A TWO PARTY SYSTEM, GENERAL
EQUILIBRIUM AND THE VOTERS' PARADOX

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1. Introduction

This work was helped considerably by the work of Anthony Downs. [1]
Although the models constructed are somewhat more formal and the
conclusions reached are somewhat different the approach to the study
of the political-economy of a democracy with competitive markets is
much the same.

The investigation is limited to the study of a two party system in
a very rudimentary form operating in an environment in which the
average individual acts as a passive voter (i.e. does not form coalitions,
or act in any way other than to individually use his vote to maximize his
own welfare). The individual also acts as a price taker in that part of
society which can be represented as a competitive market.

2. A Politico-economic Model

The model presented here is stripped down to its most abstract
form. No attempt is made to defend its "realism". Obviously much
more detail and many more "facts" of political and economic life
must be added if it is to be presented as a model with immediate
worth in application. This is not the intention. The goal is to explore
the implications of the highly simplified political model added to a
general equilibrium economic model. Because the assumptions made
to construct the model are critical and may be controversial a detailed
discussion of them is now given.

2.1. The Model is static

Much of the writing on political parties, elections and government has
been devoted to detailed description of process. Attention is paid here to
static models only. Tactical details are completely supressed. The
following assumptions are made:

(1) All preferences are known and fixed.

(2) All laws and other rules of the game are known and fixed.

(3) The political process can be represented by a simultaneous move one period game (i. e. a game in normal form 2j).

(4) Depending upon the model, various assumptions concerning preferences are made; in particular they can be represented by:

   (I) an ordinal measure,

   (II) a cardinal measure, without comparability or trans- ferability,

   (III) a cardial measure with comparability without trans- ferability,

or (IV) a cardial measure with comparability and trans- ferability.

By fixing preferences we are ignoring problems of persuasion, the effects of education and the many other aspects of change in socio-political affairs.

Holding laws fixed heavily limits the scope of government. In the models only some laws are held fixed while others may be changed within bounds. The ability to change laws defines the strategie scope of the government. The bounds serve as a link to a dynamic analysis. They represent a limit on how far things may be changed in the period of one election. Obviously they may be a function of the size of the majority, but this problem is ignored.

2.2. Economic Assumptions

The economy is assumed to be split into

   (1) a private sector

and (2) a public sector.

A competitive market prevails in the private sector. All individuals are constrained to be price-takers. All markets in the private sector satisfy the technical conditions [3] for the existence of an efficient price system.

We assume no oligopoly power and no strategic role by any individual in the economy.

The public sector is run by the government which taxes, pays subsidies and supplies public goods and services as specified below.

2.3. Political Assumptions

Although some of these assumptions will be modified to start with we assume:

   (1) Each individual has a vote;

   (2) Votes cannot be sold;

   (3) All individuals are passive vote-casters;

   (4) There are two political parties;

   (5) Party politician attach a value to their own and their party's election;

   (6) Each party selects as its strategy a policy which, if elected it will follow.

   (7) The political contest can be represented as a game of strict opposition.

3. Discussion of the Politico-economic Assumptions

A view of the relationship between government and individuals in a democratic, free enterprise economy is that the political mechanism is used to solve the problems of production and distribution of resources which cannot be handled by the individualistic market mechanism and price system. There are a whole array of indivisible goods, "natural" monopolies and public services which cannot be handled by an open market (a taxonomy of many different types of public goods has been given previously by this author [4]).

The use of the vote in a democratic country is in and of itself a value judgement. "One man one vote", "One adult one vote", "one poll tax paying adult, one vote" are all different specifications of this type of value judgement. In England until recently certain individuals might have as many as three votes (one from his domicile, another

from his place of business and another from his university).

In this paper, one adult, one vote is accepted as a fundamental democratic value. The definition of adult has some leeway and there are also many different voting systems but these problems are ignored.

It is assumed that votes are not directly sold. All that this means is that one cannot legally trade a vote. Within the "rules of the game" however there is nothing to stop log-rolling or any payoff arrangements.

