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Finite Sample Correction Factors for Panel Cointegration Tests

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Institut für Höhere Studien
Institute for Advanced Studies
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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share "work in progress" in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

In this paper we present finite T mean and variance correction factors and corresponding response surface regressions for the panel cointegration tests presented in Pedroni (1999, 2004), Westerlund (2005), Larsson et al. (2001), and Breitung (2005). For the single equation tests we consider up to 12 regressors and for the system tests vector autoregression dimensions up to 12 variables. All commonly used specifications for the deterministic components are considered. The sample sizes considered are $T \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 500\}$.

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Panel cointegration test, correction factor, response surface, simulation

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1 Introduction

In this paper we present finite T mean and variance correction factors and corresponding response surface regressions for the panel cointegration tests presented in Pedroni (1999, 2004), Westerlund (2005), Larsson *et al.* (2001) and Breitung (2005). For the single equation tests we consider up to 12 regressors and for the system tests vector autoregression (VAR) dimensions up to 12 variables. The considered sample sizes are $T \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 500\}$. The results are based on 100,000 replications of approximating the moments of the functionals of Brownian motions required in the different tests by the corresponding functionals of random walk series of length T. A detailed description of the simulation of the correction factors as well as the computer code in `GAUSS` are available upon request from the authors.

It has to be noted that the use of finite sample critical values or finite sample correction factors comes at a certain price for any unit root or cointegration test, both in a time series and panel setting. For finite time series dimension the exact finite sample test statistics typically depend on all characteristics of the data generating processes. The dependence upon these nuisance parameters vanishes only as $T \to \infty$. Thus, the advantages of using small sample correction factors based on small sample approximations of the limiting test statistics is to a certain extent offset by the finite sample nuisance parameter dependency. With respect to this tradeoff no general results are available in either the literature or in this paper. The general tendency in the literature is to value the gains due to the small sample approximations higher than the detrimental effects incurred due to nuisance parameter dependencies. Nevertheless, the potential drawbacks have to be kept in mind when using the correction factors and response surface regressions provided in this paper.

In the following two sections we briefly describe the implemented tests with a focus on the actual computation of the test statistics and the usage of the mean and variance correction factors. The single equation tests are discussed in Section 2 and the system tests in Section 3. A detailed description of the tests including a thorough discussion of the assumptions as well as simulation performance is contained in Wagner and Hlouskova (2010). Section 4 contains the results from response surface regressions to efficiently, albeit only approximately, summarize the information from the large set of correction factors. The full set of tables with the correction factors is given in the appendix.

2 The single equation tests

The single equation methods are panel extensions of the Engle and Granger (1987) approach to cointegration analysis with the DGP given by

$$
\begin{aligned}
y_{it} &= \alpha_i + \delta_i t + X_{it}'\beta_i + u_{it} & (1)\\
X_{it} &= X_{it-1} + \varepsilon_{it} & (2)
\end{aligned}
$$

observed for $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Here $y_{it}, u_{it} \in \mathbb{R}$, X_{it} and $\varepsilon_{it} \in \mathbb{R}^l$, $\alpha_i, \delta_i \in \mathbb{R}$ and $\beta_i \in \mathbb{R}^l$. Cointegration prevails in unit i, if u_{it} is stationary in which case the single cointegrating vector is given by $[1, -\beta_i']'$. To be able to write the DGP in a unified fashion for both cointegration and no cointegration we set $\beta_i = 0$ in (1) if there is no cointegration in unit i with the corresponding process u_{it} being integrated of order 1, see below.

The null hypothesis of all considered single equation tests is that of no cointegration in all units, i.e. the u_{it} are I(1) processes for $i = 1, \ldots, N$. In this case, when viewed as regressions to be estimated, equations (1) are spurious regressions under the null hypothesis.[1]

We consider the usual three cases for the deterministic variables: Case 1 without deterministic components, case 2 with only fixed effects α_i and case 3 with both fixed effects α_i and individual specific linear time trends $\delta_i t$.

Pedroni tests

The tests developed by Pedroni (1999, 2004) are based on testing for unit roots in the residuals of equation (1), $\hat{u}_{it}$ say, where in particular the OLS residuals of (1) can be chosen, see Phillips and Ouliaris (1990) for a discussion in the time series setting. Similarly to the time series case both regressor endogeneity and serial correlation have to be taken into account. Denote with $v_{it} = [\Delta u_{it}, \varepsilon_{it}']' \in \mathbb{R}^{l+1}$ the stacked stationary error processes in (1) and (2), with $\Omega_i = \begin{bmatrix} \omega_{ui}^2 & \Omega_{u\varepsilon i} \\ \Omega_{u\varepsilon i}' & \Omega_{\varepsilon i} \end{bmatrix}$ the (full rank) long-run covariance matrices of v_{it}, with $\omega_{u.\varepsilon i}^2 = \omega_{ui}^2 - \Omega_{u\varepsilon i}\Omega_{\varepsilon i}^{-1}\Omega_{u\varepsilon i}'$ the conditional long-run variances and with $\Lambda_i = \sum_{j=0}^{\infty} \mathbb{E} v_{it} v_{it-j}'$ the one-sided long-run variances, which are partitioned according to the partitioning of Ω_i.

The estimates $\hat{\omega}_{u.\varepsilon i}^2$ are given by the estimates of the long-run variances of the residuals, $\hat{\eta}_{it}$ say, of OLS regressions of Δy_{it} on the differenced deterministic components and ΔX_{it}. The

[1] The fact that equations (1) are spurious regressions in case of no cointegration implies, as is well known, that e.g. OLS estimation of β in (1) will generally lead to a non-zero limit even if the true $\beta = 0$ (in our notational convention).

estimates $\hat{\omega}^2_{u.\varepsilon i}$ can be obtained by using a kernel estimator, see Andrews (1991) or Newey and West (1987) or alternatively by fitting autoregressive moving average or autoregressive models to $\hat{\eta}_{it}$ (and computing the long-run variances model based).

The correction for serial correlation can be handled either non-parametrically (following Phillips and Perron, 1988) or by using ADF type regressions. Let us start with the non-parametric tests. Denote the residuals of the OLS regressions $\hat{u}_{it} = \rho_i \hat{u}_{it-1} + \mu_{it}$ by $\hat{\mu}_{it}$. Further, denote their estimated variances by $\hat{\sigma}^2_{\mu i}$ and their estimated long-run variances by $\hat{\omega}^2_{\mu i}$. Then, the serial correlation correction factors are given by $\hat{\lambda}_i = \frac{1}{2}(\hat{\omega}^2_{\mu i} - \hat{\sigma}^2_{\mu i})$. For later use we also define $\hat{\omega}^2_{NT} = \frac{1}{N}\sum_{i=1}^N \hat{\omega}^2_{\mu i}/\hat{\omega}^2_{u.\varepsilon i}$.

With the defined quantities the following pooled test statistics can be computed: the variance ratio statistic PP_σ, the test based on the autoregressive coefficient PP_ρ, and the test based on the t-value of the autoregressive coefficient PP_t.[2] The *essential* parts (i.e. without centering and scaling factors, see below) of the pooled test statistics are given by

$$PP^o_\sigma = \left(N^{-3/2} \sum_{i=1}^N \hat{\omega}^{-2}_{u.\varepsilon i} \left(T^{-2} \sum_{t=2}^T \hat{u}^2_{it-1} \right) \right)^{-1} \tag{3}$$

$$PP^o_\rho = N^{-1/2} \frac{\sum_{i=1}^N \hat{\omega}^{-2}_{u.\varepsilon i} \left(T^{-1} \sum_{t=2}^T \hat{u}_{it-1} \Delta \hat{u}_{it} - \hat{\lambda}_i \right)}{N^{-1} \sum_{i=1}^N \hat{\omega}^{-2}_{u.\varepsilon i} \left(T^{-2} \sum_{t=2}^T \hat{u}^2_{it-1} \right)} \tag{4}$$

$$PP^o_t = N^{-1/2} \frac{\sum_{i=1}^N \hat{\omega}^{-2}_{u.\varepsilon i} \left(T^{-1} \sum_{t=2}^T \hat{u}_{it-1} \Delta \hat{u}_{it} - \hat{\lambda}_i \right)}{\hat{\omega}_{NT} \left(N^{-1} \sum_{i=1}^N \hat{\omega}^{-2}_{u.\varepsilon i} \left(T^{-2} \sum_{t=2}^T \hat{u}^2_{it-1} \right) \right)^{1/2}} \tag{5}$$

The ADF-type test PP_{df} is based on autoregressions to correct for serial correlation, where two auxiliary regressions are performed

$$\Delta \hat{u}_{it} = \sum_{k=1}^{K_i} \gamma_{1ik} \Delta \hat{u}_{it-k} + \zeta_{1it} \tag{6}$$

$$\hat{u}_{it-1} = \sum_{k=1}^{K_i} \gamma_{2ik} \Delta \hat{u}_{it-k} + \zeta_{2it} \tag{7}$$

with the lag lengths K_i chosen in practice by e.g. minimizing the Akaike (1969) information criterion (`AIC`) in $\Delta \hat{u}_{it} = \rho_i \hat{u}_{it-1} + \sum_{k=1}^{K_i} \tilde{\gamma}_{ik} \Delta \hat{u}_{it-k} + \tilde{\zeta}_{it}$. Denote the OLS residuals of the above regressions (6) and (7) by $\hat{\zeta}_{1it}$ and $\hat{\zeta}_{2it}$, the residuals from the regressions $\hat{\zeta}_{1it} = \rho_i \hat{\zeta}_{2it} + \theta_{it}$ by $\hat{\theta}_{it}$ and their estimated variances (needed later) by $\hat{\sigma}^2_{\theta_i}$. Furthermore define $\hat{\sigma}^2_{NT} =$

[2] PP is used as acronym for Pedroni pooled test. Below we use PG as acronym for Pedroni group-mean test.

$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=K_i+2}^{T}\hat{\theta}_{it}^2$. The essential part of the pooled ADF-type statistic is then given by

$$PP_{df}^o = N^{-1/2}\frac{\sum_{i=1}^{N}\hat{\omega}_{u.\varepsilon i}^{-2}\left(T^{-1}\sum_{t=K_i+2}^{T}\hat{\zeta}_{1it}\hat{\zeta}_{2it}\right)}{\hat{\sigma}_{NT}\left(N^{-1}\sum_{i=1}^{N}\hat{\omega}_{u.\varepsilon i}^{-2}\left(T^{-2}\sum_{t=K_i+2}^{T}\hat{\zeta}_{2it}^2\right)\right)^{1/2}} \tag{8}$$

Asymptotic normality using sequential limit theory, with $T\rightarrow\infty$ followed by $N\rightarrow\infty$ can be established for the above test statistics by applying the so called Delta method (this requires knowledge of the asymptotic means and variances of the building blocks, which are obtained for practical purposes by simulation). The mean and variance correction factors, $\mu_{PP}(r,s,l)$ and $\sigma_{PP}^2(r,s,l)$ depend upon the test considered ($r\in\{\sigma,\rho,t,df\}$),[3] the deterministic variables ($s\in\{1,\ 2,\ 3\}$) and upon the number of regressors l, i.e.:

$$PP_r = \frac{PP_r^o - N^{1/2}\mu_{PP}(r,s,l)}{\left(\sigma_{PP}^2(r,s,l)\right)^{1/2}} \quad \Rightarrow \quad N(0,1)$$

Pedroni develops three group-mean tests against the heterogeneous alternative. These are: a test based on the first order autoregressive coefficient PG_ρ, a test based on its t-value PG_t and again an ADF-type test PG_{df}. The essential parts of the test statistics are given by

$$\begin{aligned} PG_\rho^o &= N^{-1/2}\sum_{i=1}^{N}PG_{\rho,i}^o = N^{-1/2}\sum_{i=1}^{N}\frac{T^{-1}\sum_{t=2}^{T}\left(\hat{u}_{it-1}\Delta\hat{u}_{it}-\hat{\lambda}_i\right)}{T^{-2}\sum_{t=2}^{T}\hat{u}_{it-1}^2} \\ PG_t^o &= N^{-1/2}\sum_{i=1}^{N}PG_{t,i}^o = N^{-1/2}\sum_{i=1}^{N}\frac{T^{-1}\sum_{t=2}^{T}\left(\hat{u}_{it-1}\Delta\hat{u}_{it}-\hat{\lambda}_i\right)}{\hat{\omega}_{\mu_i}(T^{-2}\sum_{t=2}^{T}\hat{u}_{it-1}^2)^{1/2}} \\ PG_{df}^o &= N^{-1/2}\sum_{i=1}^{N}PG_{df,i}^o = N^{-1/2}\sum_{i=1}^{N}\frac{T^{-1}\sum_{t=K_i+2}^{T}\hat{\zeta}_{1it}\hat{\zeta}_{2it}}{\hat{\sigma}_{\theta_i}(T^{-2}\sum_{t=K_i+2}^{T}\hat{\zeta}_{2it}^2)^{1/2}} \end{aligned} \tag{9}$$

Appropriately centered and scaled group-mean test statistics converge to the standard normal distribution in the sequential limit by applying a central limit theorem to the i.i.d. (across N) sequences, i.e.:

$$PG_r = \frac{PG_r^o - N^{1/2}\mu_{PG}(r,s,l)}{\left(\sigma_{PG}^2(r,s,l)\right)^{1/2}} = N^{-1/2}\sum_{i=1}^{N}\frac{PG_{r,i}^o - \mu_{PG}(r,s,l)}{\left(\sigma_{PG}^2(r,s,l)\right)^{1/2}} \quad \Rightarrow \quad N(0,1) \tag{10}$$

with $r\in\{\rho,t,df\}$,[4] $s\in\{1,\ 2,\ 3\}$ and l the number of regressors.

The five sets of different correction factors needed for the seven tests and three deterministic specifications are displayed in the appendix in Tables 5 to 19. The results from the response surface regressions are displayed in Table 1 for the pooled tests and in Table 2 for the group-mean tests in Section 4.

[3] The correction factors coincide for the PP_t test and the PP_{df} test.

[4] The correction factors coincide for the PG_t test and the PG_{df} test.

Westerlund tests

Westerlund (2005) develops two simple non-parametric tests that extend the Breitung (2002) approach from the time series to the panel case. He proposes both a pooled test, referred to as WP, and a group-mean test, referred to as WG. As for Pedroni's tests, the residuals $\hat{u}_{it}$ from estimating (1) by OLS are the starting point. Define $\hat{r}_i = \sum_{t=1}^{T} \hat{u}_{it}^2$, $\bar{r} = N^{-1} \sum_{i=1}^{N} \hat{r}_i$ and $\hat{e}_{it} = \sum_{j=1}^{t} \hat{u}_{ij}$. Using these quantities the essential parts of the test statistics are given by

$$WP^o = N^{-1/2} \left(\sum_{i=1}^{N} \frac{T^2}{\bar{r}} \left(T^{-4} \sum_{t=1}^{T} \hat{e}_{it}^2 \right) \right) \tag{11}$$

$$WG^o = N^{-1/2} \sum_{i=1}^{N} \frac{T^2}{\hat{r}_i} \left(T^{-4} \sum_{t=1}^{T} \hat{e}_{it}^2 \right) \tag{12}$$

Applying the Delta method to WP^o and a central limit theorem to WG^o (seen by writing $WG^o = N^{-1/2} \sum_{i=1}^{N} WG_i^o$) leads to asymptotic standard normality under the null hypothesis in the sequential limit when using appropriate mean and variance correction factors, i.e.:

$$WP = \frac{WP^o - N^{1/2} \mu_{WP}(s,l)}{\left(\sigma_{WP}^2(s,l)\right)^{1/2}} \quad \Rightarrow \quad N(0,1) \tag{13}$$

$$WG = \frac{WG^o - N^{1/2} \mu_{WG}(s,l)}{\left(\sigma_{WG}^2(s,l)\right)^{1/2}} \quad \Rightarrow \quad N(0,1) \tag{14}$$

with $s \in \{1,\ 2,\ 3\}$ denoting the specification of deterministic variables and l the number of regressors. The correction factors are displayed in the appendix in Tables 20 to 25 and the results from the response surface regressions are displayed in Table 3 in Section 4.

