This implication of an efficient bond market is tested in section IV.

IV. Empirical Results

The first set of empirical results in table 1 represents estimates of the term structure equation (12). Some assumptions concerning the infinite distributed lags for \( R_{lt} \) and \( P_{lt} \) in equation (12) have to be made, before actual estimation is possible. For the lag length, eight quarters were chosen after comparing the \( R^2 \)'s of unconstrained regressions using lag lengths between 5 and 10. Two estimation results are reported. The first equation (E-1) has been estimated by Ordinary Least Squares to have an unconstrained benchmark estimate for the chosen 8-quarter lag. The second equation (E-2) used the polynomial distributed lag technique developed by Almon (1965). Before discussing the results, a short comment on the data used may be helpful.

The long term interest rate (RL) is derived from a yields series of Federal Government Bonds with maturity 7-8 years. This is certainly not a very long term rate, but seems currently to be the only relevant rate available for Austria. The short rate is also derived from federal bond yields, but with a maturity of 1-2 years. The non-existence of analogues to US-Treasury Bill rates for Austria
Table 1: Estimation Results for Term Structure Equation (1976.1-1984.4)

\[ R_t = K + \sum_{j=0}^{3} b_j R_{t-j} + \sum_{j=0}^{6} c_j P_{t-j} \]

<table>
<thead>
<tr>
<th>Lag</th>
<th>RS</th>
<th>P</th>
<th>RS</th>
<th>P</th>
</tr>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>.658</td>
<td>.055</td>
<td>.649</td>
<td>.041</td>
</tr>
<tr>
<td></td>
<td>(10.3)</td>
<td>(2.0)</td>
<td>(12.5)</td>
<td>(2.3)</td>
</tr>
<tr>
<td>1</td>
<td>.130</td>
<td>.003</td>
<td>.129</td>
<td>.027</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(.11)</td>
<td>(3.1)</td>
<td>(2.6)</td>
</tr>
<tr>
<td>2</td>
<td>-.167</td>
<td>.044</td>
<td>-.080</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td>(1.8)</td>
<td>(1.3)</td>
<td>(2.2)</td>
<td>(2.5)</td>
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<tr>
<td>3</td>
<td>.022</td>
<td>.018</td>
<td>-.106</td>
<td>.031</td>
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<td></td>
<td>(.26)</td>
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<td>(3.5)</td>
<td>(2.9)</td>
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<td>.057</td>
<td>-.050</td>
<td>.038</td>
</tr>
<tr>
<td></td>
<td>(.81)</td>
<td>(1.9)</td>
<td>(1.5)</td>
<td>(3.1)</td>
</tr>
<tr>
<td>5</td>
<td>-.050</td>
<td>.022</td>
<td>.019</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>(.72)</td>
<td>(.69)</td>
<td>(.76)</td>
<td>(4.6)</td>
</tr>
<tr>
<td>6</td>
<td>.029</td>
<td>.041</td>
<td>.061</td>
<td>.031</td>
</tr>
<tr>
<td></td>
<td>(.41)</td>
<td>(1.3)</td>
<td>(2.0)</td>
<td>(3.4)</td>
</tr>
<tr>
<td>7</td>
<td>.169</td>
<td>.009</td>
<td>.066</td>
<td>.009</td>
</tr>
<tr>
<td></td>
<td>(2.2)</td>
<td>(.31)</td>
<td>(1.9)</td>
<td>(.82)</td>
</tr>
<tr>
<td>8</td>
<td>-.003</td>
<td>-.031</td>
<td>.053</td>
<td>-.033</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
<td>(1.4)</td>
<td>(1.5)</td>
<td>(2.2)</td>
</tr>
<tr>
<td>Sum</td>
<td>.725</td>
<td>.218</td>
<td>.741</td>
<td>.209</td>
</tr>
<tr>
<td>Const.</td>
<td>1.397</td>
<td></td>
<td>1.309</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.9)</td>
<td></td>
<td>(2.7)</td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>.110</td>
<td></td>
<td>1.115</td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>.987</td>
<td></td>
<td>.986</td>
<td></td>
</tr>
<tr>
<td>D.W.</td>
<td>2.106</td>
<td></td>
<td>2.190</td>
<td></td>
</tr>
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</table>

Number in parenthesis are t-ratios.
enforced the choice of this series for representation of the short region of the maturity spectrum. I did some experiments substituting money market rates for the short term rates. Although the results are quite respectable, the insertion of money market rates in term structure equations is hard to justify theoretically. The data on the inflation rate are based on the consumer price index. All the data and their sources are described in the data appendix.