In this model all individuals are assumed to be passive voters. This is a direct analogue of the passive, or mechanistic price-taker in a competitive economic market and the reasons are approximately the same. The costs of communication and organization are implicitly assumed to be too great so that even though we might assume that the individual knows his preferences and is well informed about the issues at stake he merely carries out his voting decision individually (it is assumed for now that voting per se is costless). In actuality both in politics and economics groupings exist and the "free competition" passive or mechanistic model of man is not adequate.

The analysis presented here is limited to the presence of two political parties which are represented in a most elementary manner. Three or more parties bring in game theoretic complications which are not currently analysed.

Two parties are assumed to exist. In spite of the fact that although usually elections are non-symmetric affairs inasmuch as the incumbent-party usually starts with a considerable advantage, the model presented abstracts from this nonsymmetry and assumes that each party is confronted with a symmetric game in its struggle for power.

What differentiates a politician, a candidate or a party activist from the ordinary man on the street? Let us make the naive assumption that the professional politician attaches a positive utility to being in power. He likes winning. As a good first approximation others who choose various occupations can be assumed to have a preference for the occupations of their choice, however their "market" is economic and social. Except in some formal sense in a few professions (such as the election to a membership of an honorific society) success does not depend upon a competitive voting mechanism.

The reasons why a politician chooses to stand for one party rather than another are complex, heavily social and dynamic in nature. They have to do with his social views, values, group loyalties and training. As a first approximation we assume that out of n individuals in society there are 2k candidates (leaving out the preliminary fight in each party among the group of would-be candidates for the limited number of seats). Beyond that, each set of k candidates has a value on their own election and on the election of their party to power, their preferences can be assumed to be much the same as the rest of the population. The preferences of politicians for economic goods may be less than those exhibited by their peer groups such as business executives.

Let us assume that a party goes before the electorate with a program. We make a drastic simplification by regarding the party as presenting a well defined program which may be viewed as a specification of all taxes and subsidies and the manufacture and distribution of all public goods and services. We do not discuss the less directly tangible aspects of a program such as anti-trust legislation, changes in the constitution or statements of attitude on public policy.

If a party wins an election it is assumed that by the rules of the game it is bound to put its policy into action. The machinery of government is viewed merely as a policy execution device. A party is viewed as a group of individuals with positively correlated political preferences. By the nature of the political institutions their <u>political interests</u> (not necessarily social interests) are diametrically opposed. If party I wins a seat then party II must lose the seat. A strategy of a party is the statement of a policy. Its goal is to get itself elected and its role when elected is to carry out its policy.

4. Values, Pareto Optimality and the Impossibility Theorem

It has been suggested that for at least some members of society a value can be attached to the achievement of political office. Similarly in describing the initial asset holdings of an individual in a democratic society we must include the possession of a vote. If this cannot be changed by governmental action and the vote cannot be sold then it need not appear explicitly as its distribution is the same for any state of society. However this does not prevent us from asking the question "how much is a vote worth?"

Restricting ourselves to public and private goods and services there are three diagrams which are of concern. Suppose that there are $\underline{\text{in toto}}$ m types of public or private goods and services in existence. The production transformation space (m dimensions), the distribution space (m x n dimensions) and the preference or utility space (n dimensions) must all be considered in a study of welfare. Figures 1, 2 and 3 show their relationships. Figure 1 shows the set of feasible production transformations. I is the initial point or the
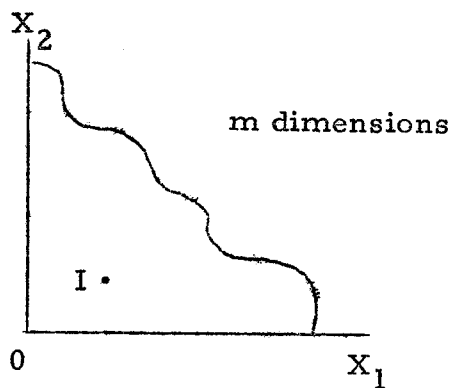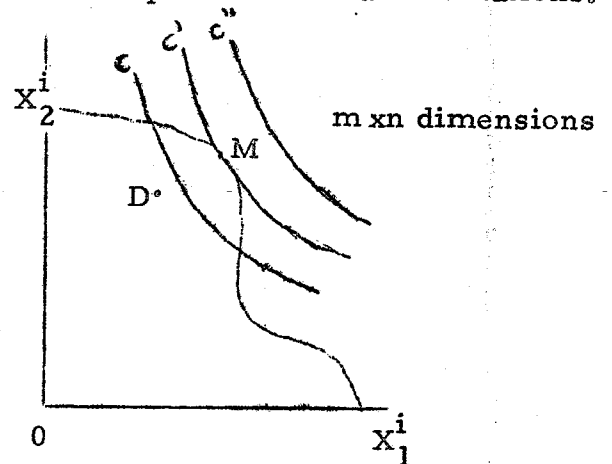