3 The system tests

The second strand of the panel cointegration literature is based on panel extensions of VAR cointegration analysis (see Johansen, 1995). Without imposing any homogeneity assumption the panel VAR DGP is given in *error correction form* by

$$\Delta Y_{it} = C_{1i} + C_{2i} t + \alpha_i \beta_i' Y_{it-1} + \sum_{j=1}^{p_i} \Gamma_{ij} \Delta Y_{it-j} + w_{it} \tag{15}$$

with $Y_{it} \in \mathbb{R}^m$, $C_{1i}, C_{2i} \in \mathbb{R}^m$, $\alpha_i, \beta_i \in \mathbb{R}^{m \times k_i}$ with full rank, $\Gamma_{ij} \in \mathbb{R}^{m \times m}$ and w_{it} cross-sectionally independent m-dimensional white noise processes with covariance matrices $\Sigma_i > 0$. To ensure that the processes described by (15) are (up to the deterministic components)

I(1) processes, the matrices $\alpha_{i\perp}'\Gamma_i\beta_{i\perp}$ have to be invertible, where $\alpha_{i\perp} \in \mathbb{R}^{m\times(m-k_i)}$, $\beta_{i\perp} \in \mathbb{R}^{m\times(m-k_i)}$ are full rank matrices such that $\alpha_{i\perp}'\alpha_i = 0$ and $\beta_{i\perp}'\beta_i = 0$ and $\Gamma_i = I_m - \sum_{j=1}^{p_i}\Gamma_{ij}$. In this case the space spanned by the columns of the matrix β_i is the k_i-dimensional cointegrating space of unit i.[5]

In the VAR cointegration literature the following five specifications concerning the deterministic components are usually discussed. Case 1 is without any deterministic components. In case 2 restricted intercepts of the form $C_{1i} = \alpha_i\tau_i$ are contained (in the cointegrating space) and case 3 includes unrestricted intercepts C_{1i} that induce linear time trends in Y_{it}. In case 4 unrestricted intercepts and restricted trend coefficients $C_{2i} = \alpha_i\kappa_i$ are included. This allows for linear trends in both the data and the cointegrating relationships. Finally in case 5 unrestricted intercepts and trend coefficients are included.

The statistical analysis, i.e. parameter estimation (via reduced rank regression) as well as testing for the cointegrating rank (using the so called trace or max tests), is well-developed and known for all five listed cases (see Johansen, 1995). Therefore we do not repeat a discussion of the procedure here.

Larsson, Lyhagen, and Löthgren test

Larsson *et al.* (2001) consider testing for cointegration in the above framework under the assumption that $\Pi_i = \alpha_i\beta_i' = \alpha\beta' = \Pi$ for $i = 1,\ldots,N$. The null hypothesis of their test is H_0: $rk(\Pi_i) = k$ for $i = 1,\ldots,N$. The test is consistent against the alternative hypothesis H_1: $rk(\Pi_i) > k$ for a non-vanishing (as $N \to \infty$) fraction of cross-section members. The construction of their test statistic is similar to Im *et al.* (2003) and hence the test statistic is given by a suitably centered and scaled version of the cross-sectional average of the individual trace statistics. Denote with $LR_i^s(k,m)$ the trace statistic for the null hypothesis of a k-dimensional cointegrating space for unit i, where the superscript s indicates the specification of the deterministic components. Using a central limit theorem in the cross-sectional dimension and the appropriate mean and variance correction factors implies that under the null hypothesis

$$LLL^s(k,m) = N^{-1/2}\sum_{i=1}^{N}\frac{LR_i^s(k,m) - \mu_{LR}^s(k,m)}{(\sigma_{LR}^{2,s}(k,m))^{1/2}} \quad \Rightarrow \quad N(0,1) \tag{16}$$

[5]The integer k_i is often referred to as cointegrating rank. Please note that β_i as used in this sub-section does not coincide with β_i used in the description of the single equation methods, where – in case of cointegration – the single cointegrating vector of unit i is given by $[1, -\beta_i']'$.

in the sequential limit with $T \to \infty$ followed by $N \to \infty$. The (asymptotic) mean and variance correction factors are given by the mean and variance of the limiting distribution of the *trace statistic* of Johansen (1995). As is well known the trace statistic depends upon $m-k$, and the correction factors are displayed in this way in the appendix in Tables 26 to 30 for the five different specifications of the deterministic variables. The results from the corresponding response surface regressions are displayed in Table 4 in Section 4.

Breitung test

Breitung (2005) proposes a 2-step estimation (and related test) procedure in a panel VAR where only the cointegrating spaces are assumed to be identical for all cross-section members.[6] In the first step of his procedure the parameters are estimated individual specifically and in the second step the common cointegrating space β is estimated in a pooled fashion.[7]

For simplicity we describe the method here for the VAR(1) model without deterministic components. In the general case lagged differences as well as (restricted) deterministic components are treated in the usual way and are concentrated out in the first step, as described in Johansen (1995). Thus, consider

$$\Delta Y_{it} = \alpha_i \beta' Y_{it-1} + w_{it} \tag{17}$$

Pre-multiplying equations (17) by $(\alpha_i'\Sigma_i^{-1}\alpha_i)^{-1}\alpha_i'\Sigma_i^{-1}$ leads to

$$(\alpha_i'\Sigma_i^{-1}\alpha_i)^{-1}\alpha_i'\Sigma_i^{-1}\Delta Y_{it} = \beta' Y_{it-1} + (\alpha_i'\Sigma_i^{-1}\alpha_i)^{-1}\alpha_i'\Sigma_i^{-1} w_{it} \tag{18}$$

$$\Delta Y_{it}^{+} = \beta' Y_{it-1} + w_{it}^{+} \tag{19}$$

where the last equation defines the variables with superscript $^+$. Note also that $\mathbb{E}w_{it}^{+}(w_{it}^{+})' = (\alpha_i'\Sigma_i^{-1}\alpha_i)^{-1}$. Now, under the assumption of its feasibility, use the normalization $\beta = [I_k, \beta_2']'$ and partition $Y_{it} = [(Y_{it}^1)', (Y_{it}^2)']'$ with $Y_{it}^1 \in \mathbb{R}^k$ and $Y_{it}^2 \in \mathbb{R}^{m-k}$. Using this notation we can rewrite the above equation (19) as

$$\Delta Y_{it}^{+} - Y_{it-1}^{1} = \beta_2' Y_{it-1}^{2} + w_{it}^{+} \tag{20}$$

Breitung suggests to obtain the estimate of β_2 by estimating (20) with pooled OLS using the estimate $(\hat{\alpha}_i'\hat{\Sigma}_i^{-1}\hat{\alpha}_i)^{-1}\hat{\alpha}_i'\hat{\Sigma}_i^{-1}$ based on the Johansen estimates. Note that, given the known

[6] This procedure extends the Ahn and Reinsel (1990) and Engle and Yoo (1991) approach from the time series to the panel case. Since this procedure is less well-known we include a short description here.

[7] Note that in the first step individual specific estimates of all parameters are obtained and used, including first step estimates of β_i.

covariance structure of the errors in (20) (and an estimate being available), pooled feasible generalized least squares (GLS) estimation of (20) is also an option.

The test Breitung proposes for the null hypothesis of $rk(\beta) = k$ is based on Saikkonen (1999). The difference from the Larsson *et al.* (2001) test is that Breitung's test incorporates the homogeneity restriction $\beta_i = \beta$ for $i = 1, \ldots, N$ in the construction of the test statistic. The discussion is again for the VAR(1) case without deterministic components. Denote with $\gamma_i \in \mathbb{R}^{m \times (m-k)}$ matrices with full column rank and consider

$$\Delta Y_{it} = \alpha_i \beta' Y_{it-1} + \gamma_i \beta_\perp' Y_{it-1} + w_{it} \tag{21}$$

Under the null hypothesis of a k-dimensional cointegrating space, $\gamma_i = 0$ for $i = 1, \ldots, N$ and under the alternative (of an m-dimensional cointegrating space) γ_i is unrestricted (in a non-vanishing fraction of panel members to imply consistency of the test against this alternative) to allow for $\Pi_i = \alpha_i \beta' + \gamma_i \beta_\perp'$ of full rank. Pre-multiply (21) with $\alpha_{i\perp}'$ to obtain

$$\begin{aligned} \alpha_{i\perp}' \Delta Y_{it} &= \alpha_{i\perp}' \gamma_i \beta_\perp' Y_{it-1} + \alpha_{i\perp}' w_{it} & (22) \\ \alpha_{i\perp}' \Delta Y_{it} &= \phi_i (\beta_\perp' Y_{it-1}) + \tilde{w}_{it} & (23) \end{aligned}$$

where the last equation defines the coefficients and variables. Replacing $\alpha_{i\perp}$ and $\beta_\perp$ by the estimates discussed above allows the estimation of equations (23) separately by OLS and the construction of test statistics for the hypotheses H_0: $\phi_i = 0$ for $i = 1, \ldots, N$. Any of the Lagrange multiplier, likelihood ratio or Wald test statistics can be used. We display here the Lagrange multiplier test statistic, which has the advantage that it only requires estimation under the null hypothesis. Denote with $\hat{f}_{it} = \hat{\alpha}_{i\perp}' \Delta Y_{it}$ and with $\hat{g}_{it} = \hat{\beta}_\perp' Y_{it}$, then the Lagrange multiplier test statistic for unit i is given by

$$LM_i(k,m) = T \text{tr} \left[\sum_{t=2}^{T} \hat{f}_{it} \hat{g}_{it-1}' \left(\sum_{t=2}^{T} \hat{g}_{it-1} \hat{g}_{it-1}' \right)^{-1} \sum_{t=2}^{T} \hat{g}_{it-1} \hat{f}_{it}' \left(\sum_{t=2}^{T} \hat{f}_{it} \hat{f}_{it}' \right)^{-1} \right] \tag{24}$$

which is sequentially computed for the different values of $k = 0, \ldots, m$. The panel test statistic is then, as usual, given by the corresponding centered and scaled cross-sectional average (putting again the superscript s to indicate the dependence upon the deterministic components). Thus, under the null hypothesis

$$B^s(k,m) = N^{-1/2} \sum_{i=1}^{N} \frac{LM_i^s(k,m) - \mu_{LM}^s(k,m))}{(\sigma_{LM}^{2,s}(k,m))^{1/2}} \quad \Rightarrow \quad N(0,1) \tag{25}$$

The correction factors μ_{LM}^{s} and $\sigma_{LM}^{2,s}$ coincide with the Larsson *et al.* (2001) correction factors, i.e. they are also given by the first two moments of the trace statistic and hence are as for the Larsson *et al.* (2001) tests given in Tables 26 to 30 in the appendix. Consequently, the results from the response surface regressions are those displayed in Table 4 in Section 4.

4 Response surface regressions

In this section we consider response surface regressions, as popularized in the unit root and cointegration context primarily by the work of James MacKinnon (see e.g. MacKinnon, 1996). In the time series unit root and cointegration literature response surfaces are performed for the critical values of the non-standard limiting distributions. In the panel unit root and cointegration literature often the appropriately centered and scaled limiting distributions for both $N, T \to \infty$ are standard normal. Thus, in the nonstationary panel context it is the centering and scaling factors as functions of the time series dimension and the number of regressors (for the single equation tests) or the number of common trends (for the system tests) for which response surface regressions are performed.

The functional form of the response surface regressions we use is inspired by the response surface regressions of Banerjee and Carrion-i-Silvestre (2006). Their suggested specification also turns out to perform well for the tests considered in this paper, based on a variety of experiments. Consider any of the tests and cases for the deterministic components discussed. Then denote with $\kappa \in \{\mu, \sigma^2\}$ either the mean or variance correction factor, which is modelled as a function of the time series length $T \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 500\}$ and of $1 \leq d \leq 12$. As indicated above, for the single equation tests $d = l$, due to their dependence upon the number of regressors. For the system tests $d = m - k$, because for the trace test the limiting distribution depends upon the number of common trends, which is given by the number of variables (m) minus the number of cointegrating relationships (k). Note that for $T = 10$ we include only values of d up to 4. This implies that the total number of observations for the response surface regressions is 136. Clearly, one could argue that also for sample sizes like $T = 20$ or 30 including up to 12 variables is probably too large a number. However, as long as $T \geq d$ all the regressions necessary to perform the tests can be performed and also the finite sample correction factors can be simulated.

The baseline specification of the response surface regressions is given by

$$\kappa(T,d) = \sum_{j=0}^{2} \left(c_{0j} + c_{1j}T^{-1} + c_{2j}T^{-2} + c_{3j}T^{-3} \right) d^j + e(T,d) \tag{26}$$

where $e(T,d)$ denotes the residuals. The results in the following tables are based on OLS estimation of the above equation with model reduction performed by applying stepwise least squares. In the stepwise search procedure the coefficients c_{00}, c_{01} and c_{02}, corresponding to the asymptotic values with respect to the time series dimension, have been excluded from the set of variables for which stepwise least squares search has been performed. Given that the estimated residuals $\hat{e}(T,d)$ show some evidence of serial correlation robust standard errors as proposed by Newey and West (1987) are used in the specification analysis. In Tables 1 to 4 we only report the coefficients included in the final equation obtained, i.e. empty entries correspond to excluded variables.

TABLE 1

Results of response surface regressions for Pedroni's pooled tests

	Case 1		Case 2		Case 3	
	μ	σ^2	μ	σ^2	μ	σ^2
			PP_σ			
c_{00}	-0.9335	42.9951	5.3771	76.3677	15.0585	236.79
c_{10}	84.5553	-12,877.37	-101.27	-13,822.22	-136.04	-33,204.3
c_{20}		601,633.1	5,636.67	650,212.1	8,103.47	1,461,661
c_{30}	1,076.93	-4,034,887	-38,117	-4,307,032	-53,972.12	-9,839,589
c_{01}	3.7642	19.7761	3.1358	10.9848	2.9211	-45.8134
c_{11}		7,467.13	70.8503	8,097.15	100.65	18,508.21
c_{21}	-1,076.83	-326,056.6	-3,285.56	-355,675.2	-4,590.12	-776,026.7
c_{31}	3,062.65	1,757,638	18,850.81	1,890,962	25,131.11	4,343,904
c_{02}	0.0156	1.4981	0.0577	2.2478	0.0677	6.7963
c_{12}		-590.94	-5.6561	-642.44	-7.7481	-1,435.01
c_{22}	134.52	24,735.32	313.87	26,923.16	397.56	55,050.31
c_{32}	605.48		-696.96			
			PP_ρ			
c_{00}	1.2451	12.9593	-2.0144	11.2377	-6.7056	22.8178
c_{10}		560.19	-8.6362	411.06	-7.1086	174.40
c_{20}	-659.26		-454.21		-316.31	
c_{30}	5,831.34	-69,511.31	4,567.16	-57,078.76	3,815.7	-50,190.74
c_{01}	-3.8462	26.2931	-3.7869	24.4422	-3.654	22.2309
c_{11}	-7.4332	-520.91	5.3581	-436.32	18.1073	-519.69
c_{21}	568.4		439.74	-2,269	342.74	
c_{31}	-4,578.47	53,562.4	-3,919.49	65,449.98	-3,561.42	54,423.99
c_{02}	-0.0085	-0.8045	-0.0115	-0.5659	-0.0178	-0.3875
c_{12}	5.7543	-33.79	5.4551	-46.3982	5.2893	-45.2887
c_{22}	-101.3	1,539.47	-103.41	1,884.3	-109.52	1,903.95
c_{32}	682.52	-16,061.26	708.33	-18,102.03	749.89	-18,992.54
			PP_t			
c_{00}	-0.584	0.7882	-1.3748	1.0342	-1.9644	0.6830
c_{10}	-4.1598		-0.633	-4.1945		-4.3101
c_{20}				-35.4021	12.1282	4.5137
c_{30}	490.96			340.29	130.21	
c_{01}	-0.5492	0.2181	-0.4276	-0.0516	-0.3566	0.0050
c_{11}	3.2827		4.0863	-2.7262	5.6216	-2.3971
c_{21}	37.071			51.1496		37.6843
c_{31}	-568.12			-226.69	-134.83	-55.7596
c_{02}	0.0172	-0.0202	0.0111	0.0012	0.0077	-0.0012
c_{12}	0.3716		0.2722	0.1301	0.2374	0.0951
c_{22}	-7.9692		-2.1943	-1.3963	-4.4724	
c_{32}	86.5335				35.7399	-20.0680

