Imports, Status Preference, and Foreign Borrowing

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Abstract

This paper considers the implications of consumption and borrowing externalities in a small open economy framework. The former reflect the assumption that status conscious agents care about the relative consumption of imported goods, while the latter arise because agents do not take into account the effects of their borrowing decisions on the interest rate on debt. We analyze in the paper the impact of an increase in the degree of status preference on the saddlepath adjustment of the decentralized economy. In addition, the contrasting steady-state and dynamic properties of the social planner's economy are derived, along with the corresponding optimal tax and subsidy policies.

Keywords

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Contents

1. Introduction

One measure of an economy's financial market integration is the terms at which it can borrow in international capital markets. According to a basic definition, a country may be "perfectly integrated" if it can borrow (and lend) at the prevailing world interest rate, however the latter is specified. Due, however, to factors such as default risk, a developing economy may be subject to an external constraint that regulates the terms at which it borrows from abroad. A way of capturing this idea in a reduced-form framework is to specify that the interest rate on debt depends on a country's ability to service its existing level of international obligations. This leads to an "upward-sloping" interest rate relationship in which the rate on debt rises with the level of indebtedness, where the latter can be scaled by a measure, such as GDP, of the economy's ability to pay. An early use of this idea was employed by Bardhan (1967) and has, more recently, been taken up by researchers such as Pitchford (1989), Bhandari et. al. (1990), Fisher (1995), Agénor (1998), Fisher and Terrell (2000), and Chatterjee and Turnovsky (2004), all of whom use the representative agent framework. These authors employ this relationship to study the intertemporal impact of macroeconomic disturbances, such as domestic fiscal policy and world interest rate shocks, on indebted open economies

In analyzing economies with this type of interest rate function, an important feature is whether or not agents take into account the "upward-sloping" nature of the relationship; in other words, whether or not agents recognize that their borrowing decisions affect the equilibrium interest rate on debt instruments. The models of Pitchford (1989), Bhandari et al. (1990), and Agénor (1998) specify that agents do recognize the fact that greater foreign borrowing raises the interest rate on debt, while the work of Fisher (1995), Fisher and Terrell (2000), and Chatterjee and Turnovsky (2004) assumes, on the other hand, that agents take the interest rate on debt as given in making their optimal choices. The latter formulation can be interpreted as a model of sovereign debt, with the interest rate relationship incorporating a "country specific" interest cost function that is rising (and convex) in a measure of the economy's indebtedness.[1] Nevertheless, as Pitchford (1989), among others, points out, the "country specific" specification results in a borrowing externality. While this externality is not (necessarily) crucial for the results of the

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[1]Bhandari et. al. (1990) do, however, incorporate features of the sovereign debt model such shifts in the cost, or "risk", premium.

work cited above, we show in our framework that it does play an important role in characterizing the economy's steady-state and saddlepath dynamics. Indeed, one of the goals of this paper is to compare the dynamic properties of the two specifications of borrowing behavior, which, in certain respects, is more complex if these decisions are "internalized."

A developing economy need not only be "ridden" with one externality, however. Indeed, consumption and production externalities can also play a crucial role in influencing the evolution of developing economies. In this paper we consider how consumption externalities affect an economy subject to an external borrowing constraint. Consumption externalities arise in our framework because we assume that agents gain utility not only from their individual consumption of goods and services, but also from their relative social position, or status. Using survey data suggesting that higher levels of average income do not necessarily translate into higher levels of personal satisfaction, Easterlin (1974, 1995) and Oswald (1997) infer, in contrast, that social position is a key factor in determining overall well-being.[2]

In our reduced-form specification of instantaneous preferences, social status is conferred by relative consumption, so that the consumption externality in our model corresponds to the developing economy's average, or aggregate, level of consumption. There is a growing literature that investigates the influence of consumption externalities in dynamic macroeconomies. Representative authors who considered this issue in the closed economy context include Galí (1994), Rauscher (1997), Grossmann (1998), Fisher and Hof (2000a, b), Dupor and Liu (2003), and Liu and Turnovsky (2004).[3] We extend this work by analyzing consumption externalities in the case of a two-good, open economy, subject to an external borrowing constraint. Specifically, we assume that it is the relative consumption of *imported* goods that confers social position, an idea that is, we believe, plausible in the case of a developing economy where foreign "luxuries" represent status goods.

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[2]More general studies of the economic implications of the quest for social status are provided by Frank (1985) and Cole, Mailath, and Postlewaite (1992). To curtail potentially wasteful status competition, Frank (1997) advocates instituting a progressive consumption tax, implemented by exempting savings from taxation.

[3]An alternative branch of this line of research specifies that status depends on relative wealth, rather than on relative consumption. Recent authors who have employed this approach include Corneo and Jeanne (1997), Futagami and Shibata (1998), and Fisher (2004). Fisher (2004) shows how relative wealth preferences can be used to obtain—in the context of the small open economy Ramsey model with perfect capital mobility—an interior, steady-state saddlepoint. General discussions of the problem of obtaining interior steady states in a small open context are found in Barro and Sala-i-Martin (1995), [ch. 3, pp. 101-25] and Turnovsky (1997), [ch. 2, pp. 36-47 and ch. 3, pp. 57-77]. One way of dealing with this issue is to impose an external borrowing constraint, an approach we adopt in this paper.

The basic framework we employ closely follows Fisher (1995): (i) the developing economy is modelled as a representative consumer-producer who consumes a domestic good—produced using the single-factor labor—and a good imported from abroad; (ii) the economy is "semi-small" in the sense that it has an endogenous terms of trade (in goods) relative to the rest of the world; and (iii) international borrowing is subject to an "upward-sloping" interest rate relationship that depends on the stock of debt. We extend this framework, first, by incorporating preferences that are a function of the relative consumption of imported goods. Furthermore, in this paper we modify the borrowing relationship by specifying that interest costs are a function of the debt to GDP ratio. For convenience, we divide the exposition of the model into two parts: (i) the decentralized framework that is subject to consumption externalities and in which agents take the interest rate relationship as given; and (ii) the socially optimal framework, where the effects of the consumption externality are eliminated and in which the planner internalizes the interest costs of the economy's borrowing decision. To derive a symmetric macroeconomic equilibrium in the decentralized framework, we assume that all agents take the same actions, which is the typical procedure in models of this type. Moreover, because the interest rate relationship in this paper depends on the debt to GDP ratio, there is also, in effect, a production externality in the decentralized equilibrium in addition to a borrowing externality.[4]

To investigate how the consumption externality interacts with the borrowing constraint, we analyze how the economy responds over time to a permanent increase in the preference "weight" on the relative consumption of imported goods. We show that this causes the economy in the long-run to "expand", i.e., due to higher work effort domestic output increases, which, in turn, results in a decline in the country's terms of trade and a corresponding rise in net exports.[5] As a consequence, the economy supports a higher steady-state stock of debt and consumes more of the imported good (the long-run response of steady-state consumption is, however, ambiguous). Using a standard phase diagram apparatus, we illustrate the transitional dynamics in response to the increase in status preference. While we distinguish three separate cases, we demonstrate that the transitional adjustment of the economy in all instances involves current account deficits,

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[4]Likewise, the model of Liu and Turnovsky (2004) incorporates consumption and production externalities, although in their paper production externalities reflect spillovers from the aggregate capital stock.

[5]This result is consistent with the single-good, closed economy findings of Fisher and Hof (2000b) and Liu and Turnovsky (2004), who show that preferences for relative consumption (or the existence of negative consumption externalities) causes a rise in equilibrium employment relative to economies in which these motives are absent.

a deterioration in the terms of trade, and declines in the economy's consumption of domestic and foreign goods. In addition, the trade balance, after an initial fall, improves along the economy's saddlepath in order to support the long-run increase indebtedness. In this part of the paper we also describe the behavior of the interest rate on debt and the economy's domestic, "internal" rate of return, the latter depending on the dynamics of the terms of trade.

The remainder of the paper is structured as follows: section 2 describes the modelling framework and derives the circumstances in which the decentralized economy is characterized by (local) saddlepoint dynamics. To study the interactions between borrowing and consumption externalities, we consider in section 3 the intertemporal implications of a permanent increase in the degree of status consciousness. Section 4 is devoted to analyzing the socially optimal counterpart to the decentralized economy. In this section we analyze how the steady-state and saddlepoint properties of the planner's economy differ from those of its decentralized counterpart and calculate the optimal tax and subsidy policies that reproduce the social optimum. The paper closes with brief concluding remarks in section 5 and an appendix containing some mathematical results.

2. The Model and Intertemporal Equilibrium

We introduce the model by assuming that there are a large number of representative agents, each of whom has the following instantaneous preferences over their own consumption of domestic and foreign goods, x and y, status, s, and work effort, l:[6]

$$W(x, y, s, l) \equiv U(x, y) + \delta s(y/Y) + V(l), \qquad \delta > 0. \tag{2.1}$$

In this formulation, also employed by Rauscher (1997), Fisher and Hof (2000b), and Liu and Turnovsky (2004), preferences over own consumption are additively separable from status. In addition, both are separable from work effort. According to (2.1), status depends on the relative consumption of foreign goods, y/Y, where Y denotes the aggregate, or average, level of imported goods consumed by the small open economy and the parameter δ represents the corresponding

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[6]All variables in the model are denominated in real terms.

utility "weight."[7] We further specify that $W(x, y, s, l)$ has the following first derivative and curvature properties:[8]

$$U_x > 0, \quad U_{xx} < 0, \quad U_y > 0, \quad U_{yy} < 0, \quad U_{xy} > 0, \quad U_{xx}U_{yy} - U_{xy}^2 > 0,$$

$$s' > 0, \quad s'' < 0, \quad V_l < 0, \quad V_{ll} < 0. \tag{2.2}$$

The function $U(x, y)$ obeys the standard assumptions that utility is increasing in the consumption of own goods and strictly concave. The condition $U_{xy} > 0$ imposes Edgeworth complementarity on the own consumption of domestic and foreign goods. In addition, (2.2) implies that status is increasing and concave in the relative consumption of imports, while work effort generates disutility and is strictly concave.