Figure 1

Figure 2

total endowment of the society. The surface may not be concave if there are indivisibilities or other complications present in the production process. (In general when shown in all dimensions including inputs and outputs. I will be on the efficiency surface in Figure I).

In Figure 2 the point D is the initial distribution of goods and services prior to any production or exchange. It maps onto the point I (as does every other distribution with the same total supplies)
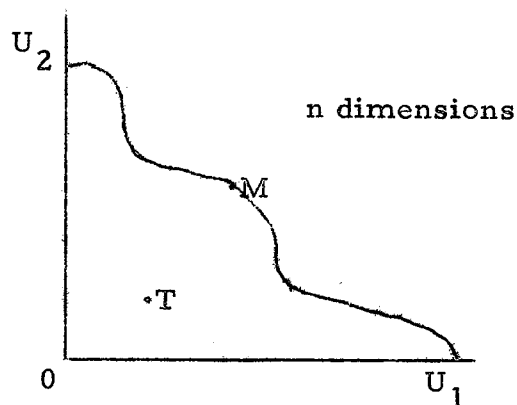

Figure 3

the curves c, c', c'' are contours of a <u>community indifference map</u>
or <u>welfare function.</u> M is the optimal point for this society. Figure 3
shows a mapping of the Pareto optimal surface in the utility space
and **T** is the mapping of D the initial point.

The Arrow impossibility theorem 5] proves that a community
indifference map cannot exist if a certain set of quite plausible conditions
are imposed. This being the case what way does society have for picking
a point on the Pareto optimal surface? It is well known that with indivisi-
bilities and public goods the competitive market fails to produce   an
optimal distribution. What method can be used, and what welfare pro-
perties does it have? The approach adopted here is to specify a choice
mechanism based upon individual behavior; see if it leads to a Pareto
optimal outcome and see if any further welfare propertus exist.

5. The Competitive Market and Two Party Election

Usually the Pareto optimal surface is regarded as having an
objective existence in the sense that it is the possibility set of all
distributions of goods and services which can be obtained by a society
which pools all resources, can act optimally in any manner and has no
constraints on distribution. No particular value is attached to <u>process.</u>
Society's evaluation or preference for how things are done is meant
to be included in the community welfare function. This implies however
that even if a community preference function existed it might not be
representable as in Figure 2. It has a far higher dimensionality than
m x n. There is no guarantee that there is a 1 to 1 mapping from the
welfare space to the distribution space since the welfare function would
include not only the evaluation of distribution but of process. For example
different values might be attached to the same distribution achieved with
or without simple majority vote; with or without control of industry or
unlimited taxation and so forth. Leaving this additional problem aside
the approach adopted here is to make specific welfare assumptions,
specific assumptions about the role and preferences of individuals and
to study the resulting game and its outcome. This involves completely
abandoning the concept of the community welfare function.

Consider a society with the following values and politico-economic considerations:

(1) Private ownership and free competitive market are regarded as a social desideratum provided that they are consistent with Pareto optimality or economic efficiency.

(2) Public ownership is accepted for goods and services which do not satisfy efficiency conditions in the open market.

(3) Any taxation scheme necessary to the achievement of any point on the optimal surface is institutionally feasible.

