MACROECONOMIC POLICY AND
THE OPTIMAL DESTRUCTION OF VAMPIRES

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Forschungsbericht/
Research Memorandum No. 162

March 1981

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Summary

This paper provides a framework for the synthesis of vampirism and macroeconomics. Vampires influence macroeconomic activity by devouring the labor force and by provoking protective human activities which divert resources from other productive uses. On the other hand, human macroeconomic activity influences vampirism by providing blood whereby vampires are nourished and stakes whereby they are destroyed. A mathematical model is built which describes the delicate ecological balance between humans and vampires. In this context, the role of stabilization policy is examined. The complete destruction of the vampire species is shown to be socially undesirable.

Zusammenfassung

Dieser Artikel präsentiert eine mathematische Synthese der vampirischen Blutsaugerei und der makroökonomischen Theorie. Vampire beeinflussen das makroökonomische Geschehen in dem sie menschliche Arbeitskräfte vernichten und ökonomische Schutzmaßnahmen provozieren. Menschen beeinflussen vampirische Aktivität, in dem sie die Vampire durch Blut ernähren und durch Spieße töten. Das empfindliche ökologische Gleichgewicht, in dem sich Menschen und Vampire befinden, wird anhand eines mathematischen Modells erläutert. Die Auswirkungen der Stabilisierungspolitik werden in diesem Zusammenhang beschrieben. Eine kontrolltheoretische Analyse beweist, daß die komplette Ausrottung der Vampire nicht menschlich wünschenswert ist.

CONTENT page

1. Introduction

Although human beings have endured the recurring ravages of vampires for centuries, scarcely any attempts have been made to analyze the macroeconomic implications of this problem and devise socially optimal policy responses. Despite the increasing incidence of vampire epidemics in recent years (in Transylvania, Hollywood, and elsewhere), vampirism remains a thoroughly neglected topic in the theory of macroeconomic policy.

The "vampires" considered in this paper are not the blood-sucking bats (e.g. Desmodus rotundus or Diphylla ecaudata) to be found in the forests of tropical America, but the bood-sucking ghosts of dead homo sapiens. The bats are comparatively innocuous; aside from taking their occasional blood sample of missionaries asleep in the jungle, they have had no measurable influence on human welfare. The blood-sucking ghosts, on the other hand, have periodically provided serious threats to human populations; their most conspicuous macroeconomic impact derives from their detrimental effect on the labor force.

While the behavior of vampires has been studied and documented over long periods of time, it was not until the recent appearance of the Hartl-Mehlmann study (1980) that the interrelation between human and vampire populations was modeled mathematically. This pathbreaking study is, essentially, operations-research advice to vampires: it indicates how vampires' utility from blood intake can be optimized through the appropriate depletion of renewable human resources. Clearly, this approach is somewhat misguided. One wonders what conceivable interest the authors could have had in helping vampires solve their intertemporal consumption problem. The implicit assumption of the Invisible Hand (whereby vampires, in pursuing their own interests, pursue those of human beings as well) is questionable in this context.

The Hartl-Mehlmann study is not concerned with the macroeconomic implications of blood-sucking behavior modes. Nor does it consider the policy instruments whereby human beings can protect themselves from vampires. Instead, humans are modeled as passive receptacles of blood, whose cultivation and harvest is left to vampire discretion.

The purpose of this paper is to provide a framework for the synthesis of vampirism and macroeconomics. Section 2 outlines a simple model of human and vampire behavior patterns. This model is devoted to a descriptive, non-optimizing analysis of the delicate ecological balance between humans and vampires. The demographic and macroeconomic impacts of a particular vampire stabilization policy are examined. Section 3 is concerned with the optimal destruction of vampires. This section demonstrates that it is socially undesirable to drive the vampire species to exinction. Furthermore, a vampire-neutrality theorem is discussed. Finally, Section 4 suggests an area for further research: the formulation of optimal macroeconomic policy when vampires have rational expectations.

2. A Model of Human-Vampire Dynamics

We assume that the "representative vampire" requires an exogenously given amount of blood per unit of time. The vampire emerges from the dead body which it inhabits and finds a human being from whose neck the requisite blood may be extracted. Immediately upon being bitten, the human turns into a vampire. A vampire may be deactivated by exposure to sunlight or by a stake driven through the dead body in which it resides.

We assume that human beings procreate at an exogenously given rate. Each human provides labor services (i.e. the labor force participation rate is 100%) and these services may be devoted to the production of either widgets or stakes. Each widget contributes directly to human wellbeing, whereas each stake is used to kill one vampire.