Regarding agents' production and financial market possibilities, we assume, first, that the production of tradable output, q, depends on the single factor employment, l, which has the standard properties of positive and declining marginal productivity: $q = F(l)$, $F' > 0$, $F'' < 0$.[9] We specify next that the interest rate at which agents borrow depends—in addition to the exogenous and time invariant world interest rate—on the economy's ability to service its outstanding level of obligations, as measured by the debt to GDP ratio. Letting $r^b[b/F(l)]$ represent the interest rate on foreign debt, this relationship is defined by the following equation

$$r^b \equiv r^b[b/F(l)] = r^* + \alpha[b/F(l)], \quad \alpha' > 0, \quad \alpha'' > 0, \tag{2.3}$$

where r^* is the given world interest rate and $\alpha[b/F(l)]$ is the country-specific interest cost, which is a positive, increasing function of $b/F(l)$, the ratio of outstanding debt to domestic output.[10] Equation (2.3) is in contrast to most of the work cited above that specifies $\alpha(\cdot)$ as a function of b alone.[11] The early study of Edwards (1984)—showing a positive relationship

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[7]Below, we employ a modified version of δ, given by $\eta \equiv \delta s'(1) > 0$, where $y \equiv Y$ in the symmetric equilibrium.

[8]The following notational conventions are observed: partial derivatives of functions are denoted by subscripts; derivatives of functions with a single argument are indicated by "primes"; and time derivatives are denoted by "dots". In general, we suppress a variable's time dependence.

[9]The implicit assumption of a fixed domestic capital stock allows us to simplify the analysis, particularly the derivation of the dynamic equilibrium, and to focus on some of the central implications of borrowing and consumption externalities.

[10]We assume for expositional purposes that the country is always a net debtor, $b > 0$, although it is straightforward to generalize the results to the case in which the country is a net creditor, $b < 0$.

[11]Exceptions are Bhandari et. al. (1990) and Chatterjee and Turnovsky (2004), who scale national indebtedness

between the interest rate spread over the LIBOR rate and the debt to GDP ratio—provides for a set of developing economies some empirical evidence for (2.3). For our purposes, a further crucial advantage of the debt to GDP specification in (2.3) is that shifts in status preference that change the level of employment (and output) lead to a non-degenerate transitional dynamics, which, on the other hand, do not occur if $\alpha(\cdot)$ is a function of debt alone (see footnote 24 below).

In the context of a perfect foresight, dynamic equilibrium, the agent's maximization problem is formulated as follows

$$\max \int_0^\infty [U(x,y) + s\,(y/Y) + V(l)]e^{-\rho t}dt, \tag{2.4a}$$

subject to

$$\dot{b} = y + [x - F(l)]/p + r^b[b/F(l)]b, \quad b(0) = b_0 > 0, \tag{2.4b}$$

where ρ is the exogenous rate of pure time preference, p is the relative price of the foreign in terms of the domestic good, and b_0 is the inherited stock of debt.[12] Observe that the flow constraint for the accumulation of debt is formulated in terms of the foreign good. In solving the optimization problem, we posit that the agent takes the average consumption of imported goods Y as given and ignores the effect of his work effort and borrowing decisions on the bond rate r^b.[13] Applying standard optimizing techniques for this class of problem, the following first order conditions obtain

$$U_x\,(x,y) = \lambda/p, \qquad U_y\,(x,y) + Y^{-1}\delta s'\,(y/Y) = \lambda, \tag{2.5a, b}$$

$$V'(l) = -\lambda F'(l)/p, \qquad \dot{\lambda} = \lambda\{\rho - r^b[b/F(l)]\}, \tag{2.5c, d}$$

where λ the current costate variable, evaluated in terms of the foreign good. Equation (2.5a) is the necessary condition for consumption of the domestic good, while (2.5b) is the corresponding condition for imports. Observe that the marginal utility of imported goods is the sum of the

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by the stock of physical capital.

[12]Since p is the relative price in terms of the domestic good, a rise (resp. fall) in p corresponds to a fall (resp. decline) in the economy's terms of trade.

[13]Since agents optimize holding r^b constant, it is unnecessary to introduce the distinction between individual and average levels of indebtedness. Also, as is usual in models of this type, the agent takes as given the terms of trade p, although it is endogenous in equilibrium.

"direct" marginal utility $U_y(x, y)$ of imports and the marginal utility $Y^{-1}\delta s'(y/Y)$ of status, where the latter is scaled by $Y^{-1}\delta$. Equation (2.5c) is the optimality condition for employment, while equation (2.5d) describes the evolution of the shadow value if international borrowing is selected optimally, given that agents neglect the effect of their choices of b and l on r^b. We next impose the following transversality condition that constrains the limiting dynamics of debt and insures that (2.5a)–(2.5d) are sufficient for optimality: $\lim_{t\to\infty} \lambda b e^{-\rho t} = 0$.

The next step is to derive the intertemporal macroeconomic equilibrium. Following the standard procedure for this type of problem, we assume that each identical individual acts in the same way and normalize the population of agents to unity. This implies $y = Y$ holds $\forall t \geq 0$ and results in the following symmetric equilibrium

$$U_x(x, y) = \lambda/p, \qquad U_y(x, y) + \delta s'(1) y^{-1} \equiv U_y(x, y) + \eta y^{-1} = \lambda, \qquad \text{(2.6a, b)}$$

$$V'(l) = -\lambda F'(l)/p, \qquad F(l) = x + Z(p), \qquad \text{(2.6c, d)}$$

$$\dot{\lambda} = \lambda\{\rho - r^b[b/F(l)]\}, \qquad \dot{b} = y - Z(p)/p + r^b[b/F(l)]b, \qquad \text{(2.6e, f)}$$

where we substitute for $\eta \equiv \delta s'(1) > 0$ in the optimality condition (2.6b) for imported goods. In our subsequent analysis, we treat η as the parameter measuring the degree, or "intensity", of agents' status preference for the relative consumption of imports. Observe that equation (2.6d) is the market clearing condition for domestic production where $Z(p)$, $Z' > 0$, represents the *exports* of the domestic good, which increase as the relative price of the domestic good falls. Equally, the (flow) accumulation equation for debt in (2.6f)—corresponding to the *negative* of the current account balance—is written in terms of $Z(p)$, so that $[y - Z(p)/p]$ is the *trade deficit*. Note further that the system (2.6a)–(2.6f) implicitly includes the transversality condition. Moreover, since the terms of trade is an endogenous variable in the macroeconomic equilibrium, we can define, under real interest rate parity, a domestic, or "internal", rate of return r^d that equals the sum of the bond rate $r^b[b/F(l)]$ and the rate of change of the terms of trade $\dot{p}/p$:

$$r^d = r^b[b/F(l)] + \dot{p}/p = r^* + \alpha[b/F(l)] + \dot{p}/p. \qquad \text{(2.7)}$$

The equations (2.6a)–(2.6d) constitute an short-run system that is solved for (x, y, p, l) in

terms of the marginal utility of wealth λ and the status parameter η

$$x = x(\lambda, \eta), \quad x_\lambda < 0, \quad x_\eta > 0; \qquad\qquad y = y(\lambda, \eta), \quad y_\lambda < 0, \quad y_\eta > 0, \qquad (2.8\text{a, b})$$

$$p = p(\lambda, \eta), \quad p_\lambda > 0, \quad p_\eta < 0; \qquad\qquad l = l(\lambda, \eta), \quad l_\lambda > 0, \quad l_\eta > 0, \qquad (2.8\text{c, d})$$

where we indicate in (2.8a)–(2.8d) the signs of the partial derivatives of the solutions (x, y, p, l) with respect to λ and η.[14] The actual expressions for the partial derivatives are stated in the appendix [see (6.1a)–(6.1d) and (6.2a)–(6.2d)] and are interpreted as follows: a rise in the shadow value λ lowers the consumption of domestic and foreign goods, x and y, and as well as the consumption of leisure. The resulting increase in employment, l, and domestic output, q, lowers, in turn, the relative price of domestic goods, i.e., p rises. In contrast, an increase in the status preference parameter η increases the demand for imported goods y. Due the assumption of Edgeworth complementarity ($U_{xy} > 0$), the higher value of η leads to an increase in the consumption of domestic goods and leisure, i.e., both x and l rise. This, in turn, raises the country's terms of trade so that p falls.