(4) One person, one vote is regarded as a fundamental democratic value which must be satisfied by the system, which elects a government by simple majority vote.

(5) Preferences of individuals are given by a complete ordering on goods, services and their own achievement of political office. Their other values are in the "rules of the game".

Condition (3) is tied in closely with the preservation of property rights and with the protection of minority rights. In some societies there is a limit to the amount of taxation that an individual will be forced to pay. In terms of this model if such a condition were imposed it might limit the production feasibility set and hence narrow the Pareto optimal possiblities. Although the condition appears to be relatively innocuous, problems concerning eminent domain, expropriation and forced labor are tied in closely with individual freedom and with taxation. For example special skills are very real factors of production, but unless the society has methods for extracting forced labor it may have to limit its distribution of resources to those points on the Pareto optimal surface which can be attained from (post tax) initial points which are individually rational. Figure 4 illustrates the problem. K is the initial endowment of resources. The domain T is the set of feasible taxation schemes. This means that taxes and subsidies of individually owned
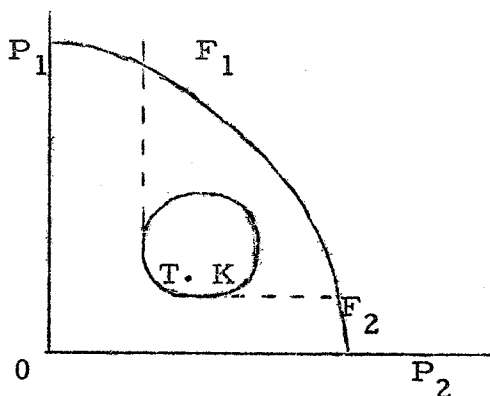

Figure 4

goods are constrained to this domain, hence the outcome from the free market sector must be limited to $F_1 F_2$.

Condition (5) imposes very little in the way of restrictions on the preference conditions of individuals in society. If one does not abandon this condition and replace it by stronger conditions no meaningful statement can be made about equity or fairness except when all preferences are identical. It appears reasonable to accept a measurable preference scale if one accepts an ordinal scale. If one does so, then even without extrinsic comparability of individual preferences a comparability intrinsic to the model may emerge 6] and statements concerning fairness and equity of distributions can be made.

5. The Voting Paradox and the Two Party System

Trivially we could imagine a completely nationalized economy. How does voting influence distribution? If nothing is left in the private sector then there is the technical problem of finding the Pareto optimal surface for the whole economy as the government will produce and distribute everything.

The strategy of a political party is to offer a policy. This policy must result in a point on the Pareto optimal surface. If it did not, the opposing party would always have a policy available that would be preferred by everyone.

Comment I   The presence of two parties each presenting a single alternative to the electorate guaranties that the outcome will be Pareto optimal.

Given that the parties are going to select a point on the Pareto optimal surface we may observe that under the socially desired rule of majority vote then either there is a set of points which dominate all others (in the sense of majority vote) and which are indifferent among themselves; or there is a set of points which are intransitive among themselves but dominate all others, or there is a set of points which are intransitive and indifferent among themselves but dominate all others.

Comment 2 (a)   If there is a unique prefered outcome both parties will offer it as their program.

(b)   If there is a set of outcomes which are prefered over all others but indifferent among themselves then any one will do as a party policy.

(c)   If there is a set of outcomes which are intransitive among themselves but are prefered to all others the resulting game of strict opposition between the two parties will call for a mixed strategy over the set of outcomes which display the intransitivity.