The allocation of labor services between widgets and stakes is of critical importance in our model. A transfer of labor services from the widget sector to the stake sector reduces human welfare at present, but may raise welfare in the future (since an increase in stake production reduces the vampire population and thereby increases the future labor force whereby future widgets may be produced). Thus humans face an intertemporal welfare tradeoff. Consequently, myopic humans, who maximize their welfare at every instant of time, may be expected to destroy a socially suboptimal number of vampires.

Let L be the size of the labor force and V the size of the vampire population. n is the (constant) rate of human procreation and ρ is the vampire's blood requirement coefficient (i.e. blood per vampire, where blood is measured in units of average homo sapien capacity). Then the rate of change of the human labor force is given by

$$(1)\quad \dot{L} = n\,L - \rho\,V,$$
$$\text{where}\quad \dot{L} = (dL/dt);\quad L, V, n, \rho \geq 0.$$

Let S be the quantity of stakes produced and σ the (constant) rate of vampire attrition (through sunlight). Then the rate of change of the vampire population is given by

$$(2)\quad \dot{V} = (\rho - \sigma)\,V - S,$$
$$\text{where}\quad \dot{V} = (dV/dt);\quad \sigma, S \geq 0.$$

This is admittedly a rather simplistic analytical framework. In reality, humans have more than one vampire-control instrument to choose from: in addition to stakes, crosses may hold vampires at bay (albeit temporarily) and even rosary beads (suitably draped about the neck) have been demonstrably effective. These are all matters from which our model abstracts.

Let there be a fixed-coefficients technology whereby labor services are transformed into widgets and stakes:

$$(3)\quad a \cdot S + b \cdot W = L,$$
$$\text{where}\quad a, b > 0;\quad W > 0,$$

and W is the quantity of widgets produced, and a and b are the labor-stake and labor-widget coefficient, respectively.

We define

$$(4)\quad x = (L/V)$$

$$(5)\quad s = (S/V),\ \text{and}$$

$$(6)\quad w = (W/L).$$

Equations (1) and (2) imply that

$$(7)\quad \dot{x} = (n - \rho + \sigma + s)\, x - \rho.$$

Equation (3) implies that

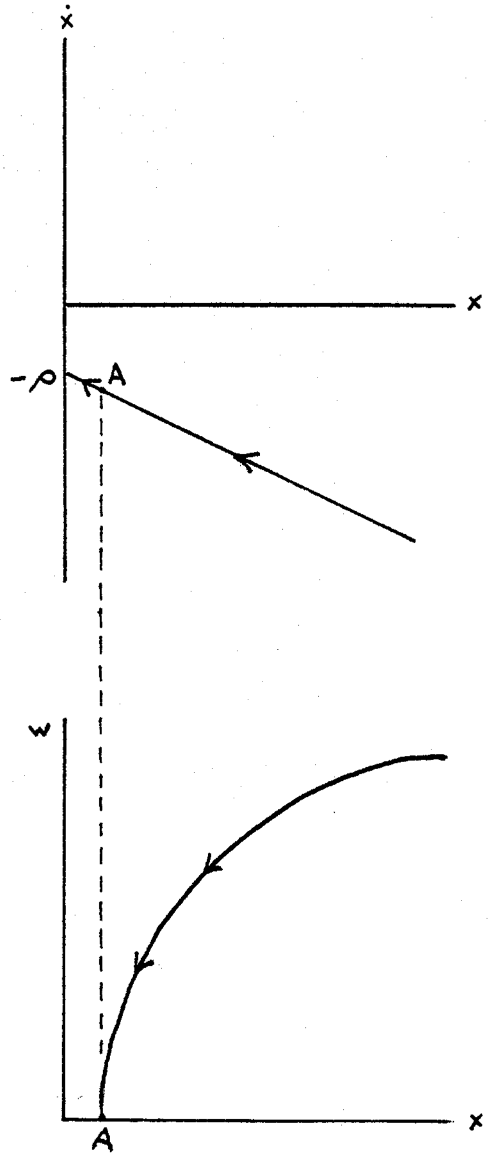
$$(8)\quad w = \frac{1}{b} - \frac{a}{b}\ \frac{s}{x}.$$