Turning the economy's dynamics, we obtain the differential equations describing the evolution of the marginal utility of wealth and stock of debt by substituting the instantaneous solutions (2.8b)–(2.8d), together with the expression (2.3) for r^b, into (2.6e)–(2.6f):

$$\dot{\lambda} = \lambda \left\{ \rho - \left[r^* + \alpha \left(\frac{b}{F[l(\lambda, \eta)]} \right) \right] \right\} \qquad (2.9\text{a})$$

$$\dot{b} = y(\lambda, \eta) - \frac{Z[p(\lambda, \eta)]}{p(\lambda, \eta)} + \left\{ r^* + \alpha \left[\frac{b}{F[l(\lambda, \eta)]} \right] \right\} b. \qquad (2.9\text{b})$$

Letting $\dot{\lambda} = \dot{b} = 0$ in (2.9a, b), the corresponding steady-state equilibrium constitutes the following set of relationships

$$U_x(\tilde{x}, \tilde{y}) = \tilde{\lambda}/\tilde{p}, \qquad\qquad U_y(\tilde{x}, \tilde{y}) + \eta \tilde{y}^{-1} = \tilde{\lambda}, \qquad (2.10\text{a, b})$$

$$V'(\tilde{l}) = -\tilde{\lambda} F'(\tilde{l})/\tilde{p}, \qquad\qquad F(\tilde{l}) = \tilde{x} + Z(\tilde{p}), \qquad (2.10\text{c, d})$$

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[14]Although it is not the focus of the analysis, the equilibrium (2.8a)–(2.8d) also depends on the parameters of the production and export functions.

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$$\tilde{r}^b = r^b[\tilde{b}/F(\tilde{l})] = r^* + \alpha[\tilde{b}/F(\tilde{l})] = \tilde{r}^d = \rho, \tag{2.10e}$$

$$Z(\tilde{p}) - \tilde{p}\tilde{y} = \tilde{p}\{r^* + \alpha[\tilde{b}/F(\tilde{l})]\}\tilde{b}, \tag{2.10f}$$

where the symbol $\tilde{\ }$ indicates a long-run variable. Equations (2.10a)–(2.10d) are the long-run versions of (2.6a)–(2.6d), while (2.10e) and (2.10f) state, respectively, that the long-run interest rates $\tilde{r}^b$ and $\tilde{r}^d$ equal the exogenous rate of time preference ρ and that steady-state interest service—in terms of the domestic good—equals the domestic trade balance (net exports).

Taking first-order approximations of (2.9a, b) around the steady-state system (2.10a)–(2.10f), the following matrix differential equation is obtained

$$\begin{pmatrix} \dot{\lambda} \\ \dot{b} \end{pmatrix} = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} \begin{pmatrix} \lambda - \tilde{\lambda} \\ b - \tilde{b} \end{pmatrix}, \tag{2.11a}$$

where

$$\theta_{11} = \frac{\tilde{\lambda}\alpha'\tilde{b}F'l_\lambda}{F^2} > 0, \quad \theta_{12} = -\frac{\tilde{\lambda}\alpha'}{F} < 0,$$

$$\theta_{21} = \left[\frac{\tilde{p}y_\lambda - \beta p_\lambda}{\tilde{p}} - \frac{\alpha'\tilde{b}^2 F'l_\lambda}{F^2}\right] < 0, \qquad \theta_{22} = \rho + \frac{\alpha'\tilde{b}}{F} > 0, \tag{2.11b}$$

and where we substitute for $\beta = Z' - Z(p)/p > 0$ in the expression for θ_{21}.[15] The stability properties of this system are determined by the signs of the trace and determinant of the Jacobian matrix $\mathbf{J}$ of (2.11a).[16] These are given, respectively, by

$$\mathrm{tr}\,(\mathbf{J}) = \mu_1 + \mu_2 = \theta_{11} + \theta_{22} = \rho + \frac{\alpha'\tilde{b}}{F}\left[1 + \frac{\tilde{\lambda}F'l_\lambda}{F}\right] > 0 \tag{2.12a}$$

$$\det\,(\mathbf{J}) = \mu_1\mu_2 = \theta_{11}\theta_{22} - \theta_{12}\theta_{21} = \frac{\tilde{\lambda}\alpha'}{F}\left[\frac{\tilde{p}y_\lambda - \beta p_\lambda}{\tilde{p}} + \frac{\rho\tilde{b}F'l_\lambda}{F}\right] \tag{2.12b}$$

where μ_1, μ_2 are the eigenvalues of $\mathbf{J}$ that satisfy the corresponding characteristic polynomial:

$$\mu^2 - [\mathrm{tr}\,(\mathbf{J})]\,\mu + \det\,(\mathbf{J}) = 0. \tag{2.12c}$$

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[15]The assumption $\beta > 0$ implies that export demand is price elastic, i.e., $(Z'p/Z) > 1$.
[16]The functions constituting the elements of J, θ_{ij}, $i,j = 1, 2$ are evaluated in the steady-state equilibrium, e.g., $F = F(\tilde{l})$.

A necessary condition for the long-run equilibrium of (2.11a) to possess a saddlepoint is $\det(\mathbf{J}) = \mu_1\mu_2 < 0$. This requires that the term in square brackets in (2.12b) be negative, i.e.:

$$\frac{\tilde{p}y_\lambda - \beta p_\lambda}{\tilde{p}} + \frac{\rho\tilde{b}F'l_\lambda}{F} < 0. \tag{2.13}$$

If (2.13) is negative, then the dynamics of (2.11a) is characterized by (local) saddlepoint stability, with $\mu_1 < 0$, $\mu_2 > 0$, $\mid \mu_1 \mid < \mu_2$.[17] Using standard methods, we then obtain the following saddlepath solutions for consumption and national debt

$$\lambda = \tilde{\lambda} + \frac{\theta_{22} - \mu_1}{\theta_{21}}(\tilde{b} - b_0)e^{\mu_1 t} = \tilde{\lambda} + \frac{\theta_{12}}{\theta_{11} - \mu_1}(\tilde{b} - b_0)e^{\mu_1 t}, \tag{2.14a}$$

$$b = \tilde{b} - (\tilde{b} - b_0)e^{\mu_1 t}, \tag{2.14b}$$

where $b(0) = b_0 > 0$.[18] Combining the solutions (2.14a, b), we obtain the stable saddlepath that describes the co-movements of the marginal utility and debt:

$$(\lambda - \tilde{\lambda}) = -\frac{\theta_{22} - \mu_1}{\theta_{21}}(b - \tilde{b}) = \frac{-\theta_{12}}{\theta_{11} - \mu_1}(b - \tilde{b}). \tag{2.15}$$

The graph of this relationship has a positive slope, which implies that b and λ and evolve in the *same* directions along the stable adjustment path, i.e., $\mathsf{sgn}\,(\dot{b}) = \mathsf{sgn}\,(\dot{\lambda})$.

We next derive the phase diagram, illustrated by Figure 1, of the dynamic system (2.14a, b). Using equations (2.9a, b), the $\dot{\lambda} = 0$ and $\dot{b} = 0$ loci are described by the following relationships:

$$r^* + \alpha\left[\frac{b}{F[l(\lambda, \eta)]}\right] = \rho, \tag{2.16a}$$

$$\frac{Z[p(\lambda, \eta)]}{p(\lambda, \eta)} - y(\lambda, \eta) = \left\{r^* + \alpha\left[\frac{b}{F[l(\lambda, \eta)]}\right]\right\}b. \tag{2.16b}$$

The slopes of (2.16a, b)—evaluated in long-run equilibrium—equal:

$$(d\lambda/db)\mid_{\dot{\lambda}=0} = F/\tilde{b}F'l_\lambda > 0, \qquad (d\lambda/db)\mid_{\dot{b}=0} = -\theta_{22}/\theta_{21} > 0. \tag{2.17a, b}$$

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[17]Loosely speaking, the condition for saddlepoint stability in (2.13) implies that a change in the marginal utility has a greater effect on the trade balance than on the bond rate. Observe also that the existence of a saddlepoint in the decentralized equilibrium does not (directly) depend on the interest-cost function $\alpha\,(\cdot)$ or its "slope", $\alpha'\,(\cdot)$.

[18]Because μ_1 is an eigenvalue of $\mathbf{J}$, $(\theta_{22} - \mu_1)/\theta_{21} = \theta_{12}/(\theta_{11} - \mu_1)$.