We may first consider a simple win, lose or draw matrix for the game of political opposition between the two parties and apply it to the simple three outcome, three voters example which is often used to illustrate non-transitivity under voting. Let there be three individuals, 1, 2 and 3 and three outcomes A, B and C. Suppose that the preferences are as shown below:

$$
\begin{array}{lll}
1 & \quad A > B > C \\
2 & \quad C > A > B \\
3 & \quad B > C > A
\end{array}
$$

It is easy to see that A wins over B by 2: 1, B over C by 2: 1 and C over A by 2: 1, hence for society $A > B > C > A$. We now define a game, of strict opposition assigning the value of 1 for a win, 0 for a draw and -1 for a loss. Each of the two parties can select one of the three social states as its policy. This gives a 3 x 3 matrix as is shown below as the representation of the competitive state between the two parties. If we interpret the numerical values as utilities then this game has the mixed strategy ($\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$) as the optimal strategy for each party. Actually

		Party 2		
		A	B	C
	A	0	1	-1
Party 1	B	-1	0	1
	C	1	-1	0

in this case we do not even need to interpret the values numerically.
Consider the n x n matrix of any size where each row or column
contains only the entries r, s, t and any row or any column is a
permutation of any other row or column; then regardless of the numeri-
cal values of r, s and t the strategy ($\frac{1}{n}$, $\frac{1}{n}$, ... , $\frac{1}{n}$) will be the saddle-
point strategy. The proof is trivial, suppose that the frequencies of
appearance of r, s and t in any column or row are n_1, n_2 and n_3 then
if a strategy with equal probabilities is played the value yielded by
playing any row or column is:

$$\frac{n_1}{n} r + \frac{n_2}{n} s + \frac{n_3}{n} t$$

hence this is a saddle point independent of any order preserving trans-
formation on r, s and t (see the Appendix for further observations
on games of strict opposition).

Comment 3   From the above we can conclude that a two party
system will not only select a Pareto optimal outcome but will choose
among them where transitivity exists and randomize when there is in-
transitivity.

Comment 4   The political game is one of pure opposition and
in this formulation is symmetric. This implies that the expected
"political profit" is zero. Furthermore each party names the same
policy or randomizes over the same set of policies. We have then
in a two party democracy where politicians value office and the
private sector is competitive, zero economic profit, zero political
profit and the same political goods offered at the same price by the
competing parties. The result of the competition is Pareto optimal
and satisfies "one person one vote" but beyond that the welfare im-
plications are slight given only ordinal preference scales (this point
is discussed further below).

Returning to the case with intransitivities, the situation may
not be as simple as the simple chain $A > B > C > A$ illustrated above,
Consider three voters with one unit to divide. Suppose there seven
alternatives: $A (\frac{1}{3} , \frac{1}{3} , \frac{1}{3})$, $B (\frac{1}{2}, \frac{1}{2}, 0)$ $C (\frac{1}{2}, 0, \frac{1}{2})$. $D (0, \frac{1}{2}, \frac{1}{2})$,
$E (1, 0, 0)$ $F (0, 1, 0)$ and $G (0, 0, 1)$; these yield the following
matrix:

	A	B	C	D	E	F	G
A	0	-1	-1	-1	1	1	1
B	1	0	0	0	0	0	1
C	1	0	0	0	0	1	0
D	1	0	0	0	1	0	0
E	-1	0	0	-1	0	0	0
F	-1	0	-1	0	0	0	0
G	-1	-1	0	0	0	0	0

This has as a solution any mix on the strategies B, C and D.[x]

5.2. The Size of the Majority and the Intensity of Preference

Two immediate modifications to the previous analysis can be made. The first is to take into account the size of the majority by which a party wins an election. The second concerns intensity of preferences.

The model of the political process presented here has all voters act in an individual mechanistic manner. There are no blocks or groups which "can deliver the vote", thus the minimal winning coalition theory of Riker [7] is not applicable to this model. His theory was based upon a cooperative model of different power groups making deals before an election. It is certainly consistent with the analysis here but concerns a different aspect of the problem. All other things being equal it is assumed that a party will prefer to win by a larger rather than a smaller majority. This being so instead of representing the conflict by a win-lose matrix we introduce the explicit size of the majority. For example consider a society with 11 voters and 3 alternatives A, B and C. Suppose the alternatives were (3, 3, 3, 3, 3, 3, 1, 1, 0, 1, 0), (2, 2, 2, 2, 2, 2, 2, 0, 1, 0, 1) and (4, 4, 1, 1, 1, 1, $\frac{3}{2}$, 2, 0, 2, 0) respectively. There is intransitivity among them but the majority sizes are different. The