It is not unrealistic to assume that the production of stakes is guided by the number of vampires in existence. According to one vampire stabilization policy, the ratio of stakes to vampires is held constant through time:

$$(9)\quad s = \bar{s}.$$

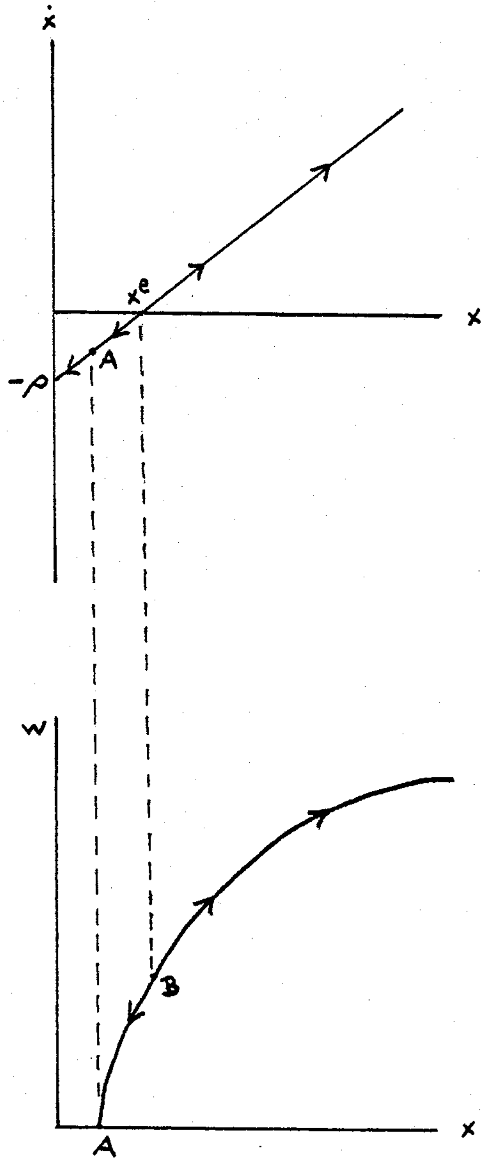
This policy may be identified as Phillips' "proportional" stabilization policy[1].

Equations (7), (8), and (9) describe the labor-vampire ratio (x) and the widget-labor ratio (W/L) through time. The behavior of this system is pictured in Figures 1a and 1b.



$(n - \rho + \sigma + s) < 0$

Figure 1a



$(n - \rho + \sigma + s) > 0$

Figure 1b

Figure 1a illustrates the Vampire Impossibility Theorem:

Theorem 1: If the numer of stakes per vampire remains below the critical level $s^c = p - n - \sigma$, it is impossible for the human race to survive.

Figure 1a shows the vampire population rising indefinitely relative to the human population. Once point A is reached, there is no longer sufficient labor available to put the stabilization policy (9) into practice. From Equation (1) it is evident that the human population declines to zero after a finite span of time. The theorem provides a clear and succinct statement of the importance of stake production for macroeconomic activity in the long run.

Figure 1b shows how fragile the ecological balance between the human and vampire population really is. If $x > x^e$, the ratio of humans to vampires rises without limit and, consequently, widgets per head (human head, that is) rise without limit as well. On the other hand, if $x < x^e$, the extinction of the human race is inevitable.

The two figures provide an explanation for a well-documented empirical observation: whenever the production of stakes is neglected for a prolonged period of time, a vampire epidemic invariably follows.

The figures also indicate that the stabilization policy (9) is unable to induce a dynamically stable stationary state compatible with the survival of human beings. Such a deficiency of proportional stabilization policies has been noted (with regard to other areas of macroeconomics) by a number of economists[2].

3. Macroeconomic Policy and Optimal Vampire Destruction

Over the past few centuries, a number of prominent investigators of vampirism have suggested that all vampires should

be destroyed. This section reaches the surprising conclusion that, under the conditions specified below, such a policy would not be socially optimal.
Suppose that human social welfare at every instant of time depends solely on the per capita consumption of widgets:

$$(9) \quad U = U(w), \qquad \text{where } U' > 0,\ U'' < 0.$$

By Equation (8),

$$(9a) \quad U = U(\tfrac{1}{b} - \tfrac{a}{b} \cdot \tfrac{s}{x}).$$

We assume that social welfare is to be maximized from the present to the infinite future. The social rate of time preference, r, is constant. Utilities at different points in time are fucntionally independent of one another.