It is straightforward to account for the positive slopes of the $\dot{\lambda} = 0$ and $\dot{b} = 0$ loci: along $\dot{\lambda} = 0$, a rise in b increases the bond rate r^b relative to the rate of time preference ρ, putting downward pressure on the marginal utility ($\dot{\lambda} < 0$). To maintain $\dot{\lambda} = 0$, a rise in λ is required in order to encourage greater work effort and output, which brings the ratio of debt to GDP back to its original level and, thus, the bond rate equal to the rate of time preference. Thus, points to the right of (resp. to the left of) the $\dot{\lambda} = 0$ locus lie on paths in which r^b exceeds (resp. is less than) ρ, with λ decreasing, $\dot{\lambda} < 0$, (resp. increasing, $\dot{\lambda} > 0$). In the case of the $\dot{b} = 0$ locus, a higher value of b leads, through higher interest service, to a deterioration in the current account balance ($\dot{b} < 0$). The latter is not offset unless λ rises, which causes a corresponding improvement in the trade balance and maintains $\dot{b} = 0$. As such, points to the right of (resp. to the left of) the $\dot{b} = 0$ locus lie on paths in which the current account balance is negative, $\dot{b} < 0$ (resp. positive, $\dot{b} > 0$). Moreover, the stability properties of the dynamic system are reflected in the *relative* slopes of the $\dot{\lambda} = 0$ and $\dot{b} = 0$ loci. In particular, the case in which the slope of the $\dot{\lambda} = 0$ locus exceeds the slope of the $\dot{b} = 0$ locus is equivalent to the condition (2.13) for saddlepoint stability:[19]

$$
(d\lambda/db)\,|_{\dot{\lambda}=0} = F/\tilde{b}F'l_\lambda > -\theta_{22}/\theta_{21} = (d\lambda/db)\,|_{\dot{b}=0} \qquad \Leftrightarrow \qquad \frac{\tilde{p}y_\lambda - \beta p_\lambda}{\tilde{p}} + \frac{\rho\tilde{b}F'l_\lambda}{F} < 0.
$$

This case is illustrated in Figure 1, where the arrows depict the directions of the phase lines and where the intersection of the $\dot{\lambda} = 0$ and $\dot{b} = 0$ loci—illustrated by point A—corresponds to the steady-state values of $\tilde{\lambda}$ and $\tilde{b}$.[20] The alternative case (not depicted) in which the slope of the $\dot{b} = 0$ locus is greater than that of the $\dot{\lambda} = 0$ locus, i.e., $(d\lambda/db)\,|_{\dot{b}=0} > (d\lambda/db)\,|_{\dot{\lambda}=0}$, results in an equilibrium that is an unstable node. Observe that Figure 1 also shows the positively sloped, stable saddlepath, based on equation (2.15) and depicted by the line SS. In terms of observable variables, what additional information can be garnered from the saddlepath SS? Consider the case in which initial stock of debt is *less* than its steady-state value, $b_0 < \tilde{b}$, so that the economy starting from point B approaches the saddlepoint A from *below*, with $\dot{\lambda} > 0$, $\dot{b} > 0$. Since $x_\lambda < 0$,

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[19]This point illustrates an important distinction between the interest rate specification (2.3) and the specification of r^b that depends on the stock of debt alone. In the latter case, studied by Fisher (1995), the $\dot{\lambda} = 0$ locus is a vertical line, $(d\lambda/db)\,|_{\dot{\lambda}=0} = \infty$, implying that the equilibrium of the linearized system is a unique, interior saddlepoint.

[20]For expositional purposes, we restrict ourselves in Figure 1 (and subsequently) to the case in which $\dot{\lambda} = 0$ and $\dot{b} = 0$ describe straight lines, although the relationships (2.16a, b) are not, in general, linear. As such, while we concentrate here on the properties of local saddlepoints, the possibility of multiple equilibria cannot be excluded.

$y_\lambda < 0$, and $p_\lambda > 0$ in (2.8a)–(2.8c), transitional adjustment along the "rising" saddlepath SS also involves *declining* consumption of domestic and foreign goods, along with a real *depreciation* in the terms of trade. Moreover, the trade balance in terms of the foreign good *improves* along the path from points B to A, eventually eliminating the current account deficit.[21]

3. Dynamics of an Increase in Status Preference

In this section we describe the dynamic response of the small open economy to an unanticipated permanent increase in the status preference parameter η. To calculate the steady-state effects of an increase in status preference, we employ the steady-state solutions, equations (2.10a)–(2.10f), derived in the previous section and differentiate with respect to η, where the expressions for the long-run multipliers are given in the appendix [see equations (6.3a)–(6.3f)]. The transitional responses of the small open economy are then calculated using the stable saddlepath solutions (2.14a, b) and illustrated with phase diagrams based on Figure 1.

3.1. Long-Run Responses

The signs of the long-run multipliers with respect to an increase in η are given by:

$$\frac{\partial \tilde{\lambda}}{\partial \eta} > 0, \qquad \frac{\partial \tilde{l}}{\partial \eta} = \left(F'\right)^{-1} \frac{\partial \tilde{q}}{\partial \eta} > 0, \qquad \frac{\partial \tilde{b}}{\partial \eta} > 0, \qquad \frac{\partial \tilde{p}}{\partial \eta} > 0, \qquad \frac{\partial \tilde{x}}{\partial \eta} \gtrless 0, \qquad \frac{\partial \tilde{y}}{\partial \eta} > 0. \quad (3.1)$$

The steady-state dynamics described by the expressions in (3.1) are explained as follows: a permanent rise in η leads to an increase in the marginal utility of wealth $\tilde{\lambda}$, which, in turn, leads to a steady-state rise in employment and output, $\tilde{l}$ and $\tilde{q}$, a result, as indicated above, comparable those derived by Fisher and Hof (2000b) and Liu and Turnovsky (2004) for single good, the closed economy. With a higher resource base, the economy supports a greater stock of long-run debt, $\tilde{b}$. Nevertheless, these adjustments do *not* lead to a change in the steady-state debt to GDP ratio, since, according to the long-run Euler relationship (2.10e), $\tilde{b}/F(\tilde{l})$ is *independent* (as are the interest rates $\tilde{r}^b$ and $\tilde{r}^d$) of the status parameter η. In addition, the increase in steady-state interest service, due to the rise in $\tilde{b}$, requires that net exports, whether

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[21]This is straightforward to show by calculating the time derivative of $Z[p(\lambda, \eta)]/p(\lambda, \eta) - y(\lambda, \eta)$.

in terms of the foreign or of the domestic good, increase to maintain long-run current account equilibrium. The improvement in the trade balance is, moreover, accompanied by a long-run deterioration in the terms of trade, i.e., by a rise in $\tilde{p}$. Regarding the long-run consumption of foreign goods, a permanent increase in the preference weight on the relative consumption of imports causes $\tilde{y}$ to rise. On the other hand, we can show that the restrictions placed on the (differential of) the steady-state system (2.10a)–(2.10f) are insufficient to determine the sign of the long-run change of the domestic good $\tilde{x}$.

3.2. Transitional Dynamics

Using the solution (2.14a), the initial change in the marginal utility in response to an increase in status preference equals:

$$\frac{\partial \lambda(0)}{\partial \eta} = \frac{\partial \tilde{\lambda}}{\partial \eta} + \frac{\theta_{22} - \mu_1}{\theta_{21}} \frac{\partial \tilde{b}}{\partial \eta} \gtrless 0. \tag{3.2}$$

The response of the marginal utility in (3.2) is ambiguous: while the steady-state rise in λ tends to increase the value of $\lambda(0)$, transitional factors involving the adjustment of (b, λ) along the saddlepath tend to lower $\lambda(0)$. Nevertheless, we can identify, using the $\dot{\lambda} = 0$ and $\dot{b} = 0$ loci depicted in Figure 1, three distinct cases. In all three cases, illustrated, respectively, in Figures 2a–c, a permanent increase in η causes $\dot{\lambda} = 0$ to shift down: a higher value of η raises work effort, lowering the debt to output ratio and, thus, the bond rate relative to the rate of time discount. Given the initial value of b, a fall in λ is then required to maintain $\dot{\lambda} = 0$.[22] In contrast, the vertical shift in the $\dot{b} = 0$ locus in response to a rise in η can be positive or negative.[23] The reason for this ambiguity is due to the fact that an increase in the degree of status preference has two, offsetting effects on the current account balance. On the one hand, a rise in η lowers, through higher consumption of the foreign good, net exports. This puts downward pressure on the current account balance and causes $\dot{b} = 0$ to shift-up. On the other hand, a higher value

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[22]From (2.16a), the vertical shift in the $\dot{\lambda} = 0$ locus equals $d\lambda = -\left(l_\eta/l_\lambda\right) d\eta < 0$.

[23]Using (2.16b), the vertical shift in the $\dot{b} = 0$ locus equals:

$$d\lambda = -\left[\frac{\tilde{p}y_\lambda - \beta p_\lambda}{\tilde{p}} - \frac{\alpha' \tilde{b}^2 F' l_\lambda}{F^2}\right]^{-1} \left\{\frac{\tilde{p}y_\eta - \beta p_\eta}{\tilde{p}} - \frac{\alpha' \tilde{b}^2 F' l_\eta}{F^2}\right\} d\eta,$$

where the term in $\{\cdot\}$ incorporates the offsetting trade balance and interest service effects of a rise in η.