[x] These three imputations form the von Neumann and Morgenstern stable set (symmetric) solutions to the three person game. For three people domination and majority voting amount to the same phenomenon.

political conflict may be described by the matrix:

$$\begin{matrix} 0 & 5 & -1 \\ -5 & 0 & 3 \\ 1 & -3 & 0 \end{matrix}$$

In this simple case majority size makes a difference to the strategy employed. Here the optimal strategy is ($\frac{3}{9}$, $\frac{1}{9}$, $\frac{5}{9}$). If we had used only +1 or -1 for win or loss the strategy would have been ($\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$). Even for this case however we must assume that there is a cardinal measure to the valuation of the size of a majority (See the Appendix).

What does the mixed strategy mean? All outcomes will be Pareto optimal which implies that no outcome will be unanimously prefered over any other outcome. As there may be intransitivity among the optimal outcomes, a party protects itself by randomizing over several policies. The way it randomizes depends upon the majorities to be expected. The randomization may be interpreted as an attempt by each party to be "most things to most men" at the same time.

Suppose that each individual had a measurable scale for his preferences. Furthermore suppose that there are only three outcomes and as each individual has two free scale factors on his preference scale we select them so that for each the least desired has a value of 1 and the most desired has a value of 1,000. Let the alternatives be A (which yields (1,000, 2, 1) to the three individuals, B yielding (999, 1, 1,000) and C yielding (1, 1,000, 2). There is intransitivity among the three alternatives and the political game will appear to be completely symmetric, yet when A is compared with B a yielding of 1 unit in his own scale by each of the first and second voters would yield 999 units to the third. Even if the units could be directly compared the voting system would ignore this assymmetry. "Passionate minorities" are treated in the same manner as almost indifferent minorities by majority voting. This does not appear to agree with experience or common sense. The reason may be that log-rolling or other means of side-payments are often used. No possibility for them exists in this model. In many politico-economic situations interpersonal comparisons are made and money transfers are used as a first

order approximation to a transferable utility. The larger the problem,
the poorer the approximation will be, but without a transferable
commodity or an ethical code that calls for individuals to give up
a small amount because others gain a great amount, then voting does
not solve conflicts involving intensity of preference.

In summary beyond guaranteeing Pareto optimality the ethical
content of majority voting is only slightly more than of the competitive
market. It calls for an equal distribution of political assets in the form
of one vote per individual.

5.3. The Size of the Private Sector

If the whole economy were nationalised we have shown that a
two party system would result in a government which chooses a policy
that is Pareto optimal. The possibility for the existence of a free
market sector can be checked backwards as follows. Suppose that by
vote a point were selected on the Pareto optimal surface. We can rule
out all processes causing external economies or diseconomies; public
goods in the sense of items which enter in the same amount into each
individual's endowment such as the law, national parks monuments
etc... indivisibilities, unique items and other goods or processes
which violate the conditions needed for the functioning of an efficient
price system. Limiting ourselves to individually ownable goods
produced by production processes representable by production cones
the possibility of running a competitive market amounts to asking is
it possible to construct a price ray from the Pareto optimal surface
to the interior such that a point on that ray in the interior would
correspond to the "initial" holdings point of all members of society
after they had been taxed sufficiently for the production of the publically
owned goods. This is shown in Figure 5 which shows the type of Pareto
optimal surface caused by set up costs 8} . Suppose E is selected,
this could be the result of a mixed controlled government economy
and free economy if the feasible tax schemes include the point T
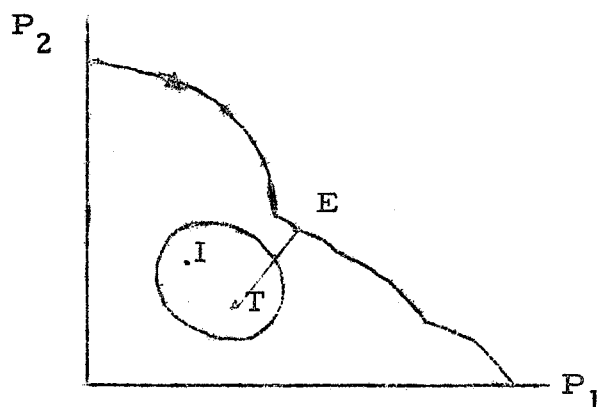which can be reached by taxes from the initial (before tax) distri-
bution point I.