TABLE 2

Results of response surface regressions for Pedroni's group-mean tests

	Case 1		Case 2		Case 3	
	μ	σ^2	μ	σ^2	μ	σ^2
			PG_ρ			
c_{00}	-1.857	10.0917	-5.0569	20.7052	-9.7333	34.5197
c_{10}		273.18			14.8251	-164.55
c_{20}	-631.9		-417.09	2,991.73	-369.35	
c_{30}	6,305.02	-41,440.06	4,731.35	-59,919.38	4,206.14	-33,598.25
c_{01}	-4.0361	16.5257	-3.9849	16.1141	-3.8462	16.6875
c_{11}	4.0981	-320.57	16.5067	-375.36	29.0607	-560.98
c_{21}	531.03	-1,999.36	377.59	-1,221.65	246.92	3,051.01
c_{31}	-5,011.37	53,749.31	-4,211.11	55,487.74	-3,608.18	34,978.93
c_{02}	0.0023	-0.0813	-0.0004	-0.062	-0.0071	-0.1171
c_{12}	5.7298	-48.7525	5.4381	-48.9814	5.2212	-40.9663
c_{22}	-115.05	1,749.99	-116.04	1,867.89	-119.18	1,776.41
c_{32}	837.71	-16,456.08	860.13	-18,369.3	881.89	-19,043.07
			PG_t			
c_{00}	-1.0445	0.8162	-1.6823	0.6907	-2.2175	0.5782
c_{10}		-5.4311		-3.828	2.7886	-4.6666
c_{20}		68.54				17.4594
c_{30}		-337	219.01	101.55	182.24	
c_{01}	-0.4921	-0.0698	-0.4038	-0.0336	-0.3418	-0.0076
c_{11}	3.6238	-0.5843	4.4786	-1.5976	5.7424	-1.6642
c_{21}			12.7381	32.565		35.4064
c_{31}	-33.1355		-270.68	-150.81	-168.44	-143.70
c_{02}	0.0148	0.0039	0.0101	0.0016	0.0073	0.0001
c_{12}	0.2846	-0.0261	0.3093	0.0457	0.2501	0.0522
c_{22}	-1.8769	1.7478	-5.9623		-5.1892	
c_{32}		-11.9386	51.3486	-11.9608	41.4558	-17.6730

TABLE 3

Results of response surface regressions for Westerlund's pooled and group-mean tests

	Case 1		Case 2		Case 3	
	μ	σ^2	μ	σ^2	μ	σ^2
			WP			
c_{00}	0.1444	0.0311	0.049	0.0021	0.0118	0.00006
c_{10}						
c_{20}			0.9693		0.2957	
c_{30}		1.8842		0.1287		
c_{01}	-0.0305	-0.0069	-0.0105	-0.0005	-0.002	-0.00001
c_{11}						
c_{21}						
c_{31}			-3.7713		-0.2667	0.0028
c_{02}	0.0016	0.0004	0.0006	0.00002	0.00009	0.0000005
c_{12}						
c_{22}						
c_{32}			0.2362		0.0183	-0.0002
			WG			
c_{00}	0.1027	0.0068	0.0333	0.0005	0.0095	0.00002
c_{10}						
c_{20}			0.8192		0.3095	0.0003
c_{30}		1.9565		0.1746		
c_{01}	-0.0217	-0.0016	-0.0071	-0.0001	-0.0016	-0.000005
c_{11}						
c_{21}						
c_{31}		-1.0981	-2.9565	-0.0956	-0.2774	-0.0011
c_{02}	0.0012	0.00009	0.0004	0.000008	0.00007	0.0000003
c_{12}						
c_{22}						
c_{32}		0.0895	0.1855	0.0078	0.0166	0.00005

TABLE 4

Results of response surface regressions for the system tests

	Case 1		Case 2		Case 3		Case 4		Case 5	
	μ	σ^2	μ	σ^2	μ	σ^2	μ	σ^2	μ	σ^2
c_{00}	-1.0124	-2.3879	-0.0243	-1.5651	-2.184	-4.8058	0.1337	0.4188	-5.4796	-5.9341
c_{10}	-101.74		-242.37		-235.46	66.6212	-234.05		-51.3523	
c_{20}			4,350.82		4,401.9		4,597.55			2,331.04
c_{30}		-19,287.2	-28,646.51	-19,399.69	-29,050.07	-23,324.16	-30,816.74	-20,528.59		-39,432.01
c_{01}	-0.6331	0.6479	2.0627	4.5342	1.2779	3.9522	4.2288	7.2123	3.8053	6.3635
c_{11}	149.41	90.1135	185.51	70.4094	189.33		165.39	-51.6168	121.36	
c_{21}	-1,655.23	-2,463.05	-3,121.19	-2,142.29	-3,199.05	-1,139.62	-3,321.95		-1,937.43	-2,067.07
c_{31}	9,940.27	28,674.14	19,432.14	26,410.64	19,887.52	22,045.71	21,530.66	15,799.62	11,998.95	30,393.17
c_{02}	1.9716	2.8732	1.9902	2.8791	1.9793	2.7616	1.979	2.7938	1.9455	2.7438
c_{12}	-33.8317	-39.4413	-40.9204	-42.723	-41.1	-35.551	-45.8386	-38.405	-42.6005	-43.0956
c_{22}	346.81	759.47	498.34	799.33	502.55	681.47	592.83	723.59	497.25	881.95
c_{32}	-1,840.87	-5,946.75	-2,694.02	-6,108.67	-2,720.62	-5,524.73	-3,258.84	-5,996.83	-2,616.74	-7,067.01

Appendix: The correction factors

Correction factors for Pedroni tests

TABLE 5

Finite T correction factors $\mu_{PP}(\sigma)$ and $\sigma^2_{PP}(\sigma)$ for the pooled variance ratio test statistic of Pedroni (PP_σ) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	5.50	9.79	15.51	23.11	–	–	–	–	–	–	–	–
	σ^2	57.50	173.79	370.51	671.09	–	–	–	–	–	–	–	–
20	μ	4.67	8.18	12.36	16.98	22.21	28.00	34.42	41.43	49.38	59.51	70.57	85.70
	σ^2	38.93	109.40	208.27	294.40	379.62	478.52	596.56	814.25	1,111.49	1,511.97	2,187.00	3,290.95
30	μ	4.46	7.65	11.66	15.96	20.53	25.32	30.25	35.48	41.00	46.83	53.31	60.23
	σ^2	35.39	98.26	174.68	234.59	310.07	353.27	417.97	494.12	584.15	694.90	837.92	1,020.30
40	μ	4.33	7.49	11.41	15.57	19.82	24.29	28.84	33.59	38.46	43.36	48.67	54.18
	σ^2	33.62	94.48	169.58	224.45	264.73	318.12	370.94	405.96	473.09	544.80	622.35	711.37
50	μ	4.27	7.38	11.12	15.18	19.51	23.81	28.06	32.72	37.09	41.93	46.74	51.58
	σ^2	32.29	91.96	164.05	211.10	252.66	296.17	330.69	382.52	424.96	475.31	537.22	599.55
60	μ	4.21	7.37	10.99	15.10	19.17	23.51	27.86	32.25	36.63	41.05	45.58	50.20
	σ^2	31.61	90.58	161.11	201.12	242.89	283.24	317.34	372.14	403.43	444.89	505.81	557.38
70	μ	4.18	7.28	10.93	14.97	19.03	23.32	27.50	31.88	36.15	40.41	44.84	49.40
	σ^2	31.01	86.33	156.23	202.15	259.85	285.54	313.21	354.89	389.48	435.05	473.46	519.59
80	μ	4.16	7.21	10.84	14.87	18.91	23.14	27.32	31.48	35.76	40.16	44.36	48.81
	σ^2	31.01	88.55	150.41	205.53	232.64	276.59	317.01	347.14	370.07	420.47	458.42	513.04
90	μ	4.14	7.15	10.78	14.77	18.86	23.00	27.14	31.27	35.59	39.85	44.14	48.29
	σ^2	30.80	85.25	147.18	193.24	245.29	267.29	309.83	340.14	371.72	413.29	448.31	499.86
100	μ	4.13	7.17	10.82	14.74	18.75	22.88	26.89	31.15	35.57	39.64	43.76	48.03
	σ^2	30.07	85.66	140.02	205.84	238.03	280.45	315.90	337.03	370.26	414.62	453.37	490.65
200	μ	4.08	7.08	10.66	14.51	18.47	22.44	26.48	30.68	34. 72	38.80	42.78	46.90
	σ^2	30.55	81.29	142.77	194.81	228.19	261.91	296.38	325.44	352.26	388.52	425.01	453.80
500	μ	4.03	6.95	10.57	14.43	18.24	22.29	26.27	30.40	34.34	38.38	42.42	46.43
	σ^2	29.23	82.00	141.90	186.05	237.16	259.98	287.10	315.50	345.66	376.86	399.93	447.58

TABLE 6

Finite T correction factors $\mu_{PP}(\sigma)$ and $\sigma^2_{PP}(\sigma)$ for the pooled variance ratio test statistic of Pedroni (PP_σ) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	10.63	14.88	20.57	28.06	–	–	–	–	–	–	–	–
	σ^2	100.77	204.13	358.80	657.46	–	–	–	–	–	–	–	–
20	μ	9.52	13.04	17.11	21.70	26.57	32.13	38.49	45.51	53.72	63.48	75.66	90.56
	σ^2	77.03	133.22	206.09	280.42	374.59	491.17	628.79	839.90	1,134.36	1,584.57	2,281.33	3,533.35
30	μ	9.21	12.62	16.39	20.52	24.86	29.45	34.27	39.38	44.83	50.91	57.10	64.21
	σ^2	70.56	121.62	175.33	244.54	302.47	371.91	434.73	511.87	599.60	737.16	888.72	1,065.07
40	μ	9.04	12.35	16.14	20.04	24.23	28.43	33.00	37.63	42.37	47.41	52.61	57.94
	σ^2	67.62	116.97	172.92	228.68	271.34	331.05	382.52	434.23	503.15	572.73	647.89	737.78
50	μ	9.01	12.23	15.87	19.73	23.84	28.02	32.34	36.62	41.04	45.97	50.46	55.28
	σ^2	66.21	114.32	165.88	213.82	266.85	306.81	355.64	403.38	451.49	502.72	565.62	644.61
60	μ	8.90	12.10	15.77	19.55	23.61	27.65	31.85	36.12	40.47	44.87	49.30	54.10
	σ^2	65.91	111.03	161.64	213.57	251.61	300.67	335.61	382.41	429.98	473.55	521.30	577.33
70	μ	8.90	12.10	15.69	19.51	23.48	27.39	31.63	35.77	39.98	44.31	48.59	53.16
	σ^2	65.64	110.72	157.91	206.07	250.96	291.85	336.09	380.19	418.56	453.19	502.43	552.38
80	μ	8.85	12.07	15.60	19.37	23.41	27.35	31.39	35.60	39.80	43.91	48.22	52.56
	σ^2	63.80	110.94	160.31	201.83	249.74	295.09	329.99	365.09	401.07	447.42	486.48	528.31
90	μ	8.87	12.06	15.58	19.38	23.27	27.22	31.28	35.54	39.58	43.65	48.02	52.16
	σ^2	64.61	108.90	157.15	205.65	244.57	286.92	320.40	363.09	398.62	444.11	471.84	518.11
100	μ	8.82	11.98	15.54	19.31	23.24	27.20	31.14	35.31	39.43	43.60	47.79	51.87
	σ^2	63.47	109.70	153.86	199.64	239.77	290.35	322.69	351.60	385.91	430.43	470.01	512.18
200	μ	8.71	11.86	15.41	19.09	22.93	26.74	30.82	34.75	38.77	42.76	46.96	50.99
	σ^2	61.83	106.66	151.21	195.47	238.30	272.43	304.76	344.96	382.14	414.50	444.06	482.01
500	μ	8.66	11.77	15.30	19.03	22.81	26.66	30.60	34.53	38.54	42.68	46.64	50.49
	σ^2	60.90	104.98	150.42	191.10	235.82	275.03	302.66	342.64	368.90	400.39	432.66	471.55

TABLE 7

Finite T correction factors $\mu_{PP}(\sigma)$ and $\sigma^2_{PP}(\sigma)$ for the pooled variance ratio test statistic of Pedroni (PP_σ) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	22.32	28.05	36.10	47.79	–	–	–	–	–	–	–	–
	σ^2	251.95	427.16	791.75	1,600.04	–	–	–	–	–	–	–	–
20	μ	19.75	23.63	27.92	32.96	38.31	44.55	51.62	60.51	70.33	82.87	98.76	119.26
	σ^2	160.67	224.18	300.09	396.77	522.58	704.03	929.39	1,267.69	1,773.52	2,542.41	3,887.98	6,257.30
30	μ	19.22	22.71	26.51	30.54	35.08	39.96	45.08	50.55	56.37	62.90	69.83	77.35
	σ^2	144.41	195.81	252.77	313.29	381.38	460.17	550.14	659.27	802.37	956.24	1,160.74	1,426.00
40	μ	18.80	22.21	26.00	29.95	33.88	38.28	42.93	47.52	52.58	57.70	63.01	68.85
	σ^2	137.96	178.94	233.97	282.55	339.69	401.52	462.96	534.01	607.66	689.08	803.83	912.22
50	μ	18.64	22.02	25.59	29.48	33.36	37.52	41.80	46.29	50.84	55.41	60.28	65.25
	σ^2	133.94	175.09	219.98	267.18	319.45	367.59	422.50	478.31	545.80	607.79	678.06	746.22
60	μ	18.53	21.88	25.46	29.02	33.03	37.15	41.22	45.53	49.81	54.29	58.80	63.57
	σ^2	130.39	174.47	220.18	263.49	313.30	355.17	402.17	453.16	504.30	559.72	609.49	674.61
70	μ	18.39	21.70	25.25	28.95	32.73	36.71	40.90	44.94	49.28	53.61	58.01	62.43
	σ^2	130.27	172.69	215.20	258.83	303.69	346.92	390.63	439.43	482.76	528.20	578.88	636.35
80	μ	18.31	21.58	25.14	28.83	32.66	36.57	40.57	44.75	48.78	53.16	57.39	61.76
	σ^2	127.01	170.69	214.63	258.06	297.01	344.40	380.54	423.14	469.83	508.55	558.83	602.37
90	μ	18.36	21.55	25.03	28.78	32.56	36.47	40.49	44.39	48.60	52.69	56.85	61.20
	σ^2	127.63	168.78	210.86	257.50	291.11	336.55	380.21	420.31	459.72	502.56	546.65	589.76
100	μ	18.28	21.48	25.01	28.72	32.44	36.30	40.32	44.40	48.40	52.50	56.66	60.88
	σ^2	126.71	165.63	210.66	253.20	291.40	332.92	377.97	416.28	454.09	491.40	535.19	581.14
200	μ	18.07	21.36	24.76	28.52	32.12	35.85	39.70	43.65	47.57	51.62	55.53	59.55
	σ^2	123.87	161.11	203.16	243.44	283.76	322.41	358.18	398.53	428.24	474.08	501.23	538.73
500	μ	17.97	21.16	24.62	28.21	31.91	35.72	39.46	43.29	47.25	51.27	55.08	59.03
	σ^2	121.99	162.68	201.61	235.50	276.57	319.81	355.36	385.14	422.98	453.64	480.05	526.84