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of η encourages greater work effort (and output), which, given b_0, lowers the bond rate r^b and interest service. The latter effect tends to improve the current account balance, which, in turn, leads $\dot{b} = 0$ to shift down.[24]

Figures 2a and 2b illustrate the case in which the trade balance effect dominates so that the $\dot{b} = 0$ locus shifts up in response to a permanent increase in the status preference parameter η. The distinction between the two phase diagrams is that in Figure 2a, the vertical decline in the $\dot{\lambda} = 0$ locus exceeds that of the $\dot{b} = 0$ locus in absolute value, while the opposite is true in Figure 2b. In Figure 2a this means that the new saddlepath, described by the line DE, lies *below* its original position (not depicted), implying that $\lambda(0)$ *falls* from point A to point D before proceeding up DE. In contrast, the new saddlepath GH in Figure 2b lies *above* its initial position so that $\lambda(0)$ *rises* from point A to point G at $t = 0$. The cases illustrated by Figures 2a and 2b are further distinguished by the initial responses of domestic and foreign goods consumption and the terms of trade: combined with the direct effect of a higher value of η [see (2.8a)–(2.8c)], the fall in $\lambda(0)$ in Figure 2a—and also below in Figure 2c—results in a rise in $x(0)$ and $y(0)$ and a fall in $p(0)$. In contrast, the initial response of these variables is ambiguous in the case of Figure 2b in which $\lambda(0)$ rises. Nevertheless, the transitional adjustment to long-run equilibrium in both Figures 2a and 2b along the saddlepaths DE and GH involves, as established in (2.15), increasing values of the marginal utility and the stock of debt, i.e., $\dot{\lambda} > 0$, $\dot{b} > 0$ and, thus, reductions in the levels of domestic and foreign consumption, $\dot{x} < 0$, $\dot{y} < 0$, and a depreciation in the terms of trade, $\dot{p} > 0$. Figure 2c illustrates the case in which the $\dot{b} = 0$ locus shifts down in response to a rise in η: in other words, the interest service effect described above dominates the trade balance effect. Here, the jump in $\lambda(0)$ from point A to point J is unambiguously negative. Nevertheless, as in Figures 2a and 2b, the marginal utility and the stock of debt *rise* toward their steady-state values, in this case along the new saddlepath JL.[25]

To further describe the response of the economy to a shift in status preference, we conclude this section of the paper by considering the behavior of the trade balance, the bond rate r^b,

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[24]If r^b is solely a function of b, then $\dot{\lambda} = 0$ does not shift in response to an increase in η. Moreover, since $\dot{b} = 0$ unambiguously shifts up in this case, the shadow value immediately rises to $\tilde{\lambda}$ with no transitional dynamics. Note, however, that for macroeconomic disturbances such as world interest rate shocks, the specification of r^b as a function of b alone suffices to generate an interior equilibrium with saddlepoint dynamics.

[25]The shift in $\dot{\lambda} = 0$ in Figure 2c must, however, sufficiently exceed that of $\dot{b} = 0$ in order for the long-run equilibrium at point L to correspond to higher values of $\tilde{\lambda}$ and $\tilde{b}$.

and the domestic rate of return r^d.[26] Linearizing the expression for the trade balance about the steady-state equilibrium (2.10a)–(2.10f) and substituting for (2.14a)–(2.14b), its solution path corresponds to:

$$TB = \rho\tilde{b} - \frac{(\tilde{p}y_\lambda - \beta p_\lambda)(\theta_{22} - \mu_1)}{\tilde{p}\theta_{21}}(\tilde{b} - b_0)e^{\mu_1 t}. \tag{3.3}$$

Differentiating (3.3) with respect to η and evaluating at $t = 0$, the initial response of the trade balance equals:

$$\frac{\partial TB(0)}{\partial \eta} = \left[\rho - \frac{(\tilde{p}y_\lambda - \beta p_\lambda)(\theta_{22} - \mu_1)}{\tilde{p}\theta_{21}}\right]\frac{\partial \tilde{b}}{\partial \eta}. \tag{3.4a}$$

Using the definition of θ_{21} and substituting for θ_{22} in (3.4a), we can demonstrate that the change in the trade balance at $t = 0$ is given by:

$$\frac{\partial TB(0)}{\partial \eta} = \theta_{21}^{-1}[\mu_1\tilde{p}^{-1}(\tilde{p}y_\lambda - \beta p_\lambda) - (\tilde{b}/\tilde{\lambda})\det(\mathbf{J})] < 0. \tag{3.4b}$$

Thus, a permanent increase in η causes the trade balance to deteriorate on impact, a result— true for all three cases described above—that depends on the sufficient condition that the long-run equilibrium is a saddlepoint, i.e., $\det(\mathbf{J}) < 0$. Using the solution path (3.3), it is clear, nevertheless, that subsequent to $t = 0$ the trade balance improves continuously in order to support the rising stock of debt. Regarding the interest rates r^b and r^d, we show in the appendix that their solution paths correspond, respectively, to:

$$r^b = \rho + \frac{\alpha'\mu_1}{F(\theta_{11} - \mu_1)}(\tilde{b} - b_0)e^{\mu_1 t}, \tag{3.5a}$$

$$r^d = \rho + \frac{\alpha'\mu_1}{F(\theta_{11} - \mu_1)}(1 - p_\lambda\tilde{\lambda}/\tilde{p})(\tilde{b} - b_0)e^{\mu_1 t}. \tag{3.5b}$$

Evaluating (3.5a, b) at $t = 0$ and combining, we show that both rates initially fall in response to a permanent increase in η, i.e.:

$$\frac{\partial r^d(0)}{\partial \eta} = (1 - p_\lambda\tilde{\lambda}/\tilde{p})\frac{\partial r^b(0)}{\partial \eta} < 0. \tag{3.6}$$

The expression reveals two crucial aspects of the economy's short-run adjustment. One is the

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[26]The procedure used to obtain the solutions of these variables is found, in the appendix, equations (6.4a)–(6.4b), (6.5a)–(6.5c), and (6.6a)–(6.6b), respectively.

fact that since r^b declines on impact, it must case that the debt to GDP ratio falls at $t = 0$. Since the stock of debt is given at $t = 0$, this means that employment and output initially increase in response to the rise in η, a result that obtains whether or not $\lambda(0)$ rises or falls. As a consequence, interest service declines in the short-run, which implies that the initial current account deficits are caused by the short-run deterioration (eventually reversed) in the trade balance. The other aspect is that the domestic rate of return r^d, while also *below* its long-run value of ρ, lies *above* the bond rate at $t = 0$. This is due to the fact that terms of trade depreciates at $t = 0$, i.e., $\dot{p}/\tilde{p} = p_\lambda \dot{\lambda}(0)/\tilde{p} > 0$, which, under interest rate parity, raises the domestic rate relative to the bond rate.[27] Finally, both interest rates rise for $t > 0$, converging to their common steady-state value of ρ, reflecting both accumulation of debt, which increases the bond rate, and the continued appreciation in the terms of trade, which raises the "internal" rate. The adjustment paths of the two rates are illustrated in Figure 3, which depicts the initial declines in $r^b(0)$ and $r^d(0)$ and their subsequent convergence to ρ.

4. The Planner's Problem and Optimal Taxation

4.1. Intertemporal Equilibrium

In this section of the paper we derive the solution to the model from a social planner's point of view. The key distinction between the solution of the planner's problem and that of the representative agent is that the planner "internalizes" the consumption and borrowing external-ities ignored by the representative agent. In terms of the consumption externality, the planner assigns to each identical agent the same level of imported goods, $y = Y$. Likewise, the planner sets the level of work effort and the stock of debt to take into account the fact that these choices affect the interest rate r^b at which the economy borrows from abroad. To distinguish the Pareto optimal solution from its decentralized counterpart, we denote the variables of the optimal so-lution with the superscript "o".[28] The social planner's optimization problem is thus formulated in the following way

$$\max \int_0^\infty [U(x^o, y^o) + \delta s\,(1) + V(l^o)]e^{-\rho t}dt, \tag{4.1a}$$

---

[27]We can show that the term $(1 - p_\lambda \tilde{\lambda}/p)$ in (3.5b), while less than unity, is positive.

[28]It is assumed in this section that functions are evaluated at their Pareto optimal values, e.g., $F = F(l^o)$.

subject to

$$\dot{b}^o = y^o + [x^o - F(l^o)]/p^o + \{r^* + \alpha[b^o/F(l^o)]\}b^o, \quad b^o(0) = b_0^o > 0, \qquad (4.1b)$$

where, as before, ρ is the exogenous rate of time preference and p^o is the relative price of the foreign in terms of the domestic good. We substitute expression for the bond rate $r^b[b^o/F(l^o)]$ in (4.1b) to emphasize the fact the planner takes into account the effect that work effort and borrowing decisions have on the bond rate. Solving this problem, the following socially optimal equilibrium is derived

$$U_x(x^o, y^o) = \lambda^o/p^o, \qquad U_y(x^o, y^o) = \lambda^o, \qquad (4.2a, b)$$

$$V'(l^o) + \frac{\lambda^o \alpha'(b^o)^2 F'}{F^2(l^o)} = -\lambda^o F'(l^o)/p^o, \qquad F(l^o) = x^o + Z(p^o), \qquad (4.2c, d)$$

$$\dot{\lambda}^o = \lambda^o\{\rho - r^b[b^o/F(l^o)] - \alpha'b^o/F(l^o)\}, \qquad \dot{b}^o = y^o - Z(p^o)/p^o + r^b[b^o/F(l^o)]b^o, \quad (4.2e, f)$$

where λ^o is the current Pareto optimal costate variable and where the market clearing condition (4.2d) and current account relationship (4.2f) are added to complete the system. Examining the optimality conditions in equations (4.2a)–(4.2f), we see the crucial differences between the Pareto and decentralized economies. In (4.2b), there is no externality from the consumption of imports. Indeed, because the planner sets $y = Y$ *prior* to the calculating the optimality conditions, status considerations and, in particular, the parameter η, play no role in the social optimum. In (4.2c) the planner incorporates into his evaluation of the disutility of work effort the fact that a greater level of employment *lowers* the interest cost of borrowing. Similarly, in (4.2e) the planner takes into account that additional indebtedness *raises* the interest cost of borrowing. In addition, the equilibrium (4.2a)–(4.2f) implicitly incorporates, as in the decentralized case, a transversality condition $\lim_{t\to\infty} \lambda^o b^o e^{-\rho t} = 0$ guaranteeing that the necessary conditions are sufficient for an optimum.