Figure 5

There are several difficulties with the above scheme. First, is the tax to be in money or kind? If it is in actual commodities there are no extra conditions to satisfy, if it is in money then the prices and tax system must satisfy supply and demand conditions to the public sector. Second as the competitive equilibrium may not be unique there is no guarantee that the government can attain outcome E by taxing from I to T, unless we regard the slope at E as defining an intrinsic comparison of utilities. Third, in some casis taxation may be virtual expropriation and may certainly leave the individual worse off than before taxation; i. e. individual rationality conditions only apply after taxation, In the final outcome an individual may be poorer than when he started.

Another major difficulty with the scheme above is that it requires considerable knowledge by the government for it to be able to select an optimal tax scheme.

5.4. One Person One Vote and Equal Incomes for All

Although many countries have the equal voting principle, none have the equal income principle. Probably the main reason for this is because of the great difficulties in enforcement. Ownership of major assets whose value can change makes calculation difficult. The relationship between income and assets is hard to sort out.

Individuals are born with different endowments which are tantamount
to personal "earning-power assets". They are hard to assess and hard
to tax. The opportunities for concealment and distortion of fact are large.

Suppose that all of the technical difficulties in defining income were
solved. We must make a distinction concerning the use of public goods
with no sale price and private goods. Do we include in each individual's
income his supply of public goods especially if the public goods are used
with different intensities? If we exclude them then income and the post
tax market value of an individual's endowment are the same.

Foley $\underline{9}$ has shown that under conditions similar to those above,
a point on the Pareto optimal surface does exist which can be arrived
at by unanimous vote over other possiblities involving taxation schemes
yielding equal incomes but not Pareto optimal final distributions.

Pareto optimality can be achieved with the conditions of one person one
vote and equal incomes without assuming more than an ordinal measure
on utility. However the welfare implications are not as large as they
may seem. Suppose a bore majority likes almost only public goods and
vice-versa for the minority. Consider outcomes A and B (1000, 1000,
1000, 1000, 1, 1, 1) and (1, 1, 1, 1, 1000, 1000, 1000). A will always be
chosen. There is no protection of the minority. Without measurability
there is no definition of a passionate minority and with it this scheme
does not protect it. In outcome A above equal incomes can be preserved
but almost all incomes may go to the state to provide public goods
favored by a bare majority.

6. Politics, Economics and Sociology

These few simple models have been used to show that when a
market mechanism fails, a two party voting process can be used to
guide a mixed economy to achieve a Pareto optimal outcome. The pro-
cess depends upon a model of the individual as a price-taker and a passive
voter. It also requires that the parties be well informed of the preferen-
ces of the public, that the public understand the policies offered and
that the elected party carry out its program.

Without a more strategic model of the individual and, or stronger conditions than ordinality on preferences virtually no further welfare conclusion beyond Pareto optimality can be obtained except for the added values such as "one man one vote" which may be regarded as goals in themselves.

A possible way to interpret this approach to a welfare theory is that the economics deals with the distribution in the relatively short run of more or less measurable goods and services which have individual utility and are suited to individual consumption and ownership. Political-economy and politics deal with the less short run problems of the selection and distribution of public goods and services and the design and modification of rules or processes which may have values associated with them in and of themselves. Thus "one man one vote" or "equal opportunity for all" or "no taxation without representation" or other aspects of process which reflect explicit desires concerning the qualities of existence are part of the domain of politico-economic and political choice. Given this possibility we must now add a fourth diagram to Figures 1, 2 and 3. This diagram intervenes between the mapping of the distribution space of dimensions to the preference space of n dimensions. It includes not only individual utility for the distribution of goods and services but also the individuals' social values for process and principles. These may modify an individual's preferences in a manner that cannot be represented by a complete ordering, for example, one man one vote, equality and Pareto optimality might be ordered lexicographically so that political freedom is weighted more heavily than the distribution of wordly goods.