TABLE 8

Finite T correction factors $\mu_{PP}(\rho)$ and $\sigma^2_{PP}(\rho)$ for the pooled autoregressive coefficient test statistic of Pedroni (PP_ρ) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-2.76	-5.54	-7.85	-9.66	–	–	–	–	–	–	–	–
	σ^2	21.04	37.37	40.04	32.17	–	–	–	–	–	–	–	–
20	μ	-2.76	-5.94	-8.90	-11.53	-13.87	-15.95	-17.80	-19.38	-20.79	-22.12	-23.23	-24.31
	σ^2	22.61	46.56	60.15	58.85	52.25	46.84	40.76	39.70	36.36	33.74	32.30	28.73
30	μ	-2.79	-6.03	-9.35	-12.37	-15.13	-17.64	-19.96	-22.11	-24.07	-25.88	-27.58	-29.15
	σ^2	23.79	52.57	67.63	64.50	69.73	63.51	60.18	56.85	54.45	52.57	51.09	49.19
40	μ	-2.79	-6.12	-9.59	-12.77	-15.81	-18.63	-21.28	-23.76	-26.02	-28.22	-30.32	-32.19
	σ^2	24.49	55.45	73.68	77.82	75.09	78.77	77.42	72.41	72.92	72.61	69.45	68.13
50	μ	-2.78	-6.17	-9.68	-13.05	-16.30	-19.27	-22.04	-24.83	-27.37	-29.85	-32.18	-34.36
	σ^2	24.59	56.81	79.69	81.65	81.94	83.83	83.10	85.79	84.07	82.57	84.06	83.09
60	μ	-2.77	-6.28	-9.78	-13.25	-16.54	-19.70	-22.74	-25.64	-28.39	-31.00	-33.54	-35.89
	σ^2	24.62	59.07	82.57	84.70	85.72	87.33	90.94	93.23	94.24	95.29	98.01	96.24
70	μ	-2.77	-6.27	-9.86	-13.34	-16.80	-20.03	-23.17	-26.22	-29.08	-31.82	-34.50	-37.20
	σ^2	24.25	56.11	80.22	89.90	97.42	96.87	98.27	101.12	103.13	105.37	107.84	107.14
80	μ	-2.77	-6.26	-9.86	-13.48	-16.95	-20.28	-23.54	-26.59	-29.61	-32.57	-35.35	-38.11
	σ^2	24.64	60.03	84.30	92.63	91.82	98.15	106.88	109.70	107.85	113.27	115.36	117.82
90	μ	-2.78	-6.25	-9.91	-13.58	-17.06	-20.47	-23.75	-26.89	-30.12	-33.12	-36.05	-38.83
	σ^2	24.85	59.91	80.01	91.85	104.73	103.42	108.83	114. 26	116.51	121.57	122.33	126.70
100	μ	-2.79	-6.29	-10.01	-13.62	-17.17	-20.59	-23.90	-27.22	-30.51	-33.57	-36.51	-39.49
	σ^2	24.73	59.72	79.75	99.46	102.86	115.42	115.12	118. 50	120.87	126.87	133.36	134.48
200	μ	-2.78	-6.35	-10.15	-13.94	-17.68	-21.31	-24.98	-28.66	-32.19	-35.68	-39.16	-42.55
	σ^2	26.04	60.45	88.65	103.95	110.50	119.34	129.80	139.39	146.30	154.60	165.56	173.37
500	μ	-2.78	-6.35	-10.24	-14.12	-17.95	-21.85	-25.63	-29.54	-33.35	-37.08	-40.96	-44.63
	σ^2	25.56	63.45	93.60	107.61	128.75	131.49	141.45	152.83	165.74	177.31	187.24	200.99

TABLE 9

Finite T correction factors $\mu_{PP}(\rho)$ and $\sigma^2_{PP}(\rho)$ for the pooled autoregressive coefficient test statistic of Pedroni (PP_ρ) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-5.48	-7.48	-9.26	-10.68	–	–	–	–	–	–	–	–
	σ^2	16.46	22.68	22.23	21.75	–	–	–	–	–	–	–	–
20	μ	-5.72	-8.42	-10.99	-13.32	-15.37	-17.23	-18.90	-20.35	-21.66	-22.87	-23.96	-24.91
	σ^2	22.83	34.37	40.60	41.30	41.88	39.79	36.47	33.91	31.94	29.66	27.69	26.60
30	μ	-5.84	-8.78	-11.71	-14.43	-17.00	-19.30	-21.49	-23.46	-25.35	-27.07	-28.63	-30.13
	σ^2	25.37	40.89	50.86	57.36	57.11	58.21	56.37	53.58	51.56	49.35	49.00	45.94
40	μ	-5.85	-8.95	-12.11	-15.04	-17.93	-20.51	-23.09	-25.40	-27.61	-29.70	-31.70	-33.51
	σ^2	26.82	43.97	56.30	66.54	67.02	72.55	69.10	69.53	68.18	67.18	66.53	63.80
50	μ	-5.92	-9.08	-12.30	-15.43	-18.49	-21.34	-24.07	-26.70	-29.14	-31.55	-33.73	-35.86
	σ^2	28.08	47.13	61.70	70.62	77.22	80.68	80.63	80.47	82.29	81.40	81.16	82.05
60	μ	-5.93	-9.11	-12.46	-15.69	-18.87	-21.92	-24.84	-27.57	-30.29	-32.80	-35.25	-37.62
	σ^2	28.28	48.85	64.47	75.85	81.34	86.77	88.36	91.17	93.95	93.11	93.66	95.47
70	μ	-5.94	-9.24	-12.57	-15.95	-19.19	-22.30	-25.39	-28.24	-31.10	-33.80	-36.39	-38.92
	σ^2	28.95	49.39	66.00	77.28	85.18	93.77	95.81	101.17	103.79	103.61	106.31	107.24
80	μ	-5.95	-9.28	-12.63	-16.04	-19.45	-22.62	-25.78	-28.84	-31.75	-34.57	-37.31	-39.99
	σ^2	28.80	50.86	68.20	79.76	90.45	100.04	103.86	105.08	109.02	112.19	115.73	115.65
90	μ	-5.97	-9.31	-12.74	-16.23	-19.62	-22.91	-26.12	-29.31	-32.28	-35.19	-38.09	-40.77
	σ^2	29.54	51.33	68.94	83.35	93.87	101.73	106.19	111.71	114.86	121.24	120.69	125.18
100	μ	-6.00	-9.29	-12.82	-16.31	-19.78	-23.16	-26.35	-29.63	-32.70	-35.73	-38.77	-41.49
	σ^2	30.10	52.53	69.56	83.67	93.94	105.44	111.35	115.67	119.04	124.47	130.80	132.69
200	μ	-5.99	-9.43	-13.08	-16.78	-20.44	-24.04	-27.71	-31.24	-34.74	-38.23	-41.74	-45.06
	σ^2	30.81	54.91	76.24	93.00	108.65	118.31	128.08	141.83	151.01	159.67	167.31	174.69
500	μ	-5.99	-9.50	-13.22	-17.04	-20.91	-24.76	-28.58	-32.36	-36.18	-40.03	-43.68	-47.42
	σ^2	31.02	56.74	80.44	97.27	116.91	133.62	142.39	159.57	169.89	178.96	192.00	206.90

TABLE 10

Finite T correction factors $\mu_{PP}(\rho)$ and $\sigma^2_{PP}(\rho)$ for the pooled autoregressive coefficient test statistic of Pedroni (PP_ρ) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-8.66	-9.96	-11.10	-12.14	–	–	–	–	–	–	–	–
	σ^2	13.70	15.49	16.13	16.06	–	–	–	–	–	–	–	–
20	μ	-9.59	-11.75	-13.80	-15.73	-17.45	-19.02	-20.43	-21.79	-22.92	-24.01	-24.99	-25.87
	σ^2	24.85	30.55	33.79	34.82	34.88	35.20	32.80	29.93	28.57	26.63	25.57	24.94
30	μ	-9.96	-12.47	-14.93	-17.27	-19.57	-21.69	-23.61	-25.45	-27.10	-28.71	-30.18	-31.52
	σ^2	30.13	39.52	46.35	50.13	51.10	51.61	50.83	49.96	48.56	46.64	44.71	43.53
40	μ	-10.10	-12.83	-15.57	-18.27	-20.76	-23.25	-25.54	-27.69	-29.81	-31.70	-33.51	-35.25
	σ^2	33.27	43.29	53.05	58.44	62.65	64.91	65.22	66.83	65.15	64.28	64.03	61.82
50	μ	-10.19	-13.08	-15.96	-18.84	-21.58	-24.28	-26.82	-29.25	-31.60	-33.75	-35.88	-37.89
	σ^2	35.33	47.06	57.30	64.37	71.62	75.31	77.55	78.87	80.47	79.73	79.82	77.96
60	μ	-10.26	-13.26	-16.24	-19.21	-22.17	-25.03	-27.75	-30.38	-32.90	-35.31	-37.65	-39.86
	σ^2	36.38	50.49	62.65	71.26	79.65	82.64	87.53	89.94	90.98	92.33	92.89	92.40
70	μ	-10.30	-13.29	-16.41	-19.50	-22.50	-25.51	-28.38	-31.19	-33.87	-36.47	-39.05	-41.34
	σ^2	38.03	52.47	65.25	75.64	84.38	90.35	94.46	98.34	100.74	102.65	104.54	105.49
80	μ	-10.34	-13.35	-16.55	-19.73	-22.88	-25.94	-28.94	-31.84	-34.65	-37.44	-40.07	-42.63
	σ^2	38.25	54.43	67.87	78.88	87.49	96.83	100.23	104.67	108.64	110.99	114.24	113.63
90	μ	-10.40	-13.46	-16.67	-19.94	-23.15	-26.32	-29.41	-32.32	-35.32	-38.14	-40.87	-43.60
	σ^2	38.99	54.95	69.03	82.23	91.06	99.91	105.26	112.20	114.96	119.97	124.00	124.81
100	μ	-10.41	-13.48	-16.78	-20.08	-23.32	-26.53	-29.73	-32.83	-35.84	-38.74	-41.58	-44.36
	σ^2	39.07	55.42	70.66	83.99	93.82	103.20	111.73	117.25	121.31	125.55	130.09	134.37
200	μ	-10.51	-13.79	-17.21	-20.82	-24.33	-27.87	-31.42	-34.84	-38.39	-41.80	-45.08	-48.42
	σ^2	41.68	59.75	77.91	94.44	109.66	120.36	133.30	143.45	152.21	163.89	171.56	178.23
500	μ	-10.54	-13.92	-17.50	-21.20	-24.96	-28.70	-32.47	-36.22	-40.01	-43.78	-47.41	-51.20
	σ^2	42.79	63.72	82.80	100.19	118.49	135.85	148.41	162.62	178.04	187.21	197.97	215.35

TABLE 11

Finite T correction factors $\mu_{PP}(t)$ and $\sigma^2_{PP}(t)$ for the pooled t-value test statistic of Pedroni (PP_t) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-0.83	-1.15	-1.23	-1.17	–	–	–	–	–	–	–	–
	σ^2	1.06	0.85	0.64	0.52	–	–	–	–	–	–	–	–
20	μ	-0.92	-1.41	-1.69	-1.85	-1.93	-1.94	-1.92	-1.86	-1.77	-1.65	-1.51	-1.36
	σ^2	1.24	1.09	0.89	0.66	0.51	0.42	0.37	0.35	0.34	0.32	0.31	0.29
30	μ	-0.96	-1.49	-1.85	-2.09	-2.25	-2.36	-2.42	-2.46	-2.47	-2.45	-2.41	-2.36
	σ^2	1.33	1.25	0.97	0.69	0.61	0.49	0.43	0.39	0.36	0.34	0.33	0.32
40	μ	-0.97	-1.53	-1.93	-2.21	-2.42	-2.57	-2.69	-2.78	-2.83	-2.87	-2.89	-2.89
	σ^2	1.39	1.31	1.06	0.82	0.64	0.56	0.49	0.44	0.40	0.38	0.36	0.35
50	μ	-0.98	-1.56	-1.97	-2.28	-2.52	-2.71	-2.85	-2.97	-3.07	-3.14	-3.19	-3.23
	σ^2	1.42	1.34	1.13	0.85	0.67	0.59	0.51	0.47	0.43	0.40	0.38	0.37
60	μ	-0.98	-1.58	-2.00	-2.33	-2.58	-2.79	-2.97	-3.11	-3.23	-3.32	-3.40	-3.46
	σ^2	1.44	1.40	1.18	0.88	0.70	0.60	0.54	0.49	0.46	0.43	0.41	0.39
70	μ	-0.98	-1.59	-2.03	-2.36	-2.64	-2.86	-3.04	-3.21	-3.34	-3.45	-3.55	-3.64
	σ^2	1.42	1.33	1.14	0.92	0.77	0.64	0.58	0.52	0.48	0.46	0.44	0.41
80	μ	-0.99	-1.60	-2.04	-2.39	-2.67	-2.91	-3.11	-3.28	-3.43	-3.56	-3.67	-3.77
	σ^2	1.44	1.44	1.22	0.96	0.73	0.65	0.62	0.55	0.50	0.47	0.45	0.43
90	μ	-0.99	-1.60	-2.05	-2.41	-2.70	-2.95	-3.15	-3.33	-3.50	-3.64	-3.76	-3.87
	σ^2	1.46	1.44	1.16	0.95	0.83	0.68	0.61	0.57	0.52	0.49	0.46	0.44
100	μ	-1.00	-1.61	-2.07	-2.43	-2.72	-2.97	-3.19	-3.38	-3.55	-3.70	-3.83	-3.95
	σ^2	1.45	1.43	1.14	1.01	0.81	0.75	0.64	0.59	0.53	0.51	0.49	0.46
200	μ	-1.00	-1.64	-2.12	-2.50	-2.83	-3.11	-3.36	-3.60	-3.80	-4.00	-4.17	-4.34
	σ^2	1.55	1.45	1.26	1.05	0.86	0.76	0.70	0.65	0.60	0.57	0.56	0.54
500	μ	-1.01	-1.65	-2.15	-2.55	-2.89	-3.20	-3.47	-3.73	-3.96	-4.18	-4.39	-4.58
	σ^2	1.52	1.52	1.33	1.08	0.98	0.82	0.74	0.69	0.66	0.63	0.60	0.59