An important implication of the optimality condition (4.2c) for work effort is that the instantaneous solutions in the planner's equilibrium depend, in addition to the marginal utility λ^o, on the stock of debt b^o. In other words, equations (4.2a)–(4.2d) constitute a short-run system that is solved for as follows

$$x^o = x(\lambda^o, b^o), \quad x_\lambda^o < 0, \quad x_b^o > 0; \qquad y^o = y(\lambda^o, b^o), \quad y_\lambda^o < 0, \quad y_b^o > 0, \qquad (4.3a, b)$$

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$$p^o = p(\lambda^o, b^o), \quad p_\lambda^o > 0 \quad p_b^o > 0; \qquad l^o = l(\lambda^o, b^o), \qquad l_\lambda^o > 0, \quad l_b^o > 0, \qquad \text{(4.3c, d)}$$

where the signs of the partial derivatives of (x^o, y^o, p^o, l^o) with respect to the marginal utility of wealth λ^o and the stock of debt b^o are indicated in (4.3a)–(4.3d).[29] The partial derivatives with respect to λ^o have an interpretation similar to that the decentralized model. In contrast, the partial derivatives with respect to debt b^o in the social equilibrium are explained as follows: a higher stock of debt lowers the disutility of labor, since greater work effort implies that a larger stock of debt is less "costly" in terms of debt service. As such, employment rises, $l_b^o > 0$, which results in an expansion in domestic output that causes a fall in the terms of trade, $p_b^o > 0$. The latter implies, in turn, an increase in domestic consumption, $x_b^o > 0$, and, because $U_{xy} > 0$, a rise in foreign consumption, $y_b^o > 0$.

Substituting the instantaneous solutions (4.3b)–(4.3d) into (4.2e, f), we obtain the differential equation system that describes the evolution of the marginal utility of wealth and stock of debt

$$\dot{\lambda}^o = \lambda^o \left\{ \rho - \left[r^* + \alpha \left(\frac{b^o}{F\left[l(\lambda^o, b^o)\right]} \right) \right] - \frac{b^o}{F\left[l(\lambda^o, b^o)\right]} \alpha' \left(\frac{b^o}{F\left[l(\lambda^o, b^o)\right]} \right) \right\}, \qquad \text{(4.4a)}$$

$$\dot{b}^o = y\left(\lambda^o, b^o\right) - \frac{Z\left[p\left(\lambda^o, b^o\right)\right]}{p\left(\lambda^o, b^o\right)} + \left\{ r^* + \alpha \left[\frac{b^o}{F\left[l(\lambda^o, b^o)\right]} \right] \right\} b^o, \qquad \text{(4.4b)}$$

where we substitute in (4.4a, b) for the bond rate. Setting $\dot{\lambda}^o = \dot{b}^o = 0$ in (4.4a, b), the steady-state equilibrium of the social planner corresponds to

$$U_x(\tilde{x}^o, \tilde{y}^o) = \tilde{\lambda}^o / \tilde{p}^o, \qquad U_y\left(\tilde{x}^o, \tilde{y}^o\right) = \tilde{\lambda}^o, \qquad \text{(4.5a, b)}$$

$$V'(\tilde{l}^o) + \frac{\tilde{\lambda}^o \alpha'(\tilde{b}^o)^2 F'}{F^2(\tilde{l}^o)} = -\tilde{\lambda}^o F'(\tilde{l}^o)/\tilde{p}^o, \qquad F(\tilde{l}^o) = \tilde{x}^o + Z(\tilde{p}^o), \qquad \text{(4.5c, d)}$$

$$r^b[\tilde{b}^o/F(\tilde{l}^o)] + \alpha'\tilde{b}^o/F(\tilde{l}^o) = r^* + \alpha[\tilde{b}^o/F(\tilde{l}^o)] + \alpha'\tilde{b}^o/F(\tilde{l}^o) = \rho, \qquad \text{(4.5e)}$$

$$Z\left(\tilde{p}^o\right) - \tilde{p}^o\tilde{y}^o = \tilde{p}^o\{r^* + \alpha[\tilde{b}^o/F(\tilde{l}^o)]\}\tilde{b}^o, \qquad \text{(4.5f)}$$

where, as before, the symbol ~ indicates a long-run variable. Similar to the decentralized framework, equations (4.5a)–(4.5d) are the long-run counterparts to (4.2a)–(4.2d), while (4.5e) and (4.5f) are, respectively, the steady-state Euler and current account relationships in the social

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[29]The expressions for the partial derivatives of the socially optimal economy are stated in the appendix, equations (6.7a)–(6.7d) and (6.8a)–(6.8d).

optimum. Moreover, comparing (4.5e) to (2.10e), it is straightforward to show that the long-run debt to GDP ratio in the decentralized economy, $\tilde{b}/F(\tilde{l})$, is "too high" compared the optimal ratio, $\tilde{b}^o/F(\tilde{l}^o)$.[30] This implies, in turn, that the steady-state bond rate in the decentralized economy *exceeds* its socially optimal counterpart, i.e., $\tilde{r}^b > (\tilde{r}^o)^b$. Since, as we show above, $\tilde{b}/F(\tilde{l})$ is *independent* of the status preference parameter η, the deviations of $\tilde{b}/F(\tilde{l})$ and $\tilde{r}^b$ from their social optima are due to the spillovers arising from the external borrowing constraint.

How do the other decentralized variables compare to those of the Pareto optimal equilibrium? We consider this question by employing a technique, recently used by Liu and Turnovsky (2004), that involves linearizing (2.10a)–(2.10f) about (4.5a)–(4.5f). The resulting system is then solved for the steady-state *difference* between the decentralized and socially optimal values of the economy [see equations (6.9) and (6.10a)–(6.10e) in the appendix]. Moreover, this approach also allows us to isolate the influence of the consumption externality from that the borrowing externality. Consistent with our results in section 3, we can show that the decentralized values of employment $\tilde{l}$ (and output $\tilde{q}$), debt $\tilde{b}$, the consumption of imported goods $\tilde{y}$, and the marginal utility $\tilde{\lambda}$ *exceed* their socially optimal counterparts the *larger* is status preference parameter η. In addition, the decentralized terms of trade are "too low" (i.e., p is "too high") in the presence of the relative consumption externality.

In contrast to these results, the borrowing externality has a more ambiguous influence on the decentralized relative to the planning equilibrium. Two basic effects can be identified: (i) in the first, amounting to a relative price distortion, agents in the decentralized economy ignore the effects of work effort on borrowing costs, which implies that employment (and output) are not only "too low" relative to the social optimum, $(\tilde{l} - \tilde{l}^o) < 0$, $(\tilde{q} - \tilde{q}^o) < 0$, but also that the terms of trade are "too high", $(\tilde{p} - \tilde{p}^o) > 0$; (ii) in the second, the fact that the bond rate in the decentralized equilibrium exceeds its socially optimal counterpart implies—due to the negative wealth effect of a higher interest rate—that employment (and output) are "too high" relative to the planner's equilibrium, $(\tilde{l} - \tilde{l}^o) > 0$, $(\tilde{q} - \tilde{q}^o) > 0$ and that the terms of trade are "too low", $(\tilde{p} - \tilde{p}^o) < 0$. In fact, the only variable for which the two effects work in the same direction is the consumption of domestic goods, which we can show—under our assumptions—is less than its socially optimal level, $(\tilde{x} - \tilde{x}^o) < 0$. For all other decentralized

---

[30]Equating (4.5e) and (2.10e), we obtain $\alpha[\tilde{b}/F(\tilde{l})] - \alpha[\tilde{b}^o/F(\tilde{l}^o)] = a'\tilde{b}^o/F(\tilde{l}^o) > 0$, which implies, since $\alpha(\cdot)$ is increasing, $\tilde{b}/F(\tilde{l}) > \tilde{b}^o/F(\tilde{l}^o)$.

variables, including the consumption of foreign goods, the stock of debt, and the marginal utility—it is ambiguous whether they are higher or lower than their corresponding values in the socially optimal economy.[31]