Following a suggestion by Maddala (1977) for the estimation of distributed lag models, equation (12) has first been estimated by OLS. This unconstrained "garbage equation" E-1 is mainly useful for comparison with the ultimately "preferred" equation E-2, because it offers some first insights into the peculiarities of the data and the effects of the constraints imposed on the distributed lag. The high degree of multicollinearity among the regressors is likely to result in imprecise OLS-estimates of the lag coefficients, so that an interpretation of the estimates will not be very informative.

Equation E-2 has been estimated by the Almon-technique assuming the lag weights to follow a $4^{th}$-degree polynomial for both independent variables with no start or end restrictions imposed. Assuming the lag length of 8 quarters being appropriate, the $4^{th}$-degree polynomial has been selected among the possible specifications as the one that minimizes the standard error of regression. Generally, the results for different polynomials are surprisingly robust as far as the magnitude of the regression standard errors are concerned but the weighting pattern for the coefficients after the first two lags at times change drastically.

The estimated coefficients for the short rate exhibit a pattern typical for an extrapolative-regressive expectation formation process. The current value of the short rate gets a relatively high weight (.65) and the coefficients turn
negative from the second to fourth lag and become positive again. The sharp dip
to the negative after two lags depicts the extrapolative component, whereas the
leveling out at positive weights is sign of a regressive component, i.e. people
expect the short rate to return to some "normal level". The inflation rate reveals
a uniform lag pattern, the different lag coefficients getting about the same
weights. The positive constant should not be interpreted as representing the
liquidity premium. I tried to proxy a variable liquidity premium by inserting a
standard moving average of the short rate (stretching 8 quarters back) as a
measure of interest rate uncertainty, but with no sensible estimation results
turning up. As the estimation period has been characterized by unprecedented
fluctuations in interest rates, the fit of the equation is reasonably good. The
question is, of course, whether the equation would stay in "good shape" if longer
data series were available.

The results presented so far should not be considered as spectacularly convincing
evidence for the theory used to derive the term structure equation. Especially,
the relatively short estimation period and the choice of the short interest rate
series leaves some doubts open. On the other hand, the available data seem not
to be obscenely at variance with the notion of representing the long term
interest rate as a weighted average of expected short rates. The problem with
evaluating evidence of this kind is the absence of an explicitly stated alternative
hypothesis.

The next part of this section reports on tests of the market efficiency
implication described in section III. Put simply, the implication to be tested is
Table 2: Tests for Market Efficiency

The change in the long term interest rate ($\Delta R_t$) has been regressed on known information about economic variables at time $t-1$. The typical regression is:

$$\Delta R_t = a_0 + a_1 X_{t-1} + \ldots + a_4 X_{t-4} + e_t \quad e_t \sim N(0, \sigma^2)$$

where $X_t$ denotes the information variable. Four lags of the respective variable were included in each regression.

Results:

<table>
<thead>
<tr>
<th>Information Variable</th>
<th>F-Statistik</th>
<th>Prob. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Run Interest Rate</td>
<td>$F(5,31) = 3.53^*$</td>
<td>1.55%</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>$F(5,30) = 3.09^*$</td>
<td>2.23%</td>
</tr>
<tr>
<td>Eurodollar Bond Rate</td>
<td>$F(5,30) = 2.88^*$</td>
<td>7.15%</td>
</tr>
<tr>
<td>GNP-Growth Rate (real)</td>
<td>$F(5,31) = 1.83$</td>
<td>13.44%</td>
</tr>
<tr>
<td>Money Growth Rate (M2)</td>
<td>$F(5,30) = 1.28$</td>
<td>29.50%</td>
</tr>
</tbody>
</table>

All data quarterly.

*Significant at the 10% level.
that information known to market participants at time t-1 should not be correlated with $R_L$. Using ordinary regressions and F-tests, this implication may be tested for different variables from a possibly infinite information universe.  

Table 2 contains the results of the tests. Both variables on the right side part of the term structure equation, the inflation rate and the short rate, turn out to be significantly correlated with changes in the long term rate at the 5% significance level. The other three variables chosen for the test (GNP-growth, money-growth and the eurodollar bond rate) are not significant.

These results are prima facie evidence that the bond market does not use all available information at time t-1 to predict long term rates for time t: But it has to be kept in mind, that the test implication has been derived from rather restrictive assumptions, e. g., the assumption of a constant liquidity premium may not be justified, although experiments by the present author with standard moving deviations of the short rate as uncertainty proxies did not turn out to be significant. Begg (1982) has pointed out that a time dependent liquidity premium may account for negative tests of the sort presented in this paper.
V. Conclusion

The paper applies the expectations theory on the term structure of interest rates to Austrian interest rates data. A specific version of the expectations theory has been chosen to derive a term structure equation depicting the relationship between a representative long term interest rate and a representative short term interest rate.