The the optimal production of stakes and the optimal depletion of vampires may be found by solving the following optimal control problem:

$$(10) \quad \text{Maximize} \int_0^\infty e^{-rt}\, U(\tfrac{1}{b} - \tfrac{a}{b}\tfrac{s}{x})\, dt$$

$$\text{subject to} \quad \dot{x} = (n - \rho + \sigma + s)\, x - \rho.$$

where s is the control variable and x is the state variable.

The current-value Hamiltonian is

$$H = U(\tfrac{1}{b} - \tfrac{a}{b} \cdot \tfrac{s}{x}) + \mu\left[(n - \rho + \sigma + s)\, x - \rho\right]$$

where μ is the costate variable. The first-order conditions are[3]:

$$(11) \quad \frac{\partial H}{\partial s} = 0 \Leftrightarrow U' \cdot \frac{a}{b} = \mu \cdot x^2$$

$$(12) \quad \frac{\partial H}{\partial x} = \dot{\mu} - r\mu \Leftrightarrow$$

$$U' \cdot \frac{a}{b} \cdot \frac{s}{x^2} + \mu \cdot (n - \rho + \sigma + s) = -\dot{\mu} + r\,\mu$$

We assume that the following elasticity of marginal utility is constant:

$$(13)\quad \eta = \frac{\partial U'}{\partial x}\cdot\frac{x}{u'} = \left(\frac{U''}{U'}\right)\cdot\left(\frac{a}{b}\right)\cdot\left(\frac{s}{x}\right) < 0$$

Substituting (11) into (12), we find (in the neighborhood of $\dot{x}=0$):

$$(14)\quad \frac{\dot{s}}{s} = \frac{1}{\eta}\left[2s + n - \rho + \sigma\right] + \frac{1}{\eta}\left[(n - \rho + \sigma + s)\ x - \rho\right]\cdot\left(\frac{1}{x}\right)\cdot(\eta-2)$$

Equation (7) may be rewritten as follows:

$$(15)\quad \frac{\dot{x}}{x} = (n - \rho + \sigma + s) - \frac{\rho}{x}.$$

Equations (14) and (15) describe the optimal time paths of s and x. These paths are illustrated in Figure 2. The $(\dot{s}/s) = 0$ function is upward-sloping in x-s space since

$$\left.\frac{ds}{dx}\right|_{\frac{\dot{s}}{s}=0} = \left(\frac{\eta-2}{\eta}\right)\cdot\left(\frac{\rho}{x^2}\right) > 0.$$

The $(\dot{x}/x) = 0$ function is downward-sloping in x-s space since

$$\left.\frac{ds}{dx}\right|_{\frac{\dot{x}}{x}=0} = -\frac{\rho}{x^2} < 0.$$

The second-order conditions[4] are satisfied only along the "saddle-point path", SPP. Given an initial human-to-vampire ratio, x(0), the initial stake-to-vampire ratio, s(0), should be set so as to place the economy on the saddle-point path. All movements of x and s thereafter should be such as to maintain the economy on this path. In the long run, the economy approaches a stationary equilibrium at which the human-to-vampire ratio is x^* and the stake-to-vampire ratio is s^*.

If all vampires were to be destroyed, x would be infinitely large. Yet such a state is never socially optimal. Given an