Taking first-order approximations of (4.4a, b) around the steady state (45a)–(4.5f), the following matrix differential equation is obtained

$$
\begin{pmatrix} \dot{\lambda}^o \\ \dot{b}^o \end{pmatrix} = \begin{pmatrix} \theta_{11}^o & \theta_{12}^o \\ \theta_{21}^o & \theta_{22}^o \end{pmatrix} \begin{pmatrix} \lambda^o - \tilde{\lambda}^o \\ b^o - \tilde{b}^o \end{pmatrix},
\tag{4.6a}
$$

where

$$
\theta_{11}^o = \frac{\tilde{\lambda}^o F' l_\lambda^o \tilde{b}^o}{F^2}[2\alpha' + \alpha''\tilde{b}^o/F] > 0, \quad \theta_{12}^o = -\frac{\tilde{\lambda}^o(F - F' l_b^o \tilde{b}^o)}{F^2}[2\alpha' + \alpha''\tilde{b}^o/F],
$$

$$
\theta_{21}^o = \left[\frac{\tilde{p}^o y_\lambda^o - \beta^o p_\lambda^o}{\tilde{p}^o} - \frac{\alpha'(\tilde{b}^o)^2 F' l_\lambda^o}{F^2}\right] < 0, \quad \theta_{22}^o = \left[\rho + \frac{\alpha'\tilde{b}^o[F - F' l_b^o \tilde{b}^o]}{F^2} + \frac{\tilde{p}^o y_b^o - \beta^o p_b^o}{\tilde{p}^o}\right],
\tag{4.6b}
$$

and where $\beta^o = Z' - Z(p^o)/p^o$ is substituted into the element θ_{21}^o. The signs of the trace and determinant of the Jacobian matrix $\mathbf{J}^o$ of (4.6a) determine the local dynamics of the social optimum. These relationships correspond to

$$
\text{tr}(\mathbf{J}^o) = \mu_1^o + \mu_2^o = \theta_{11}^o + \theta_{22}^o
$$

$$
= \rho + \frac{\tilde{\lambda}^o F' l_\lambda^o \tilde{b}^o}{F^2}(2\alpha' + \alpha''\tilde{b}^o/F) + \frac{\alpha'\tilde{b}^o(F - F' l_b^o \tilde{b}^o)}{F^2} + \frac{\tilde{p}^o y_b^o - \beta^o p_b^o}{\tilde{p}^o},
\tag{4.7a}
$$

$$
\det(\mathbf{J}^o) = \mu_1^o \mu_2^o = \theta_{11}^o \theta_{22}^o - \theta_{12}^o \theta_{21}^o
$$

$$
= \frac{\tilde{\lambda}^o}{F^2}(2\alpha' + \alpha''\tilde{b}^o/F)\left\{(F - F' l_b^o \tilde{b}^o)\frac{\tilde{p}^o y_\lambda^o - \beta^o p_\lambda^o}{\tilde{p}^o} + F' l_\lambda^o \tilde{b}^o\left[\rho + \frac{\tilde{p}^o y_b^o - \beta^o p_b^o}{\tilde{p}^o}\right]\right\},
\tag{4.7b}
$$

where μ_1^o, μ_2^o are the eigenvalues of $\mathbf{J}^o$ such the following characteristic equation is satisfied: $(\mu^o)^2 - [\text{tr}(\mathbf{J}^o)]\mu^o + \det(\mathbf{J}^o) = 0$. For planner's economy to possess a saddlepoint equilibrium, $\det(\mathbf{J}^o) = \mu_1^o \mu_2^o < 0$. This obtains if the term in $\{\cdot\}$ brackets in (4.7b) is negative, i.e.:

$$
\det(\mathbf{J}^o) = \mu_1^o \mu_2^o < 0 \quad \Leftrightarrow \quad (F - F' l_b^o \tilde{b}^o)\frac{\tilde{p}^o y_\lambda^o - \beta^o p_\lambda^o}{\tilde{p}^o} + F' l_\lambda^o \tilde{b}^o\left[\rho + \frac{\tilde{p}^o y_b^o - \beta^o p_b^o}{\tilde{p}^o}\right] < 0.
$$

---

[31]We can show, nevertheless, that the wealth effect lowers $\tilde{y}$ relative to $\tilde{y}^o$ and raises $\tilde{\lambda}$ relative to $\tilde{\lambda}^o$.

If this condition is satisfied, then the equilibrium of the planner's problem, like its decentralized counterpart, is a saddlepoint, with $\mathbf{J}^o$ possessing a negative and a positive eigenvalue: $\mu_1^o < 0$, $\mu_2^o > 0$, $|\mu_1^o| < \mu_2^o$.[32] As in the decentralized framework, we can solve for the following stable saddlepath that describes the transitional adjustment of (b, λ):

$$(\lambda^o - \tilde{\lambda}^o) = -\frac{\theta_{22}^o - \mu_1^o}{\theta_{21}^o}(b^o - \tilde{b}^o) = \frac{-\theta_{12}^o}{\theta_{11}^o - \mu_1^o}(b^o - \tilde{b}^o) = \frac{\theta_{12}^o}{\theta_{11}^o - \mu_1^o}(\tilde{b}^o - b_0^o)e^{\mu_1^o t}. \qquad (4.8)$$

Using equations (4.4a, b), the $\dot{\lambda}^o = 0$ and $\dot{b}^o = 0$ loci in the socially optimal framework are described by the following relationships:

$$r^* + \alpha\left[\frac{b^o}{F[l(\lambda^o, b^o)]}\right] + \frac{b^o}{F[l(\lambda^o, b^o)]}\alpha'\left[\frac{b^o}{F[l(\lambda^o, b^o)]}\right] = \rho, \qquad (4.9a)$$

$$\frac{Z[p(\lambda^o, b^o)]}{p(\lambda^o, b^o)} - y(\lambda^o, b^o) = \left\{r^* + \alpha\left[\frac{b^o}{F[l(\lambda^o, b^o)]}\right]\right\}b^o. \qquad (4.9b)$$

In contrast to the decentralized model, however, the $\dot{\lambda}^o = 0$ and $\dot{b}^o = 0$ loci in the planning framework are *not* unambiguously positive relationships. This is evident from the expressions for their slopes, which correspond, respectively, to

$$(d\lambda/db)^o\,|_{\dot{\lambda}=0} = (F - F'l_b^o\tilde{b}^o)/\tilde{b}^o F'l_\lambda^o, \qquad (d\lambda/db)^o\,|_{\dot{b}=0} = -\theta_{22}^o/\theta_{21}^o,$$

where the term $(F - F'l_b^o\tilde{b}^o)$ in the expression for $(d\lambda/db)^o\,|_{\dot{\lambda}^o=0}$ is ambiguous in sign, as is the element θ_{22}^o in $(d\lambda/db)^o\,|_{\dot{b}^o=0}$.[33] Given the ambiguity of the slopes of the $\dot{\lambda}^o = 0$ and $\dot{b}^o = 0$ loci, there are six possible (local) cases, yielding six distinct (local) equilibria. Here, we briefly focus on the three cases that yield saddlepoints (the equilibria in the other three cases correspond to unstable nodes). The first case we consider—illustrated in Figure 4a—is the one in which the $\dot{\lambda}^o = 0$ locus is positively sloped, while the $\dot{b}^o = 0$ locus, unlike in the decentralized framework, is *negatively* sloped:

$$(d\lambda/db)^o\,|_{\dot{\lambda}^o=0} > 0 > (d\lambda/db)^o\,|_{\dot{b}^o=0}.$$

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[32]In contrast to the decentralized model, the stability properties of the planner's equilibrium depend, through the partial derivatives (y_b^o, p_b^o, l_b^o) directly on the slope and curvature properties of $\alpha(\cdot)$.

[33]The term $(F - F'l_b^o\tilde{b}^o)$ in $(d\lambda/db)^o\,|_{\dot{\lambda}^o=0}$ can be rewritten as $(F - F'l_b^o\tilde{b}^o) = (1 - \omega_{ql}^o\omega_{lb}^o)F$, where $\omega_{ql}^o = (\partial F/\partial l)(l/F)$ and $\omega_{lb}^o = (\partial l/\partial b)(b/l)$ are, respectively, the elasticities of output with respect to employment and employment with respect to debt. Thus, if $\omega_{ql}^o\omega_{lb}^o$ exceeds unity, then $\dot{\lambda}^o = 0$ is negatively sloped.

What is the intuition behind a negatively-sloped $\dot{b}^o = 0$ locus? Recall that in the social optimum [see (4.3d)] an increase in b^o encourages work effort. This lowers, in turn, the debt to GDP ratio and the bond rate, which tends to improve the current account ($\dot{b}^o < 0$). Moreover, in this context, a higher level of b^o has relative price and trade balance effects that also lead to $\dot{b}^o < 0$. If together these effects are sufficiently strong, they then dominate the direct negative implications for the current account balance that a higher value of b^o has on interest service, thus requiring a *fall* in λ^o to maintain $\dot{b}^o = 0$, as is depicted in Figure 4a. As in the decentralized framework, saddlepath VV leading (b^o, λ^o) to the equilibrium Q in Figure 4a is positive relationship, although its slope, in general, differs from SS in Figure 1.

The second case—depicted in Figure 4b—illustrates the situation in which *both* the $\dot{\lambda}^o = 0$ and $\dot{b}^o = 0$ loci are negatively sloped, with $\dot{\lambda}^o = 0$ steeper in absolute value:

$$(d\lambda/db)^o \,|_{\dot{\lambda}^o=0} < (d\lambda/db)^o \,|_{\dot{b}^o=0} < 0.$$

How do we account for a negatively sloped $\dot{\lambda}^o = 0$ locus, which is the case as long as $(F - F'l_b^o\tilde{b}^o) < 0$? As indicated, a rise in b^o leads to greater employment and, thus, to a fall in $r^b[b^o/F(l^o)]$. From the Euler equation (4.2e), this leads to $\dot{\lambda}^o > 0$ unless the *level* of λ^o also declines, the latter causing a rise in leisure that keeps $\dot{\lambda}^o = 0$. An important implication of this case is that—in contrast to our previous examples—the stable saddlepath WW is *negatively* sloped, implying that the stock of debt and its shadow value move in *opposite* directions in the transition to steady-state equilibrium, i.e., sgn $(\dot{b}^o) = -$sgn $(\dot{\lambda}^o)$. What are the implications of the negatively sloped WW locus? Consider the situation in which the initial stock of debt, as in the decentralized economy in Figure 1, is less than its long-run value, $b_0^o < \tilde{b}^o$. The planner then chooses a "declining" path along WW starting at point R, with $\dot{\lambda}^o < 0$, $\dot{b}^o > 0$. From (4.3a, b), it is clear that adjustment toward point Q involves *rising*—instead of falling—domestic and foreign consumption: $\dot{x}^o = x_\lambda^o\dot{\lambda}^o + x_b^o\dot{b}^o > 0$, $\dot{y}^o = y_\lambda^o\dot{\lambda}^o + y_b^o\dot{b}^o > 0$. Nevertheless, to close the current account deficit ($\dot{b} > 0$) between points R and Q, we can show that there must be a corresponding surplus on the trade balance, reflecting in the social optimum the direct positive effects of growing levels of debt on the relative price and output.[34] Finally, the third saddlepoint case in the planner's economy is qualitatively identical to that of the

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[34]The solution path of the trade balance in the social optimum is derived in appendix, equations (6.11a)–(6.11b)

decentralized economy, i.e., both $\dot{\lambda}^o = 0$ and $\dot{b}^o = 0$ are positively sloped, with $\dot{\lambda}^o = 0$ steeper than $\dot{b}^o = 0$: $(d\lambda/db)^o|_{\dot{\lambda}^o=0} > (d\lambda/db)^o|_{\dot{b}^o=0} > 0$. The corresponding phase diagram in this instance is qualitatively the same as Figure 1 and is not reproduced here.