TABLE 12

Finite T correction factors $\mu_{PP}(t)$ and $\sigma^2_{PP}(t)$ for the pooled t-value test statistic of Pedroni (PP_t) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-1.39	-1.43	-1.38	-1.24	–	–	–	–	–	–	–	–
	σ^2	0.57	0.52	0.46	0.47	–	–	–	–	–	–	–	–
20	μ	-1.56	-1.80	-1.96	-2.04	-2.06	-2.05	-2.00	-1.91	-1.80	-1.68	-1.53	-1.37
	σ^2	0.70	0.63	0.55	0.47	0.44	0.40	0.36	0.35	0.33	0.32	0.31	0.32
30	μ	-1.62	-1.92	-2.16	-2.32	-2.44	-2.51	-2.55	-2.56	-2.55	-2.52	-2.47	-2.40
	σ^2	0.75	0.71	0.64	0.57	0.49	0.44	0.40	0.38	0.36	0.34	0.33	0.32
40	μ	-1.64	-1.99	-2.27	-2.47	-2.63	-2.75	-2.84	-2.90	-2.95	-2.97	-2.97	-2.97
	σ^2	0.80	0.76	0.67	0.62	0.53	0.50	0.44	0.41	0.38	0.37	0.35	0.34
50	μ	-1.67	-2.03	-2.32	-2.56	-2.75	-2.90	-3.02	-3.12	-3.20	-3.25	-3.30	-3.32
	σ^2	0.85	0.80	0.73	0.65	0.58	0.54	0.47	0.44	0.42	0.39	0.38	0.36
60	μ	-1.68	-2.05	-2.36	-2.62	-2.83	-3.01	-3.15	-3.27	-3.37	-3.45	-3.52	-3.57
	σ^2	0.84	0.84	0.75	0.68	0.61	0.56	0.50	0.46	0.45	0.42	0.39	0.39
70	μ	-1.69	-2.08	-2.39	-2.67	-2.89	-3.08	-3.24	-3.37	-3.49	-3.59	-3.68	-3.75
	σ^2	0.86	0.83	0.77	0.69	0.62	0.58	0.52	0.50	0.47	0.44	0.42	0.40
80	μ	-1.69	-2.09	-2.41	-2.69	-2.93	-3.13	-3.31	-3.46	-3.59	-3.70	-3.80	-3.89
	σ^2	0.86	0.85	0.79	0.70	0.65	0.61	0.55	0.51	0.48	0.46	0.44	0.41
90	μ	-1.70	-2.10	-2.44	-2.73	-2.97	-3.18	-3.36	-3.53	-3.67	-3.79	-3.90	-3.99
	σ^2	0.88	0.86	0.80	0.73	0.67	0.61	0.56	0.53	0.49	0.48	0.44	0.44
100	μ	-1.71	-2.10	-2.45	-2.75	-3.00	-3.22	-3.40	-3.58	-3.72	-3.86	-3.99	-4.08
	σ^2	0.89	0.88	0.80	0.74	0.67	0.63	0.58	0.54	0.50	0.48	0.47	0.45
200	μ	-1.72	-2.15	-2.52	-2.84	-3.12	-3.37	-3.61	-3.82	-4.01	-4.19	-4.36	-4.51
	σ^2	0.91	0.92	0.87	0.80	0.74	0.68	0.64	0.61	0.59	0.56	0.54	0.52
500	μ	-1.72	-2.17	-2.56	-2.90	-3.20	-3.48	-3.73	-3.97	-4.19	-4.40	-4.59	-4.77
	σ^2	0.92	0.95	0.91	0.83	0.78	0.74	0.68	0.66	0.63	0.60	0.58	0.58

TABLE 13

Finite T correction factors $\mu_{PP}(t)$ and $\sigma^2_{PP}(t)$ for the pooled t-value test statistic of Pedroni (PP_t) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-1.62	-1.48	-1.30	-1.07	–	–	–	–	–	–	–	–
	σ^2	0.39	0.40	0.43	0.48	–	–	–	–	–	–	–	–
20	μ	-1.97	-2.05	-2.10	-2.11	-2.07	-2.01	-1.93	-1.81	-1.68	-1.54	-1.37	-1.20
	σ^2	0.45	0.43	0.41	0.38	0.36	0.35	0.34	0.32	0.31	0.30	0.30	0.29
30	μ	-2.08	-2.25	-2.38	-2.47	-2.53	-2.57	-2.57	-2.56	-2.52	-2.47	-2.41	-2.33
	σ^2	0.50	0.49	0.47	0.43	0.40	0.38	0.36	0.34	0.33	0.32	0.31	0.31
40	μ	-2.14	-2.35	-2.52	-2.66	-2.77	-2.86	-2.91	-2.96	-2.98	-2.98	-2.97	-2.95
	σ^2	0.53	0.52	0.50	0.47	0.44	0.41	0.39	0.38	0.36	0.34	0.33	0.33
50	μ	-2.17	-2.41	-2.61	-2.78	-2.92	-3.04	-3.13	-3.21	-3.26	-3.30	-3.33	-3.34
	σ^2	0.56	0.54	0.52	0.49	0.47	0.45	0.43	0.40	0.39	0.37	0.36	0.34
60	μ	-2.19	-2.45	-2.67	-2.86	-3.03	-3.17	-3.28	-3.38	-3.46	-3.52	-3.58	-3.61
	σ^2	0.57	0.57	0.55	0.53	0.50	0.47	0.45	0.43	0.41	0.39	0.38	0.36
70	μ	-2.21	-2.47	-2.71	-2.92	-3.09	-3.25	-3.39	-3.50	-3.60	-3.68	-3.76	-3.81
	σ^2	0.59	0.59	0.57	0.54	0.52	0.49	0.46	0.44	0.42	0.41	0.39	0.38
80	μ	-2.22	-2.49	-2.74	-2.96	-3.15	-3.32	-3.47	-3.60	-3.71	-3.81	-3.90	-3.97
	σ^2	0.59	0.61	0.59	0.56	0.53	0.52	0.48	0.46	0.44	0.42	0.41	0.40
90	μ	-2.23	-2.51	-2.77	-3.00	-3.20	-3.38	-3.54	-3.67	-3.80	-3.91	-4.00	-4.09
	σ^2	0.60	0.61	0.59	0.58	0.55	0.53	0.49	0.48	0.46	0.44	0.43	0.41
100	μ	-2.24	-2.52	-2.79	-3.02	-3.23	-3.42	-3.59	-3.74	-3.87	-3.99	-4.09	-4.19
	σ^2	0.60	0.62	0.60	0.59	0.56	0.53	0.52	0.49	0.47	0.45	0.44	0.42
200	μ	-2.27	-2.59	-2.88	-3.16	-3.40	-3.62	-3.83	-4.01	-4.20	-4.36	-4.51	-4.65
	σ^2	0.63	0.65	0.65	0.64	0.62	0.59	0.58	0.56	0.53	0.53	0.51	0.49
500	μ	-2.29	-2.62	-2.94	-3.23	-3.50	-3.74	-3.97	-4.19	-4.40	-4.59	-4.77	-4.95
	σ^2	0.65	0.69	0.68	0.67	0.66	0.65	0.62	0.61	0.60	0.57	0.55	0.56

TABLE 14

Finite T correction factors $\mu_{PG}(\rho)$ and $\sigma^2_{PG}(\rho)$ for the group-mean autoregressive coefficient test statistic of Pedroni (PG_ρ) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-4.91	-7.52	-9.49	-10.96	–	–	–	–	–	–	–	–
	σ^2	13.30	13.21	12.03	10.44	–	–	–	–	–	–	–	–
20	μ	-5.33	-8.57	-11.33	-13.68	-15.82	-17.65	-19.29	-20.75	-22.04	-23.24	-24.27	-25.23
	σ^2	18.30	22.99	24.97	25.46	25.48	24.43	23.22	22.15	20.68	19.22	18.07	16.59
30	μ	-5.49	-8.94	-12.09	-14.92	-17.50	-19.82	-21.99	-23.97	-25.80	-27.49	-29.06	-30.56
	σ^2	20.57	27.82	32.70	35.96	37.15	37.97	38.51	37.82	37.05	36.12	34.77	33.82
40	μ	-5.56	-9.21	-12.56	-15.61	-18.47	-21.12	-23.63	-25.97	-28.12	-30.22	-32.18	-33.96
	σ^2	21.88	31.22	37.89	42.46	46.13	48.55	50.32	51.05	51.62	51.60	50.61	50.22
50	μ	-5.64	-9.36	-12.76	-16.03	-19.12	-21.99	-24.64	-27.29	-29.70	-32.06	-34.29	-36.40
	σ^2	22.98	33.17	40.92	47.44	52.18	56.81	59.58	61.52	63.19	63.59	63.72	64.34
60	μ	-5.67	-9.47	-13.02	-16.33	-19.51	-22.54	-25.49	-28.28	-30.92	-33.41	-35.89	-38.16
	σ^2	23.63	34.48	43.94	51.40	57.41	62.10	66.54	69.84	72.18	74.18	75.78	76.85
70	μ	-5.68	-9.52	-13.12	-16.54	-19.90	-23.02	-26.02	-29.00	-31.78	-34.41	-37.02	-39.62
	σ^2	23.85	35.67	45.69	53.96	61.41	67.07	71.97	76.44	80.35	82.29	85.84	87.40
80	μ	-5.70	-9.57	-13.18	-16.76	-20.06	-23.33	-26.54	-29.50	-32.39	-35.29	-37.97	-40.67
	σ^2	24.10	36.52	47.01	56.60	63.64	70.19	77.30	82.48	86.28	90.26	92.54	96.50
90	μ	-5.72	-9.58	-13.25	-16.88	-20.31	-23.60	-26.84	-29.93	-33.00	-35.96	-38.79	-41.53
	σ^2	24.48	37.15	47.63	57.83	66.37	74.57	80.73	87.99	92.10	96.92	100.98	103.53
100	μ	-5.73	-9.61	-13.37	-16.96	-20.47	-23.83	-27.08	-30.31	-33.49	-36.51	-39.39	-42.28
	σ^2	24.55	37.38	49.21	58.89	68.96	77.32	84.53	91.22	96.94	102.00	106.12	111.35
200	μ	-5.81	-9.80	-13.70	-17.47	-21.19	-24.86	-28.50	-32.11	-35.60	-39.03	-42.50	-45.83
	σ^2	25.97	40.28	54.19	66.51	79.02	90.74	102.48	111.58	121.99	130.65	140.24	148.82
500	μ	-5.83	-9.88	-13.86	-17.75	-21.61	-25.54	-29.35	-33.25	-37.00	-40.77	-44.55	-48.31
	σ^2	26.75	41.79	56.44	70.76	85.11	99.77	113.86	128.33	140.89	154.79	165.24	179.11

TABLE 15

Finite T correction factors $\mu_{PG}(\rho)$ and $\sigma^2_{PG}(\rho)$ for the group-mean autoregressive coefficient test statistic of Pedroni (PG_ρ) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-7.08	-9.00	-10.54	-11.77	–	–	–	–	–	–	–	–
	σ^2	11.71	11.36	10.13	8.94	–	–	–	–	–	–	–	–
20	μ	-7.92	-10.66	-13.11	-15.23	-17.11	-18.79	-20.29	-21.60	-22.81	-23.90	-24.89	-25.80
	σ^2	19.76	22.99	24.21	24.22	23.85	23.00	21.63	20.40	19.31	17.81	16.63	15.51
30	μ	-8.31	-11.35	-14.21	-16.81	-19.22	-21.37	-23.42	-25.24	-26.98	-28.59	-30.07	-31.46
	σ^2	24.07	29.96	33.44	35.75	37.04	37.26	37.27	36.59	35.66	34.59	33.53	32.43
40	μ	-8.46	-11.73	-14.82	-17.70	-20.41	-22.89	-25.32	-27.51	-29.61	-31.59	-33.48	-35.19
	σ^2	26.67	34.26	39.46	43.73	46.61	48.40	49.53	49.94	50.06	50.15	49.50	48.44
50	μ	-8.61	-11.97	-15.21	-18.24	-21.21	-23.95	-26.56	-29.06	-31.42	-33.68	-35.78	-37.83
	σ^2	28.34	37.06	44.32	49.31	53.74	57.28	59.42	61.24	62.66	62.92	63.50	63.48
60	μ	-8.67	-12.13	-15.47	-18.67	-21.75	-24.70	-27.48	-30.14	-32.74	-35.17	-37.50	-39.80
	σ^2	29.31	39.39	47.09	53.92	59.75	64.21	67.41	70.59	72.32	74.09	74.79	76.30
70	μ	-8.74	-12.28	-15.67	-19.03	-22.16	-25.20	-28.20	-30.98	-33.73	-36.32	-38.83	-41.29
	σ^2	30.36	40.93	49.75	58.15	63.87	69.30	73.67	77.93	81.03	83.03	85.25	87.38
80	μ	-8.77	-12.38	-15.82	-19.16	-22.52	-25.63	-28.70	-31.64	-34.49	-37.22	-39.91	-42.48
	σ^2	31.13	42.27	51.62	59.89	67.51	73.32	79.68	83.68	87.66	91.30	94.43	95.60
90	μ	-8.83	-12.43	-15.96	-19.43	-22.76	-25.98	-29.10	-32.23	-35.13	-37.98	-40.76	-43.40
	σ^2	31.49	43.24	53.33	62.70	70.70	77.83	83.99	89.42	94.11	98.59	101.49	104.60
100	μ	-8.88	-12.46	-16.05	-19.55	-22.99	-26.30	-29.43	-32.62	-35.61	-38.59	-41.56	-44.22
	σ^2	32.28	44.15	54.66	64.73	73.14	81.10	87.31	93.60	99.19	103.79	109.05	111.77
200	μ	-8.97	-12.74	-16.49	-20.26	-23.95	-27.52	-31.15	-34.67	-38.14	-41.55	-45.02	-48.30
	σ^2	34.17	48.00	61.26	73.46	86.09	97.09	107.89	117.79	127.42	134.85	143.97	151.98
500	μ	-9.04	-12.92	-16.77	-20.67	-24.56	-28.42	-32.23	-36.06	-39.84	-43.65	-47.32	-51.07
	σ^2	35.79	50.70	65.24	79.92	93.91	107.60	120.80	135.53	148.01	159.64	173.89	187.52

TABLE 16

Finite T correction factors $\mu_{PG}(\rho)$ and $\sigma^2_{PG}(\rho)$ for the group-mean autoregressive coefficient test statistic of Pedroni (PG_ρ) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-9.72	-10.95	-12.01	-12.92	–	–	–	–	–	–	–	–
	σ^2	9.59	9.12	8.30	7.29	–	–	–	–	–	–	–	–
20	μ	-11.43	-13.58	-15.53	-17.33	-18.93	-20.40	-21.68	-22.91	-23.95	-24.93	-25.84	-26.64
	σ^2	21.56	23.33	23.83	23.39	22.66	21.64	20.34	18.99	17.75	16.55	15.36	14.31
30	μ	-12.15	-14.71	-17.12	-19.38	-21.54	-23.55	-25.34	-27.08	-28.63	-30.11	-31.51	-32.77
	σ^2	28.83	33.01	35.35	36.78	36.98	37.09	36.25	35.51	34.53	33.35	32.17	30.79
40	μ	-12.49	-15.27	-18.04	-20.65	-23.07	-25.45	-27.62	-29.67	-31.68	-33.47	-35.21	-36.83
	σ^2	33.14	38.74	43.52	46.38	48.38	49.37	49.98	50.27	49.88	49.54	48.74	47.52
50	μ	-12.71	-15.70	-18.61	-21.43	-24.10	-26.73	-29.15	-31.49	-33.74	-35.81	-37.85	-39.76
	σ^2	36.16	43.74	49.47	53.63	56.99	59.67	61.37	62.39	62.90	63.42	63.63	62.69
60	μ	-12.88	-15.99	-19.02	-21.97	-24.88	-27.64	-30.27	-32.83	-35.25	-37.58	-39.81	-41.93
	σ^2	38.55	46.60	53.26	59.15	63.94	67.13	70.16	72.37	74.03	75.68	75.44	75.57
70	μ	-12.99	-16.12	-19.31	-22.38	-25.33	-28.28	-31.07	-33.78	-36.36	-38.91	-41.40	-43.64
	σ^2	40.25	48.80	57.20	63.73	69.06	74.20	78.24	80.81	83.05	85.66	86.93	88.53
80	μ	-13.06	-16.26	-19.54	-22.72	-25.80	-28.85	-31.73	-34.56	-37.30	-39.98	-42.56	-45.02
	σ^2	41.18	51.38	60.10	67.90	73.83	80.13	84.01	88.30	91.53	93.99	96.20	97.64
90	μ	-13.16	-16.43	-19.69	-22.99	-26.15	-29.29	-32.29	-35.16	-38.07	-40.81	-43.49	-46.14
	σ^2	42.33	52.88	61.97	70.25	78.00	84.40	89.26	93.74	98.36	101.00	104.65	107.30
100	μ	-13.20	-16.47	-19.85	-23.17	-26.40	-29.57	-32.72	-35.75	-38.69	-41.51	-44.31	-47.03
	σ^2	43.01	54.09	64.23	72.92	80.92	88.01	93.72	99.31	103.73	108.48	111.93	116.28
200	μ	-13.46	-16.95	-20.51	-24.17	-27.72	-31.27	-34.80	-38.20	-41.70	-45.09	-48.33	-51.60
	σ^2	47.43	60.47	73.07	85.14	96.12	107.48	117.94	127.18	136.67	145.11	153.91	159.01
500	μ	-13.60	-17.24	-20.97	-24.75	-28.58	-32.33	-36.11	-39.84	-43.63	-47.39	-50.99	-54.80
	σ^2	50.00	64.85	79.48	94.51	108.19	121.00	134.85	147.31	160.24	173.52	183.94	197.79