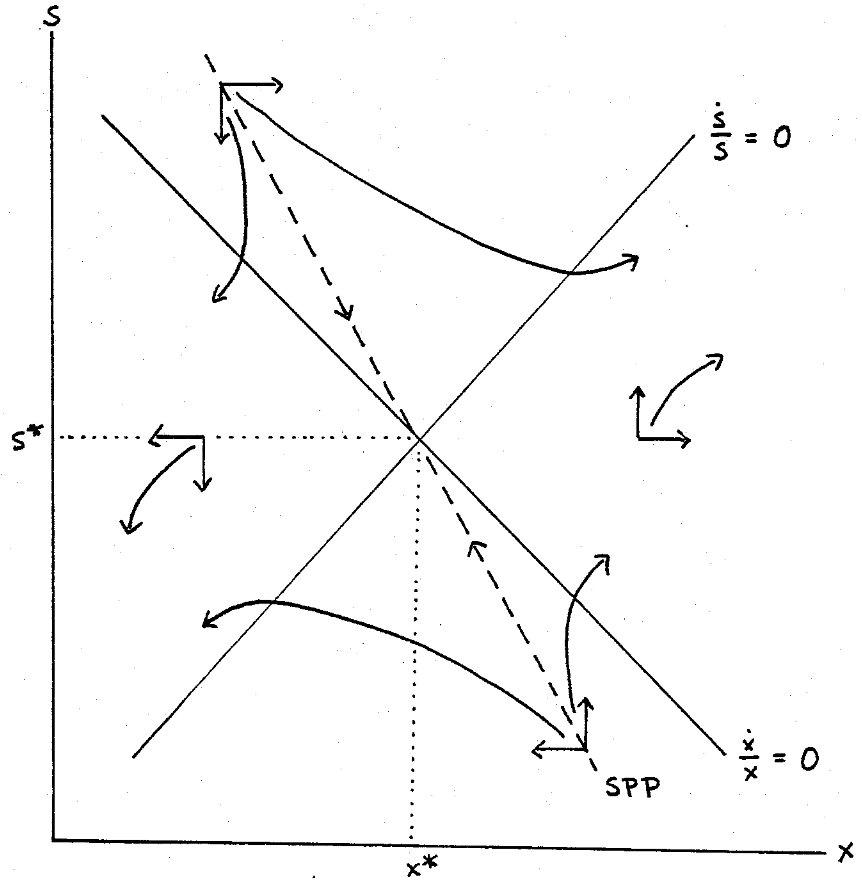


Figure 2

initial state in which the populations of humans and vampires are both finite, the economy steadily approaches (x^*, s^*) and x^* is finite. Thus, the complete destruction of vampires is not desirable.

If $x(0) > x^*$, then vampires are an endangered species. Consequently, the production of stakes (per vampire) should be sufficiently low to permit the regeneration of the vampire population. Conversely, if $x(0) < x^*$, then humans are an endangered species; thus, the production of stakes (per vampire) should be high enough to reduce the vampire population.

Figure 2 provides an illustration of the <u>Vampire Neutrality Theorem:</u>

Theorem 2: The spontaneous generation of vampires (i.e. the appearance of vampires ex nihilo) affects the optimal x and s in the short run, but not in the long run. In other words, vampires are neutral in the long run.

This theorem follows immediately from the fact that the long run, socially optimal stationary state (x^*, s^*) does not depend on the initial conditions $(x(0), s(0))$. The spontaneous generation of vampires may be viewed as an exogenous shock which lowers the initial level of x. Consequently, the initial level of s must be riased. In the long run, however, the optimal x and s approach the same values (i.e. x^* and s^*, respectively) they would bave approached in the absence of the exogenous shock. This principle may be summarized by the aphorism "Vampires are a veil".

4. A Suggestion for Further Research

The analysis above presupposes that vampires' blood-sucking behavior patterns may be described by a constant blood requirement coefficient, ρ. This assumption does not do justice to the flexibility of the vampire's injestive system. In reality, vampires are likely to maximize their utility functional, which depends on their per capita blood intake through time (i.e., ρ). In doing so, they face a blood supply constraint (Equation (7)), which indicates how the supply of homo sapiens (i.e. the blood receptacles) depends on the vampire blood intake, the vampire attrition rate, the homo sapien procreation rate, and the rate of stake production. This intertemporal optimization problem is quite straightforward if vampires assume that the stake-to-vampire ratio (s) remains constant through time.

In that case, the homo sapien control problem (10) and the vampire control problem are analogous. Just as homo sapiens find their optimal s given their expectation that vampires' behavior yields a constant ρ, vampires find their optimal ρ given their expectation that homo sapiens' behavior yields a constant s. The two optimal control problems give rise to the reaction functions of homo sapiens and vampires ($s = s(\rho^E)$ and $\rho = \rho(s^E)$, respectively, where ρ is expected by homosapiens and s^E is expected by vampires). (The superscript "E" stands for "expected".) The stage is set for an investigation of whether humans and vampires grope their way towards a Cournot-Nash equilibrium.

Needless to say, the expectations underlying such a system are not rational. Under rational expectations, human beings expect vampires to use the optimal feedback control rule $\rho = \rho^E(s)$ and this rule is precisely the one vampires actually use. Similarly, vampires expect humans to use the optimal feedback control rule $s = s^E(\rho)$ which corresponds to what humans actually use. An analysis along these lines lies beyond the scope of this paper. However, since both humans and vampires are known to exploit all the economic rent at their disposal, the study of human-vampire interrelations under rational expectations promises to be an important area of future economic research.

FOOTNOTES

1. See Phillips (1954, 1957).

2. See, for example, Baumol (1961) and Turnovsky (1977).

3. We are looking for an interior optimum. The complete destruction of vampires is desirable only if the optimum is reached when x is infinitesly large.

4. See, for example, Arrow and Kurz (1970).

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