The estimation results are sensible in terms of statistical fit and provide some evidence that the expectations theory is suitable for the explanation of the Austrian term structure. A test for bond market efficiency reveals however that a particularly restrictive version of the expectations theory (i.e. the version assuming rational expectations and a constant liquidity premium) does not hold.
Appendix: Sources of Data

Empirical work on the term structure of Austrian interest rates must draw on a particularly small data base. The interest rate data used in this paper are derived from Federal Bond yields (Rendite österreichischer Bundesanleihen) series, calculated by the Österreichische Kontrollbank AG. The quarterly series stretch from 1974.1 to 1984.3. Looking at the series from a yields curve perspective, the data reveal an extremely flat yield curve being present for the period of estimation. Actually, the average spread between the long and the short rate is only 7 basis points with a standard deviation of 21 basis points between 1976.1 and 1984.3.

Long term interest rate (RL): Yields from Federal Bonds with a maturity of 7 to 8 years. Observations are end of first month of the respective quarter. Source: Österreichische Kontrollbank AG.

Short term interest rate (RS): Yields for Federal Bonds with maturity 1 to 2 years. Source: Österreichische Kontrollbank AG.

Inflation rate (P): Quarterly change of Austrian consumer price index on an annual basis. Source: WIFO-data base.

Money growth rate (M2), Eurodollar bond rate and GNP-growth rate (real): All data quarterly from the WIFO-database.
NOTES

1) Data problems are presumably the main reason. There exists a paper on yield
curves, i.e., curves depicting graphically the spectrum of yields plotted against
the respective maturities, published by Pfeffer and Sauerschnigg (1980).

2) Formula (1) is derived from the perfect foresight condition that market
equilibrium for a bond with maturity \(N\) implies:

\[
(1+R_{Nt})^N = (1+R_{1t})(1+t_1F_{1t}) \ldots (1+t_{N-1}F_{1t})
\]

Taking logarithms of both sides:

\[
N \log(1+R_{Nt}) = \log(1+R_{1t}) + \log(1+t_1F_{1t}) + \ldots + \log(1+t_{N-1}F_{1t})
\]

If interest rates are measured decimals then \(\log(1+0.1)\) is close to \(0.1\) itself, thus
formula (1) emerges as an approximation. For a more accurate approximation if
\(N\) is large, see Modigliani and Shiller (1973), pp. 15-16.

3) This statement should be qualified with respect to work published by
Culbertson (e.g. 1965), who considered the bond market as being segmented into
partial markets according to the maturities of the bonds. He hypothesized that
only supply and demand in the segmented maturity classes determines the yield
of a bond in a specific class. Expected short rates would play a non-significant
role under these circumstances.

4) Maybe this is a case in point where the methods used for testing really "call
the tune" in the interplay between theory and data. A much neglected possibility
in discussions on the methodology of economics where it is usually asserted that
either theory selects the data (historians of science adhere often to this view) or
the data will select the theory (philosophers of science view).

5) For a discussion of the (microeconomic) conditions required to derive the
expectations theory, see St. Leroy, 1982, pp. 195-203.

6) Lucas and Sargent (1981, pp. XI-XVIII) discuss rationales for the hypothesis of
rational expectations on a broad "philosophical" level. The adjectives "true" and
"relevant" have been put under quotation marks, because their operational
meaning is controversial.

7) The assumption of autoregressive processes for the variables to be forecasted is
convenient but clearly ad hoc. The empirical literature using rational
expectations abounds with ad hocery of this kind, but there seems to be no easy
way out short of specifying an elaborate econometric model determining the
expected values "truly" endogenously. Such an approach is unlikely to be
implemented easily with existing econometric methods. For the intricacies
involved see Wallis (1980).

8) For linear predictions based solely on the history of the variable to be
forecasted, the chain principle of forecasting gives optimal predictions in
the least squares sense for $m \geq 1$.

9) Modigliani and Shiller (1973) do not assume expectations to be formed rationally as an assumption of their model, but test indirectly for rational expectations by recalculating the coefficients in the term structure equation after inserting optimal linear forecasts of $R_{t+m}$ and $P_{t+m}$. If expectations are rational, the coefficients should be the "same" as in the original equation.

10) All calculations have been performed with the IAS-econometric package developed at the Institute for Advanced Studies, Vienna.

11) These tests are similar in spirit to the orthogonality tests used for checking the observed expectations of agents for rationality. See e.g. B. Friedman (1980).
REFERENCES