TABLE 17

Finite T correction factors $\mu_{PG}(t)$ and $\sigma^2_{PG}(t)$ for the group-mean t-value test statistic of Pedroni (PG_t) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-1.08	-1.33	-1.36	-1.27	–	–	–	–	–	–	–	–
	σ^2	0.54	0.34	0.28	0.25	–	–	–	–	–	–	–	–
20	μ	-1.23	-1.65	-1.88	-1.99	-2.04	-2.04	-2.00	-1.93	-1.83	-1.71	-1.57	-1.42
	σ^2	0.64	0.46	0.37	0.33	0.31	0.29	0.28	0.27	0.25	0.25	0.23	0.22
30	μ	-1.29	-1.76	-2.06	-2.26	-2.39	-2.47	-2.52	-2.55	-2.54	-2.52	-2.48	-2.41
	σ^2	0.67	0.52	0.43	0.38	0.35	0.33	0.31	0.30	0.29	0.28	0.27	0.27
40	μ	-1.31	-1.82	-2.16	-2.40	-2.58	-2.71	-2.81	-2.88	-2.92	-2.96	-2.97	-2.96
	σ^2	0.70	0.54	0.46	0.42	0.39	0.36	0.34	0.33	0.32	0.31	0.30	0.29
50	μ	-1.33	-1.86	-2.21	-2.48	-2.69	-2.85	-2.98	-3.09	-3.17	-3.23	-3.28	-3.31
	σ^2	0.72	0.57	0.49	0.44	0.41	0.39	0.37	0.36	0.34	0.33	0.32	0.31
60	μ	-1.34	-1.88	-2.25	-2.53	-2.76	-2.95	-3.10	-3.23	-3.34	-3.42	-3.50	-3.55
	σ^2	0.73	0.58	0.51	0.46	0.43	0.40	0.39	0.37	0.36	0.35	0.34	0.32
70	μ	-1.34	-1.90	-2.28	-2.57	-2.82	-3.02	-3.19	-3.34	-3.46	-3.56	-3.65	-3.73
	σ^2	0.73	0.59	0.52	0.48	0.44	0.42	0.40	0.38	0.37	0.36	0.36	0.34
80	μ	-1.35	-1.91	-2.30	-2.61	-2.86	-3.07	-3.26	-3.41	-3.55	-3.67	-3.77	-3.86
	σ^2	0.74	0.60	0.53	0.49	0.45	0.43	0.42	0.40	0.39	0.37	0.36	0.36
90	μ	-1.36	-1.91	-2.31	-2.63	-2.90	-3.11	-3.31	-3.47	-3.62	-3.76	-3.87	-3.97
	σ^2	0.74	0.61	0.53	0.50	0.47	0.45	0.43	0.42	0.40	0.39	0.38	0.37
100	μ	-1.36	-1.92	-2.33	-2.65	-2.92	-3.15	-3.35	-3.53	-3.69	-3.82	-3.95	-4.06
	σ^2	0.74	0.61	0.54	0.50	0.48	0.46	0.44	0.43	0.41	0.40	0.39	0.38
200	μ	-1.38	-1.96	-2.39	-2.74	-3.04	-3.30	-3.54	-3.76	-3.95	-4.14	-4.31	-4.46
	σ^2	0.77	0.63	0.57	0.54	0.52	0.50	0.49	0.47	0.46	0.46	0.45	0.44
500	μ	-1.39	-1.98	-2.43	-2.79	-3.11	-3.40	-3.66	-3.90	-4.13	-4.33	-4.54	-4.72
	σ^2	0.78	0.64	0.59	0.56	0.54	0.53	0.52	0.52	0.51	0.50	0.49	0.49

TABLE 18

Finite T correction factors $\mu_{PG}(t)$ and $\sigma^2_{PG}(t)$ for the group-mean t-value test statistic of Pedroni (PG_t) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-1.52	-1.54	-1.47	-1.33	–	–	–	–	–	–	–	–
	σ^2	0.40	0.33	0.29	0.26	–	–	–	–	–	–	–	–
20	μ	-1.76	-1.98	-2.10	-2.15	-2.16	-2.13	-2.07	-1.98	-1.87	-1.74	-1.59	-1.43
	σ^2	0.49	0.41	0.36	0.33	0.31	0.29	0.28	0.27	0.26	0.25	0.24	0.22
30	μ	-1.85	-2.13	-2.33	-2.47	-2.56	-2.61	-2.64	-2.64	-2.62	-2.58	-2.53	-2.46
	σ^2	0.53	0.46	0.41	0.37	0.34	0.32	0.31	0.30	0.29	0.29	0.28	0.27
40	μ	-1.89	-2.21	-2.45	-2.63	-2.77	-2.87	-2.95	-3.00	-3.03	-3.04	-3.04	-3.03
	σ^2	0.57	0.49	0.44	0.40	0.38	0.36	0.34	0.33	0.32	0.31	0.30	0.29
50	μ	-1.92	-2.26	-2.52	-2.73	-2.90	-3.03	-3.14	-3.23	-3.29	-3.34	-3.37	-3.40
	σ^2	0.59	0.51	0.47	0.43	0.40	0.38	0.36	0.35	0.34	0.33	0.32	0.31
60	μ	-1.94	-2.29	-2.57	-2.80	-2.99	-3.15	-3.28	-3.38	-3.47	-3.55	-3.60	-3.65
	σ^2	0.59	0.53	0.48	0.45	0.42	0.40	0.38	0.37	0.36	0.34	0.33	0.33
70	μ	-1.95	-2.32	-2.61	-2.85	-3.05	-3.23	-3.38	-3.50	-3.61	-3.70	-3.77	-3.84
	σ^2	0.60	0.54	0.49	0.46	0.43	0.42	0.39	0.38	0.37	0.36	0.35	0.34
80	μ	-1.96	-2.34	-2.63	-2.88	-3.11	-3.29	-3.45	-3.59	-3.71	-3.81	-3.90	-3.98
	σ^2	0.61	0.55	0.50	0.47	0.45	0.43	0.41	0.40	0.38	0.37	0.36	0.35
90	μ	-1.97	-2.35	-2.66	-2.92	-3.15	-3.34	-3.51	-3.66	-3.79	-3.90	-4.01	-4.09
	σ^2	0.62	0.56	0.51	0.48	0.46	0.44	0.42	0.41	0.39	0.39	0.37	0.37
100	μ	-1.98	-2.36	-2.68	-2.94	-3.18	-3.38	-3.55	-3.71	-3.85	-3.98	-4.09	-4.19
	σ^2	0.63	0.56	0.52	0.49	0.47	0.45	0.43	0.42	0.41	0.39	0.39	0.38
200	μ	-2.00	-2.41	-2.75	-3.05	-3.32	-3.55	-3.77	-3.97	-4.15	-4.32	-4.48	-4.63
	σ^2	0.65	0.59	0.55	0.53	0.51	0.50	0.48	0.47	0.46	0.45	0.44	0.44
500	μ	-2.02	-2.44	-2.80	-3.12	-3.41	-3.67	-3.91	-4.13	-4.34	-4.55	-4.73	-4.91
	σ^2	0.66	0.61	0.58	0.55	0.54	0.52	0.52	0.51	0.50	0.49	0.49	0.49

TABLE 19

Finite T correction factors $\mu_{PG}(t)$ and $\sigma^2_{PG}(t)$ for the group-mean t-value test statistic of Pedroni (PG_t) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	-1.67	-1.54	-1.37	-1.15	–	–	–	–	–	–	–	–
	σ^2	0.32	0.30	0.27	0.25	–	–	–	–	–	–	–	–
20	μ	-2.09	-2.16	-2.20	-2.19	-2.15	-2.08	-1.99	-1.88	-1.74	-1.60	-1.44	-1.27
	σ^2	0.37	0.35	0.33	0.31	0.30	0.28	0.27	0.26	0.25	0.24	0.23	0.21
30	μ	-2.23	-2.39	-2.51	-2.59	-2.63	-2.65	-2.65	-2.63	-2.58	-2.53	-2.47	-2.38
	σ^2	0.42	0.39	0.37	0.34	0.33	0.31	0.30	0.29	0.28	0.28	0.27	0.27
40	μ	-2.31	-2.51	-2.67	-2.79	-2.89	-2.96	-3.01	-3.04	-3.05	-3.05	-3.04	-3.00
	σ^2	0.44	0.42	0.40	0.37	0.35	0.34	0.33	0.32	0.31	0.30	0.29	0.29
50	μ	-2.35	-2.58	-2.77	-2.92	-3.05	-3.16	-3.24	-3.30	-3.35	-3.38	-3.40	-3.41
	σ^2	0.47	0.44	0.42	0.39	0.38	0.36	0.35	0.34	0.33	0.32	0.31	0.30
60	μ	-2.38	-2.63	-2.84	-3.01	-3.16	-3.29	-3.40	-3.48	-3.55	-3.61	-3.66	-3.69
	σ^2	0.48	0.46	0.43	0.41	0.40	0.38	0.37	0.35	0.35	0.33	0.33	0.32
70	μ	-2.40	-2.66	-2.88	-3.07	-3.24	-3.38	-3.51	-3.61	-3.70	-3.78	-3.85	-3.89
	σ^2	0.49	0.47	0.45	0.43	0.41	0.39	0.38	0.37	0.36	0.35	0.34	0.34
80	μ	-2.42	-2.68	-2.92	-3.12	-3.30	-3.46	-3.60	-3.71	-3.82	-3.91	-3.99	-4.06
	σ^2	0.50	0.49	0.46	0.44	0.42	0.41	0.40	0.38	0.37	0.36	0.35	0.35
90	μ	-2.44	-2.71	-2.95	-3.17	-3.35	-3.52	-3.67	-3.79	-3.91	-4.01	-4.10	-4.18
	σ^2	0.50	0.49	0.47	0.45	0.44	0.42	0.40	0.39	0.39	0.37	0.37	0.36
100	μ	-2.44	-2.72	-2.97	-3.19	-3.39	-3.56	-3.72	-3.86	-3.99	-4.09	-4.19	-4.28
	σ^2	0.51	0.50	0.48	0.46	0.44	0.43	0.42	0.41	0.39	0.38	0.38	0.37
200	μ	-2.49	-2.80	-3.08	-3.34	-3.57	-3.78	-3.98	-4.16	-4.33	-4.49	-4.63	-4.77
	σ^2	0.53	0.53	0.51	0.50	0.49	0.48	0.47	0.46	0.45	0.44	0.44	0.43
500	μ	-2.52	-2.85	-3.15	-3.42	-3.68	-3.92	-4.14	-4.35	-4.55	-4.73	-4.91	-5.08
	σ^2	0.56	0.55	0.54	0.54	0.52	0.51	0.51	0.50	0.49	0.49	0.48	0.48

Correction factors for Westerlund tests

TABLE 20

Finite T correction factors μ_{WP} and σ^2_{WP} for the pooled variance ratio test statistic of Westerlund (WP) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.134103	0.090341	0.059848	0.041406	–	–	–	–	–	–	–	–
	σ^2	0.023753	0.024009	0.017662	0.008442	–	–	–	–	–	–	–	–
20	μ	0.130021	0.088235	0.058862	0.039839	0.028563	0.020743	0.015691	0.012337	0.010055	0.008152	0.006888	0.005882
	σ^2	0.023179	0.023379	0.016530	0.007786	0.005509	0.001913	0.000898	0.000473	0.000358	0.000189	0.000116	0.000075
30	μ	0.128439	0.087240	0.057642	0.039486	0.027710	0.020734	0.015549	0.012131	0.009663	0.007863	0.006583	0.005587
	σ^2	0.022383	0.022640	0.013141	0.007704	0.003774	0.002095	0.000995	0.000485	0.000301	0.000164	0.000105	0.000074
40	μ	0.127737	0.087497	0.058260	0.039498	0.027779	0.020296	0.015351	0.011953	0.009524	0.007786	0.006434	0.005499
	σ^2	0.022774	0.024238	0.013711	0.007207	0.003176	0.001875	0.000852	0.000526	0.000259	0.000173	0.000116	0.000071
50	μ	0.127612	0.087269	0.057848	0.039780	0.028155	0.020321	0.015347	0.012032	0.009611	0.007785	0.006466	0.005431
	σ^2	0.021886	0.022561	0.014201	0.007641	0.004064	0.001679	0.000885	0.000480	0.000302	0.000172	0.000109	0.000069
60	μ	0.126132	0.086633	0.058258	0.039536	0.027969	0.020482	0.015398	0.011930	0.009605	0.007668	0.006420	0.005458
	σ^2	0.022576	0.021776	0.014674	0.006875	0.003571	0.001828	0.000855	0.000581	0.000298	0.000149	0.000109	0.000073
70	μ	0.126749	0.087798	0.058745	0.039786	0.027948	0.020570	0.015422	0.011945	0.009578	0.007713	0.006394	0.005388
	σ^2	0.022292	0.022105	0.014588	0.008070	0.003696	0.001921	0.000864	0.000504	0.000310	0.000161	0.000110	0.000064
80	μ	0.127472	0.086622	0.058596	0.039658	0.028004	0.020706	0.015450	0.011933	0.009544	0.007738	0.006446	0.005445
	σ^2	0.022729	0.022478	0.015641	0.008356	0.003693	0.002071	0.000910	0.000438	0.000282	0.000227	0.000120	0.000078
90	μ	0.126532	0.086117	0.058615	0.039483	0.027891	0.020427	0.015473	0.011944	0.009528	0.007766	0.006423	0.005404
	σ^2	0.022875	0.022878	0.014966	0.008170	0.003682	0.001673	0.000929	0.000476	0.000277	0.000176	0.000111	0.000077
100	μ	0.126276	0.087659	0.058451	0.039214	0.027963	0.020574	0.015349	0.011807	0.009502	0.007778	0.006404	0.005401
	σ^2	0.021918	0.022825	0.015706	0.006865	0.003634	0.001831	0.000905	0.000441	0.000285	0.000167	0.000103	0.000074
200	μ	0.125862	0.087576	0.058500	0.039444	0.027873	0.020609	0.015336	0.011902	0.009598	0.007752	0.006418	0.005387
	σ^2	0.021627	0.023599	0.014543	0.008208	0.003388	0.002159	0.000870	0.000459	0.000270	0.000182	0.000117	0.000073
500	μ	0.126553	0.087158	0.058470	0.039744	0.028117	0.020393	0.015524	0.012077	0.009531	0.007764	0.006423	0.005386
	σ^2	0.022755	0.022336	0.016309	0.007948	0.003348	0.001691	0.000949	0.000598	0.000283	0.000178	0.000117	0.000074