Although the classification is by no means neat or complete we might distinguish individual utilities from values by observing that the first applies to objects or items or processes which would confront an isolated individual, the other is of prime concern for two or more individuals and the preferences might best only be defined on sets. Thus operationally one needs more than one individual before ' religious freedom" becomes a problem.

Politics in general and voting in particular deal with more or less explicit values. We have left social structure out of the model. Its role is of an even more long run nature than politics. The social structure provides the institutions. Thus an adequate model would not have institution-free traders and voters but a structure that would call for a strategic analysis of groups. Furthermore the sociological background provides the basis for the implicit values and the modification and birth of explicit values.

The Welfare Function was a Philosopher's Stone, it tried to explain far toomuch with far too little. The voting paradox is not necessarily a paradox when viewed differently. A partially competitive, partially nationalized economy can be run efficiently by a partial price system and two party voting. The structurally noncompetitive part of the economy must be controlled unless legal constraints can correct the flaws in physical structure. Any amount of the remainder of the economy can be left to the competitive market or can be controlled to achieve Pareto optimality (for example liquor, tobacco, housing).

APPENDIX

A few technical points concerning the models and ommisions in models are noted here.

A game of strict opposition is one in which the payoffs of one player a directly negatively related to the payoffs of the other. Suppose the first has payoffs $a_1 > b_1 > c_1 > d_1$ then related to a_1 is the payoff d_2; b_1 and c_2, c_1 and b_2, d_1 and a_2. There are six two by two games of strict opposition.

(1) a b (2) a c (3) a d (4) a b (5) a c (6) a d
      c d   b d   c b   d c   d b   b c,

(1), (2) and (4) have pure strategy solutions. The other three call for mixed strategies. If in the political struggle each party tries to maximize its expected majority the game is strategically equivalent to a zero-sum game.

Games of strict oppositions which have saddle points are equivalent to zero sum games. The ones with mixed strategies are not generally equivalent. When we wish to study election majorities we may want to consider the struggle as strategically equivalent to a zero-sum game. This involves the assumption that although the parties' preferences for seats are not comparable they are the same up to a linear transformation.

All frictions and features which might differentiate the parties are left out here. For example no uncertainty or persuasion exists in the model. There is no marginal cost to obtaining an extra vote. Complete symmetry was assumed. Elaborations of the model are needed to do justice to the "product variation" of different parties.

We have noted values such as voting or equity. We have not discussed the possibility of the construction of measures to judge by how much a system fails to satisfy a property. How undemocratic were the British when they had university seats? Are values and principles to be always treated as 0 or 1 phenomena or do we permit gradations?

If there is unlimited taxation and lepayments then the structure of society may be viewed as a simple game 10 with "winners take all".

Footnotes

1 Downs A., "An Economic Theory of Democracy", Harper and Row, New York 1957.

2 Shapley, L. S. and Shubik M., "Competition, Welfare and the Theory of Games", unpublished manuscript, Chapter I.

3 For an excellent discussion of these conditions see Debreu G., "Theory of Value", Cowles Foundation Monogram 17, John Wiley, New York 1959.

4 Shubik M., "Towards a Taxonomy of Public Goods", Cowles Foundation, Discusiion Paper.

5 Arrow K. J., "Social Choice and Individual Values", Cowles Commission Monogram 12, John Wiley, New York 1951.

6 Shapley L. S. and Shubik M., op cit, Cahpter I

7 Riker W., "The Theory of Political Coalitions", Yale University Press, New Haven 1962.

8 Shubik M., "A Note on Taxation, Increasing Returns Due to Set-up Costs and Competitive Equilibrium," CFDP 237, Nov. 1967.

9 Foley D. K., "Resource Allocation and the Public Sector", PhD Thesis, Yale University 1966.

10 Shapley L. S. and Shubik M., op cit, Chapter III