TABLE 21

Finite T correction factors μ_{WP} and σ^2_{WP} for the pooled variance ratio test statistic of Westerlund (WP) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.047254	0.032017	0.021742	0.015676	–	–	–	–	–	–	–	–
	σ^2	0.001690	0.001583	0.001050	0.000672	–	–	–	–	–	–	–	–
20	μ	0.044920	0.029869	0.019839	0.013613	0.009629	0.007118	0.005514	0.004410	0.003651	0.003092	0.002674	0.002344
	σ^2	0.001615	0.001476	0.000988	0.000518	0.000305	0.000108	0.000053	0.000026	0.000016	0.000008	0.000004	0.000003
30	μ	0.044752	0.029176	0.019242	0.013140	0.009232	0.006730	0.005190	0.004111	0.003309	0.002753	0.002328	0.002002
	σ^2	0.001628	0.001439	0.000921	0.000520	0.000213	0.000105	0.000076	0.000034	0.000014	0.000008	0.000004	0.000003
40	μ	0.044586	0.029360	0.019189	0.012981	0.009127	0.006661	0.005080	0.003963	0.003186	0.002620	0.002187	0.001873
	σ^2	0.001638	0.001448	0.000904	0.000517	0.000239	0.000104	0.000057	0.000028	0.000014	0.000007	0.000004	0.000003
50	μ	0.044771	0.028932	0.019108	0.012929	0.009053	0.006620	0.004985	0.003878	0.003128	0.002558	0.002131	0.001827
	σ^2	0.001656	0.001425	0.000925	0.000513	0.000240	0.000122	0.000049	0.000027	0.000014	0.000008	0.000004	0.000003
60	μ	0.044430	0.028739	0.019168	0.012873	0.009049	0.006574	0.004932	0.003877	0.003106	0.002527	0.002115	0.001792
	σ^2	0.001663	0.001406	0.000940	0.000498	0.000262	0.000104	0.000045	0.000025	0.000014	0.000008	0.000004	0.000003
70	μ	0.044702	0.028932	0.018992	0.012897	0.008975	0.006604	0.004928	0.003864	0.003057	0.002517	0.002095	0.001768
	σ^2	0.001672	0.001457	0.000856	0.000480	0.000256	0.000153	0.000051	0.000026	0.000012	0.000009	0.000004	0.000002
80	μ	0.044662	0.028925	0.019105	0.012893	0.008994	0.006506	0.004935	0.003840	0.003042	0.002492	0.002076	0.001769
	σ^2	0.001641	0.001387	0.000924	0.000498	0.000241	0.000107	0.000051	0.000036	0.000013	0.000007	0.000004	0.000003
90	μ	0.044378	0.029038	0.019095	0.012799	0.008984	0.006511	0.004935	0.003818	0.003045	0.002494	0.002067	0.001753
	σ^2	0.001641	0.001402	0.000915	0.000527	0.000241	0.000124	0.000052	0.000024	0.000012	0.000007	0.000004	0.000003
100	μ	0.044509	0.029144	0.019101	0.012680	0.008978	0.006533	0.004862	0.003814	0.003042	0.002481	0.002066	0.001750
	σ^2	0.001681	0.001419	0.000948	0.000427	0.000256	0.000125	0.000044	0.000024	0.000012	0.000007	0.000004	0.000002
200	μ	0.044339	0.029044	0.018997	0.012613	0.008947	0.006467	0.004939	0.003772	0.003028	0.002464	0.002046	0.001724
	σ^2	0.001633	0.001443	0.000960	0.000456	0.000249	0.000110	0.000068	0.000026	0.000011	0.000007	0.000004	0.000003
500	μ	0.044387	0.028966	0.019123	0.012849	0.009033	0.006463	0.004891	0.003791	0.003028	0.002476	0.002031	0.001718
	σ^2	0.001657	0.001398	0.000939	0.000517	0.000276	0.000100	0.000059	0.000025	0.000029	0.000008	0.000004	0.000002

TABLE 22

Finite T correction factors μ_{WP} and σ^2_{WP} for the pooled variance ratio test statistic of Westerlund (WP) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.012730	0.010672	0.008966	0.007687	–	–	–	–	–	–	–	–
	σ^2	0.000048	0.000043	0.000035	0.000029	–	–	–	–	–	–	–	–
20	μ	0.011001	0.008786	0.007101	0.005734	0.004736	0.003982	0.003372	0.002916	0.002553	0.002266	0.002040	0.001855
	σ^2	0.000047	0.000039	0.000032	0.000022	0.000015	0.000011	0.000007	0.000004	0.000003	0.000002	0.000002	0.000001
30	μ	0.010624	0.008457	0.006722	0.005418	0.004382	0.003598	0.003018	0.002557	0.002194	0.001918	0.001692	0.001504
	σ^2	0.000046	0.000040	0.000029	0.000022	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001
40	μ	0.010505	0.008336	0.006592	0.005268	0.004257	0.003500	0.002886	0.002433	0.002087	0.001794	0.001564	0.001379
	σ^2	0.000047	0.000039	0.000031	0.000022	0.000014	0.000010	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
50	μ	0.010484	0.008274	0.006546	0.005196	0.004194	0.003424	0.002833	0.002374	0.002018	0.001737	0.001509	0.001327
	σ^2	0.000046	0.000039	0.000031	0.000021	0.000014	0.000009	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
60	μ	0.010435	0.008215	0.006538	0.005186	0.004178	0.003380	0.002807	0.002346	0.001987	0.001703	0.001477	0.001290
	σ^2	0.000045	0.000038	0.000030	0.000021	0.000015	0.000009	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
70	μ	0.010402	0.008205	0.006503	0.005139	0.004152	0.003379	0.002784	0.002326	0.001968	0.001686	0.001455	0.001278
	σ^2	0.000045	0.000039	0.000030	0.000021	0.000015	0.000009	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
80	μ	0.010388	0.008174	0.006471	0.005153	0.004144	0.003373	0.002782	0.002316	0.001958	0.001678	0.001446	0.001262
	σ^2	0.000045	0.000039	0.000030	0.000022	0.000015	0.000009	0.000006	0.000004	0.000002	0.000002	0.000001	0.000001
90	μ	0.010419	0.008218	0.006489	0.005150	0.004121	0.003355	0.002767	0.002299	0.001955	0.001666	0.001436	0.001255
	σ^2	0.000045	0.000040	0.000030	0.000022	0.000015	0.000009	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
100	μ	0.010409	0.008227	0.006459	0.005150	0.004122	0.003371	0.002761	0.002308	0.001946	0.001663	0.001427	0.001251
	σ^2	0.000046	0.000039	0.000030	0.000021	0.000014	0.000010	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
200	μ	0.010385	0.008174	0.006468	0.005101	0.004106	0.003333	0.002727	0.002277	0.001924	0.001641	0.001411	0.001233
	σ^2	0.000046	0.000039	0.000029	0.000021	0.000014	0.000009	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001
500	μ	0.010391	0.008218	0.006429	0.005109	0.004097	0.003330	0.002740	0.002287	0.001915	0.001637	0.001412	0.001225
	σ^2	0.000046	0.000039	0.000029	0.000021	0.000014	0.000009	0.000006	0.000004	0.000003	0.000002	0.000001	0.000001

TABLE 23

Finite T correction factors μ_{WG} and σ^2_{WG} for the group-mean variance ratio test statistic of Westerlund (WG) for case 1

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.097280	0.062168	0.041707	0.029765	–	–	–	–	–	–	–	–
	σ^2	0.006535	0.003618	0.001855	0.000996	–	–	–	–	–	–	–	–
20	μ	0.094159	0.060167	0.040204	0.027977	0.020670	0.015628	0.012248	0.009874	0.008156	0.006809	0.005869	0.005078
	σ^2	0.006367	0.003594	0.001898	0.000994	0.000563	0.000319	0.000188	0.000118	0.000077	0.000050	0.000035	0.000024
30	μ	0.093253	0.059739	0.039924	0.027793	0.020274	0.015393	0.011995	0.009622	0.007835	0.006528	0.005556	0.004775
	σ^2	0.006264	0.003569	0.001886	0.001007	0.000557	0.000322	0.000192	0.000120	0.000078	0.000052	0.000036	0.000026
40	μ	0.093045	0.059747	0.040104	0.027893	0.020345	0.015247	0.011901	0.009470	0.007748	0.006447	0.005409	0.004682
	σ^2	0.006313	0.003604	0.001901	0.001011	0.000566	0.000317	0.000194	0.000121	0.000077	0.000051	0.000036	0.000026
50	μ	0.093674	0.059925	0.039915	0.027897	0.020311	0.015320	0.011843	0.009528	0.007751	0.006403	0.005425	0.004624
	σ^2	0.006336	0.003620	0.001897	0.001031	0.000563	0.000324	0.000193	0.000123	0.000080	0.000053	0.000037	0.000026
60	μ	0.092438	0.059770	0.039945	0.027881	0.020247	0.015314	0.011856	0.009403	0.007746	0.006349	0.005359	0.004614
	σ^2	0.006223	0.003588	0.001909	0.001023	0.000565	0.000327	0.000194	0.000121	0.000080	0.000052	0.000037	0.000026
70	μ	0.092707	0.060086	0.040101	0.028007	0.020303	0.015356	0.011872	0.009470	0.007740	0.006353	0.005353	0.004588
	σ^2	0.006266	0.003637	0.001915	0.001039	0.000567	0.000330	0.000192	0.000122	0.000080	0.000053	0.000036	0.000026
80	μ	0.092990	0.059679	0.040178	0.027801	0.020363	0.015383	0.011912	0.009467	0.007690	0.006343	0.005377	0.004596
	σ^2	0.006286	0.003613	0.001960	0.001014	0.000564	0.000332	0.000196	0.000121	0.000080	0.000052	0.000038	0.000027
90	μ	0.092902	0.059475	0.040038	0.027731	0.020213	0.015297	0.011904	0.009457	0.007682	0.006399	0.005351	0.004564
	σ^2	0.006301	0.003555	0.001934	0.001013	0.000565	0.000326	0.000199	0.000123	0.000079	0.000054	0.000037	0.000026
100	μ	0.092761	0.060109	0.039912	0.027805	0.020303	0.015355	0.011817	0.009372	0.007663	0.006391	0.005367	0.004570
	σ^2	0.006279	0.003681	0.001899	0.001007	0.000567	0.000331	0.000196	0.000120	0.000079	0.000054	0.000037	0.000026
200	μ	0.092586	0.059996	0.040131	0.027805	0.020329	0.015301	0.011848	0.009421	0.007752	0.006360	0.005359	0.004557
	σ^2	0.006262	0.003658	0.001926	0.001021	0.000570	0.000332	0.000193	0.000121	0.000082	0.000054	0.000037	0.000027
500	μ	0.092833	0.059939	0.039880	0.027927	0.020456	0.015321	0.011953	0.009471	0.007720	0.006383	0.005357	0.004568
	σ^2	0.006280	0.003640	0.001903	0.001040	0.000576	0.000328	0.000201	0.000125	0.000080	0.000054	0.000037	0.000026

TABLE 24

Finite T correction factors μ_{WG} and σ^2_{WG} for the group-mean variance ratio test statistic of Westerlund (WG) for case 2

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.033845	0.022279	0.015499	0.011554	–	–	–	–	–	–	–	–
	σ^2	0.000534	0.000286	0.000136	0.000067	–	–	–	–	–	–	–	–
20	μ	0.031467	0.020144	0.013503	0.009566	0.007055	0.005470	0.004388	0.003619	0.003070	0.002656	0.002337	0.002086
	σ^2	0.000519	0.000278	0.000135	0.000065	0.000031	0.000016	0.000008	0.000005	0.000003	0.000002	0.000001	0.000001
30	μ	0.031119	0.019598	0.013061	0.009119	0.006714	0.005094	0.004026	0.003279	0.002725	0.002313	0.001994	0.001742
	σ^2	0.000520	0.000273	0.000131	0.000063	0.000031	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001
40	μ	0.030974	0.019634	0.012962	0.009010	0.006567	0.005006	0.003926	0.003155	0.002601	0.002183	0.001862	0.001616
	σ^2	0.000517	0.000276	0.000133	0.000063	0.000032	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001
50	μ	0.031012	0.019428	0.012872	0.008931	0.006514	0.004938	0.003848	0.003084	0.002547	0.002125	0.001801	0.001561
	σ^2	0.000521	0.000272	0.000132	0.000063	0.000031	0.000016	0.000008	0.000005	0.000003	0.000002	0.000001	0.000001
60	μ	0.030854	0.019335	0.012873	0.008909	0.006475	0.004905	0.003817	0.003065	0.002517	0.002094	0.001782	0.001531
	σ^2	0.000515	0.000267	0.000133	0.000063	0.000031	0.000016	0.000008	0.000005	0.000003	0.000002	0.000001	0.000001
70	μ	0.030937	0.019337	0.012838	0.008912	0.006459	0.004882	0.003804	0.003050	0.002484	0.002081	0.001761	0.001510
	σ^2	0.000520	0.000269	0.000130	0.000063	0.000031	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001
80	μ	0.030938	0.019390	0.012803	0.008898	0.006470	0.004861	0.003800	0.003032	0.002469	0.002064	0.001745	0.001505
	σ^2	0.000522	0.000274	0.000133	0.000064	0.000031	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001
90	μ	0.030816	0.019383	0.012828	0.008885	0.006442	0.004844	.0.003781	0.003020	0.002467	0.002059	0.001735	0.001492
	σ^2	0.000516	0.000273	0.000132	0.000063	0.000031	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001
100	μ	0.030807	0.019439	0.012813	0.008851	0.006436	0.004853	0.003763	0.003023	0.002466	0.002049	0.001732	0.001489
	σ^2	0.000518	0.000275	0.000133	0.000062	0.000031	0.000016	0.000008	0.000005	0.000003	0.000002	0.000001	0.000001
200	μ	0.030743	0.019368	0.012756	0.008795	0.006430	0.004821	0.003773	0.002985	0.002456	0.002033	0.001715	0.001464
	σ^2	0.000518	0.000273	0.000131	0.000061	0.000031	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001
500	μ	0.030787	0.019406	0.012816	0.008845	0.006434	0.004814	0.003747	0.002990	0.002443	0.002032	0.001705	0.001460
	σ^2	0.000517	0.000273	0.000133	0.000062	0.000032	0.000016	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001

TABLE 25

Finite T correction factors μ_{WG} and σ^2_{WG} for the group-mean variance ratio test statistic of Westerlund (WG) for case 3

T/l		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.010960	0.009182	0.007771	0.006731	–	–	–	–	–	–	–	–
	σ^2	0.000022	0.000016	0.000011	0.000008	–	–	–	–	–	–	–	–
20	μ	0.009083	0.007217	0.005833	0.004756	0.003979	0.003381	0.002909	0.002553	0.002270	0.002038	0.001853	0.001701
	σ^2	0.000021	0.000015	0.000011	0.000007	0.000005	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000005	0.0000003
30	μ	0.008691	0.006856	0.005458	0.004424	0.003612	0.003012	0.002557	0.002195	0.001911	0.001685	0.001503	0.001349
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000005	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000005	0.0000003
40	μ	0.008563	0.006733	0.005320	0.004267	0.003489	0.002899	0.002429	0.002071	0.001794	0.001565	0.001377	0.001226
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000005	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000004	0.0000003
50	μ	0.008540	0.006673	0.005272	0.004220	0.003431	0.002834	0.002370	0.002010	0.001732	0.001508	0.001322	0.001170
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000004	0.0000003
60	μ	0.008497	0.006632	0.005259	0.004197	0.003409	0.002795	0.002345	0.001987	0.001703	0.001471	0.001290	0.001139
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000005	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000005	0.0000003
70	μ	0.008475	0.006618	0.005230	0.004160	0.003384	0.002786	0.002319	0.001964	0.001680	0.001456	0.001270	0.001123
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000004	0.0000003
80	μ	0.008455	0.006582	0.005201	0.004154	0.003366	0.002780	0.002318	0.001953	0.001674	0.001445	0.001260	0.001109
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000005	0.0000003
90	μ	0.008481	0.006599	0.005211	0.004154	0.003359	0.002771	0.002305	0.001941	0.001666	0.001435	0.001251	0.001101
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000005	0.0000003
100	μ	0.008453	0.006606	0.005184	0.004156	0.003358	0.002768	0.002300	0.001944	0.001659	0.001432	0.001243	0.001096
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000004	0.0000003
200	μ	0.008441	0.006568	0.005193	0.004116	0.003347	0.002743	0.002273	0.001919	0.001638	0.001409	0.001226	0.001078
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000004	0.0000003
500	μ	0.008437	0.006594	0.005166	0.004120	0.003336	0.002739	0.002277	0.001921	0.001631	0.001408	0.001223	0.001072
	σ^2	0.000021	0.000015	0.000010	0.000007	0.000004	0.000003	0.000002	0.000001	0.000001	0.000001	0.0000005	0.0000003

Correction factors for system tests

TABLE 26

Finite T correction factors μ_{LR} and σ^2_{LR} for case 1

T/m-k		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	1.00	4.97	11.40	19.78	–	–	–	–	–	–	–	–
	σ^2	1.75	8.32	19.39	34.71	–	–	–	–	–	–	–	–
20	μ	1.06	5.45	12.90	23.03	35.61	50.24	66.71	84.95	104.78	125.83	148.15	171.74
	σ^2	1.92	9.05	21.03	37.62	58.62	84.27	113.31	148.14	184.39	226.09	271.19	320.41
30	μ	1.09	5.66	13.54	24.47	38.20	54.51	73.08	93.92	116.69	141.48	167.88	195.98
	σ^2	2.01	9.48	21.75	39.14	61.75	88.48	117.93	152.72	193.06	237.08	284.01	336.21
40	μ	1.10	5.74	13.88	25.30	39.71	56.88	76.86	99.28	123.98	150.95	180.01	211.09
	σ^2	2.06	9.73	22.29	40.10	63.23	89.36	121.90	158.13	198.57	244.92	293.25	346.30
50	μ	1.10	5.82	14.09	25.79	40.62	58.49	79.28	102.71	128.85	157.32	188.24	221.35
	σ^2	2.05	9.90	22.95	41.23	63.97	91.55	124.31	161.67	202.75	247.66	298.42	357.47
60	μ	1.12	5.85	14.28	26.11	41.31	59.67	80.99	105.26	132.33	161.95	194.07	228.72
	σ^2	2.15	9.83	23.44	41.90	65.21	92.74	127.47	164.55	207.47	254.13	304.52	358.61
70	μ	1.12	5.91	14.37	26.39	41.81	60.47	82.26	107.12	134.89	165.41	198.64	234.42
	σ^2	2.14	10.08	23.44	42.19	66.12	94.90	128.93	166.05	208.39	257.97	307.79	369.28
80	μ	1.12	5.94	14.45	26.56	42.16	61.08	83.33	108.53	136.97	168.09	202.06	238.98
	σ^2	2.12	10.09	23.68	42.41	66.40	95.85	129.08	169.05	211.98	262.02	313.65	371.03
90	μ	1.12	5.96	14.53	26.77	42.42	61.60	84.06	109.80	138.45	170.22	204.96	242.70
	σ^2	2.13	10.22	23.77	43.50	66.64	96.19	130.49	170.71	213.41	260.29	317.19	375.56
100	μ	1.13	5.96	14.57	26.84	42.69	62.04	84.73	110.63	139.85	171.96	207.29	245.47
	σ^2	2.16	10.27	23.97	43.18	67.53	96.97	132.94	170.64	215.28	266.49	320.99	377.86
200	μ	1.13	6.04	14.83	27.44	43.88	63.90	87.75	115.12	146.11	180.60	218.52	259.80
	σ^2	2.19	10.37	24.48	44.17	69.79	100.86	136.66	178.12	224.39	279.63	336.48	394.70
500	μ	1.14	6.09	14.96	27.82	44.56	65.17	89.67	118.03	150.13	186.00	225.74	269.25
	σ^2	2.22	10.60	24.81	45.29	71.04	103.68	141.29	183.52	233.34	289.01	344.85	413.23

TABLE 27

Finite T correction factors μ_{LR} and σ^2_{LR} for case 2

T/m-k		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	3.22	8.96	16.72	26.17	–	–	–	–	–	–	–	–
	σ^2	5.09	14.49	28.11	45.92	–	–	–	–	–	–	–	–
20	μ	3.59	10.24	19.65	31.50	45.51	61.43	79.11	98.30	118.94	140.82	163.96	188.04
	σ^2	5.81	16.01	31.04	50.38	74.02	101.92	135.14	170.36	210.06	252.84	300.82	347.16
30	μ	3.73	10.79	20.89	33.86	49.43	67.37	87.51	109.60	133.81	159.59	187.12	216.12
	σ^2	6.12	16.90	32.35	52.49	77.52	107.03	140.33	180.46	221.95	266.18	317.92	371.30
40	μ	3.80	11.09	21.60	35.21	51.65	70.82	92.55	116.59	142.83	171.25	201.69	233.86
	σ^2	6.25	17.45	33.32	54.68	79.33	110.98	144.42	184.69	226.89	275.85	328.78	381.85
50	μ	3.86	11.27	22.05	36.05	53.15	73.12	95.83	121.23	148.88	179.12	211.52	246.06
	σ^2	6.40	17.85	34.20	55.62	81.71	113.28	148.06	189.04	232.97	282.80	336.08	391.97
60	μ	3.90	11.36	22.34	36.62	54.13	74.78	98.19	124.38	153.28	184.80	218.74	255.15
	σ^2	6.51	17.89	34.40	55.75	82.79	114.80	150.95	190.61	237.51	284.44	341.12	400.51
70	μ	3.92	11.47	22.56	37.12	54.96	75.91	99.92	126.87	156.63	189.04	224.23	261.89
	σ^2	6.51	18.14	35.56	57.01	84.99	116.19	152.63	193.24	242.18	292.74	347.74	410.06
80	μ	3.93	11.56	22.77	37.46	55.53	76.82	101.30	128.67	159.18	192.45	228.51	267.12
	σ^2	6.51	18.51	35.46	57.88	85.34	117.65	154.69	194.41	243.23	293.13	351.59	408.80
90	μ	3.94	11.60	22.89	37.78	55.96	77.61	102.42	130.25	161.28	195.21	231.88	271.34
	σ^2	6.63	18.50	35.43	58.35	86.07	118.16	156.11	198.41	245.40	297.28	355.44	412.20
100	μ	3.95	11.65	23.04	37.94	56.36	78.21	103.20	131.55	162.90	197.40	234.83	275.03
	σ^2	6.66	18.48	35.90	58.56	85.85	120.17	157.46	200.23	246.99	298.91	357.41	418.69
200	μ	3.99	11.84	23.55	38.97	58.11	80.97	107.48	137.43	171.02	207.98	248.59	292.28
	σ^2	6.72	19.19	37.07	60.54	89.44	124.84	163.45	208.60	258.08	316.24	377.25	442.71
500	μ	4.03	11.97	23.84	39.61	59.32	82.79	110.16	141.31	176.29	215.07	257.55	303.71
	σ^2	6.81	19.22	37.82	62.06	91.65	128.25	168.56	214.31	269.14	328.44	391.92	461.76

TABLE 28

Finite T correction factors μ_{LR} and σ^2_{LR} for case 3

T/m-k		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	1.00	6.13	13.14	21.73	–	–	–	–	–	–	–	–
	σ^2	2.02	10.32	22.06	37.82	–	–	–	–	–	–	–	–
20	μ	1.00	7.03	15.70	26.67	39.80	54.87	71.61	89.87	109.48	130.55	152.58	175.78
	σ^2	2.01	11.59	24.80	42.09	63.89	89.64	120.57	153.31	192.50	232.73	276.72	326.25
30	μ	1.01	7.43	16.82	28.88	43.55	60.65	79.87	101.12	124.24	149.15	175.69	203.66
	σ^2	2.03	12.28	26.69	44.68	67.26	94.73	126.01	163.09	201.99	245.51	296.49	348.48
40	μ	1.00	7.62	17.41	30.16	45.70	64.01	84.74	107.86	133.14	160.56	189.96	221.29
	σ^2	2.01	12.58	27.33	46.47	69.45	97.56	130.57	167.31	210.10	255.71	307.56	362.07
50	μ	0.99	7.75	17.80	30.99	47.14	66.26	88.01	112.34	139.28	168.46	199.87	233.50
	σ^2	1.95	12.90	28.13	47.82	71.45	100.84	134.01	170.88	215.57	262.25	311.43	368.27
60	μ	1.00	7.85	18.08	31.54	48.15	67.73	90.28	115.65	143.54	174.08	207.01	242.30
	σ^2	2.04	13.18	28.88	48.80	73.02	103.21	136.37	176.03	218.73	265.96	321.32	376.13
70	μ	1.00	7.92	18.29	31.94	48.88	68.96	92.06	117.99	146.77	178.35	212.41	249.11
	σ^2	1.99	13.50	28.96	49.10	74.03	104.47	139.61	176.93	221.35	272.42	322.91	384.19
80	μ	1.00	7.97	18.39	32.29	49.42	69.79	93.35	119.85	149.31	181.56	216.61	254.26
	σ^2	1.99	13.47	29.26	49.89	75.15	105.69	139.68	179.88	224.96	274.09	326.32	386.03
90	μ	1.00	7.99	18.54	32.51	49.85	70.56	94.36	121.39	151.39	184.30	220.04	258.77
	σ^2	1.98	13.59	29.44	50.28	75.72	106.62	143.20	181.91	227.37	277.58	329.77	393.15
100	μ	1.00	8.05	18.65	32.68	50.26	71.07	95.24	122.60	152.95	186.47	222.93	262.24
	σ^2	1.98	13.62	29.93	50.67	76.15	107.02	143.69	184.28	228.62	276.88	337.40	394.47
200	μ	0.99	8.17	19.03	33.64	51.91	73.82	99.35	128.39	161.02	197.08	236.39	279.30
	σ^2	1.93	14.00	30.73	52.67	79.67	111.83	149.76	193.45	240.25	295.07	352.65	416.60
500	μ	0.99	8.25	19.34	34.29	53.01	75.59	101.99	132.18	166.19	204.00	245.43	290.75
	σ^2	1.97	14.31	31.78	54.43	82.13	115.94	155.02	201.71	253.81	307.90	367.34	434.40

TABLE 29

Finite T correction factors μ_{LR} and σ^2_{LR} for case 4

T/m-k		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	4.14	10.15	17.74	26.70	–	–	–	–	–	–	–	–
	σ^2	6.16	16.00	30.01	47.34	–	–	–	–	–	–	–	–
20	μ	5.03	12.69	22.71	34.80	48.88	64.64	81.86	100.49	120.52	141.62	163.83	186.95
	σ^2	7.53	18.67	34.11	53.81	78.23	106.66	137.53	174.30	212.12	255.28	302.04	351.78
30	μ	5.43	13.80	24.89	38.62	54.63	72.81	93.11	115.27	139.10	164.66	191.72	220.20
	σ^2	8.32	20.31	36.57	57.21	82.68	112.75	146.05	185.01	226.36	272.51	322.24	380.27
40	μ	5.61	14.41	26.17	40.80	58.00	77.72	99.86	124.17	150.66	179.01	209.23	241.36
	σ^2	8.65	21.37	38.28	59.58	86.25	117.20	151.45	192.42	235.72	283.36	335.43	394.13
50	μ	5.75	14.82	26.98	42.10	60.19	80.98	104.34	130.19	158.45	188.89	221.48	255.98
	σ^2	8.97	22.17	39.77	61.08	89.30	120.15	153.72	196.79	239.78	293.55	346.37	400.24
60	μ	5.82	15.07	27.54	43.09	61.73	83.29	107.56	134.57	164.02	195.97	230.33	266.85
	σ^2	9.06	22.59	40.70	62.76	90.35	123.03	159.36	201.46	245.69	298.38	350.03	412.23
70	μ	5.89	15.26	27.93	43.88	62.86	84.98	109.93	137.77	168.22	201.48	237.10	275.10
	σ^2	9.32	22.90	41.21	64.23	91.31	125.07	162.69	203.69	250.92	302.86	357.78	418.92
80	μ	5.96	15.41	28.25	44.47	63.81	86.28	111.88	140.32	171.66	205.64	242.26	281.56
	σ^2	9.46	23.19	41.96	65.37	94.64	125.63	164.71	205.86	254.07	306.95	359.60	423.67
90	μ	5.99	15.53	28.49	44.88	64.52	87.36	113.33	142.38	174.36	209.09	246.69	286.86
	σ^2	9.67	23.58	42.35	66.03	94.73	127.95	166.09	209.43	260.46	309.55	367.29	430.26
100	μ	6.02	15.61	28.71	45.24	65.11	88.24	114.53	143.99	176.40	211.89	250.19	291.17
	σ^2	9.60	23.93	42.48	66.36	94.86	129.49	168.54	210.94	258.85	312.01	372.00	436.40
200	μ	6.16	16.08	29.64	46.94	67.82	92.35	120.48	151.95	187.07	225.49	267.27	312.42
	σ^2	10.07	24.73	44.59	70.05	100.41	136.18	178.96	222.20	276.05	330.16	392.91	463.94
500	μ	6.26	16.33	30.26	48.03	69.62	94.97	124.17	157.26	193.96	234.45	278.66	326.68
	σ^2	10.44	25.35	46.02	71.86	104.10	141.53	185.03	234.67	286.81	345.58	411.83	481.41

TABLE 30

Finite T correction factors μ_{LR} and σ^2_{LR} for case 5

T/m-k		1	2	3	4	5	6	7	8	9	10	11	12
10	μ	0.99	6.54	13.57	21.84	–	–	–	–	–	–	–	–
	σ^2	1.97	10.79	23.04	38.66	–	–	–	–	–	–	–	–
20	μ	0.99	8.11	17.47	28.91	42.20	57.16	73.58	91.49	110.62	130.84	152.13	174.49
	σ^2	1.95	12.89	27.02	44.88	67.09	93.01	122.73	156.37	194.05	234.06	277.62	326.94
30	μ	0.99	8.76	19.23	32.17	47.42	64.87	84.35	105.57	128.60	153.27	179.50	207.03
	σ^2	2.00	14.16	29.54	48.70	72.24	100.28	132.29	167.96	206.37	252.34	303.15	352.81
40	μ	1.00	9.15	20.18	34.09	50.52	69.41	90.76	114.13	139.86	167.28	196.68	227.90
	σ^2	2.01	14.76	31.02	51.21	75.15	103.80	138.47	175.47	217.52	263.76	315.05	368.66
50	μ	1.00	9.39	20.83	35.26	52.59	72.48	94.93	120.01	147.30	176.82	208.46	242.26
	σ^2	1.99	15.36	31.91	52.78	77.99	107.68	141.87	180.82	223.07	271.82	318.77	378.48
60	μ	1.00	9.54	21.29	36.13	53.97	74.63	97.96	124.19	152.77	183.78	217.22	252.90
	σ^2	1.99	15.64	32.98	54.46	80.15	110.42	146.35	184.05	227.37	277.56	329.12	386.89
70	μ	0.99	9.65	21.61	36.73	54.98	76.26	100.36	127.27	156.89	189.04	223.76	260.91
	σ^2	1.98	15.94	33.76	54.78	81.63	112.63	146.98	188.32	234.51	283.27	337.15	394.53
80	μ	1.00	9.76	21.87	37.25	55.87	77.51	102.18	129.77	160.01	193.25	229.02	267.38
	σ^2	2.01	16.35	34.04	56.48	83.29	113.42	149.59	189.78	237.91	286.84	343.06	403.27
90	μ	1.00	9.85	22.06	37.64	56.44	78.45	103.52	131.61	162.73	196.49	233.21	272.45
	σ^2	1.99	16.34	34.49	57.02	84.09	115.80	151.94	193.11	240.19	291.46	347.52	404.28
100	μ	1.00	9.90	22.23	37.96	57.06	79.27	104.71	133.25	164.83	199.20	236.60	276.91
	σ^2	1.98	16.68	34.83	57.65	85.45	117.73	154.11	194.74	241.14	290.08	348.40	412.78
200	μ	1.01	10.19	22.98	39.41	59.48	83.09	110.16	140.93	175.10	212.55	253.55	297.61
	σ^2	2.03	17.36	36.78	60.98	88.95	123.56	161.88	207.05	257.35	311.83	370.54	436.12
500	μ	1.00	10.34	23.46	40.38	61.06	85.63	113.87	145.90	181.75	221.43	264.54	311.53
	σ^2	1.99	17.86	37.84	62.78	93.65	129.56	171.20	216.27	269.89	327.17	389.63	458.78

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