4.2. Optimal Policy

In this section of the paper, we derive the optimal policy to offset the relative consumption, work effort, and borrowing externalities characterizing the decentralized economy. Given the fact that there are three distinct externalities, three separate policy tools are required to attain the social optimum. We assume that the policy tools available to public sector include a tariff on levied on the imported good, τ_y, a tax on domestic labor, τ_l, (amounting to a tax on output in this single-factor framework), and surcharges, or "penalties", τ_b, on international interest service. The decentralized individual budget constraint (2.4b) then becomes

$$\dot{b} = (1 + \tau_y)y + \frac{x - (1 - \tau_l)\, F(l)}{p} + (1 + \tau_b)\, r^b[b/F(l)]b + T, \qquad (4.10)$$

where the government budget is closed by a continuous adjustment in lump-sum transfers (or taxes), T: $\tau_y y + \tau_l F(l)/p + \tau_b r^b[b/F(l)]b = T$. Solving the decentralized problem of section 2 under these constraints, it is straightforward to show that the necessary optimality conditions in the symmetric equilibrium become:[35]

$$U_x(x, y) = \lambda/p, \qquad U_y(x, y) + \eta y^{-1} = (1 + \tau_y)\lambda, \qquad (4.11\text{a, b})$$

$$V'(l) = -\frac{\lambda(1 - \tau_l)\, F'(l)}{p}, \qquad \dot{\lambda} = \lambda\{\rho - (1 + \tau_b)\, r^b[b/F(l)]\}. \qquad (4.11\text{c, d})$$

The decentralized economy "reproduces" the planner's optimum if the solutions for decentralized case coincide with their Pareto optimal counterparts, i.e.: $x = x^o$, $y = y^o$, $\lambda = \lambda^o$, $l = l^o$, $p = p^o$, and $b = b^o$. The latter obtains if the decentralized optimality conditions for imports, work effort, and borrowing, represented by (4.11b)–(4.11d), equal their socially optimal counterparts (4.2b,

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[35]In all other respects, the properties of the decentralized framework remain the same as in section 2. In addition to imposing a transversality condition insuring sufficiency, we assume that the introduction of distortionary taxation does not affect the saddlepoint property of the decentralized equilibrium, i.e., a condition analogous to (2.13) obtains in this context.

c, e):

$$U_y(x, y) + \eta y^{-1} = (1 + \tau_y)U_y(x, y), \tag{4.12a}$$

$$V'(l) + \frac{(1 - \tau_l)\lambda F'(l)}{p} = V'(l) + \frac{\lambda \alpha' b^2 F'}{F^2(l)} + \frac{\lambda F'(l)}{p}, \tag{4.12b}$$

$$(1 + \tau_b)[r^* + \alpha[b/F(l)]] = r^* + \alpha[b/F(l)] + \alpha' b/F(l). \tag{4.12c}$$

Solving (4.12a)–(4.12c) in terms of the relevant policy instrument, we obtain following optimal tariff, output tax, and interest surcharge $(\tau_y^*, \tau_l^*, \tau_b^*)$:

$$\tau_y^* = \frac{\eta}{U_y(x, y) y} > 0, \qquad \tau_l^* = -\frac{p\alpha' b^2}{F^2(l)} < 0, \qquad \tau_b^* = \frac{\alpha' b}{F(l) r^b[b/F(l)]} > 0.$$

Thus, the optimal policy to offset the effects of relative consumption and borrowing externalities is to levy a positive *tariff* on imports, a *subsidy*—rather than a tax—on employment (output), and a positive *charge*, or "penalty", on international interest payments. In other words, the "over-consumption" of imported goods should be deterred by an appropriate tariff, work effort should be subsidized to achieve the optimal debt to GDP ratio given the upward-sloping interest rate relationship, and, similarly, international borrowing should be penalized to internalize the interest cost externality of greater indebtedness.[36]

5. Conclusions

This paper employs a standard developing economy framework to consider the interactions of macroeconomic spillovers. The particular externalities we consider are consumption externalities and spillovers arising from an external borrowing constraint. The consumption externality reflects the assumption that agents' social status depends on their relative consumption of imported goods, while the "upward-sloping" interest rate relationship leads agents to neglect the effects of their borrowing decisions on the interest cost of their international obligations. A key feature of the external borrowing constraint is that interest costs are a positive function of the debt to GDP ratio. This specification has important implications for the economy's saddlepoint

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[36]Observe, for example, that τ_y^* rises with the degree of status preference η and that τ_l^* increases (in absolute value) the lower is the terms of trade p. Finally, note that τ_b^* coincides with the partial elasticity of the bond rate r^b with respect to the stock of debt b.

dynamics and results in a production externality as well as a "direct" borrowing externality. The goal of this paper is to analyze an economy characterized by these phenomena and contrast its behavior with that of the social planner's.

To briefly highlight some of the chief results of the paper, we analyzed the response of the open economy in the decentralized equilibrium to a permanent rise in the preference "weight" that agents place on status considerations. We show that this leads to a long-run rise in employment and output—consistent with closed economy studies of this issue—and to current account deficits, declines in the consumption of domestic and foreign goods, and a depreciation in the terms of trade during the transition to the steady-state equilibrium. We also find that the bond rate and the domestic interest rate decline on impact and then increase toward their common steady-state value, the exogenous time rate of preference. Due to the depreciation in the terms of trade, the domestic rate lies above the bond rate during the transition.

In the second half of the paper, we study the contrasting steady-state and dynamic properties of the social planner's equilibrium. Focusing on the implications of the borrowing externality, we show that the steady-state debt to GDP ratio and bond rate are "too high" and the long-run consumption of domestic goods is "too low" in the decentralized equilibrium relative to the social optimum. Regarding the other variables, it is ambiguous whether their long-run values are higher or lower than their socially optimal counterparts, due to the offsetting effects of the resulting relative price and wealth distortions. We also derive the intertemporal properties of the planner's economy and show that saddlepath for debt and the marginal utility of wealth can be negatively sloped, which is in contrast to the upward-sloping saddlepath in the decentralized case. Finally, we demonstrate that the optimal policy combination to achieve the Pareto optimum involves a tariff on imports, a subsidy on employment, and a penalty on international borrowing.

6. Appendix

6.1. Partial Derivatives in (2.8a)–(2.8d)

Differentiating the instantaneous decentralized equilibrium (2.6a)–(2.6d) with respect to λ, we calculate the following partial derivatives.

(i) For λ:

$$x_\lambda = -D^{-1} \left\{ \left[\left(U_{yy} - \eta y^{-2}\right)/p - U_{xy} \right] \Lambda Z' + U_{xy}\lambda(F')^2/p^2 \right\} < 0, \tag{6.1a}$$

$$y_\lambda = -D^{-1} \left\{ \left[(U_{xx} - U_{xy}/p)Z' - \lambda/p^2 \right] \Lambda - U_{xx}\lambda(F')^2/p^2 \right\} < 0, \tag{6.1b}$$

$$p_\lambda = D^{-1} \left\{ \left[U_{xx}(U_{yy} - \eta y^{-2}) - U_{xy}^2 \right] (F')^2/p + \left[(U_{yy} - \eta y^{-2})/p - U_{xy} \right] \Lambda \right\} > 0, \tag{6.1c}$$

$$l_\lambda = \left(F'/pD \right) \left\{ \left[U_{xx} \left(U_{yy} - \eta y^{-2} \right) - U_{xy}^2 \right] Z' - U_{xy}\lambda/p \right\} > 0. \tag{6.1d}$$

(ii) For η:

$$x_\eta = U_{xy}/yD \left\{ \lambda(F')^2/p^2 - \Lambda Z' \right\} > 0, \tag{6.2a}$$

$$y_\eta = -(yD)^{-1} \left\{ U_{xx} \left[\lambda(F')^2/p^2 - \Lambda Z' \right] + \Lambda\lambda/p^2 \right\} > 0, \tag{6.2b}$$

$$p_\eta = (yD)^{-1}U_{xy}\Lambda < 0, \qquad l_\eta = U_{xy}\lambda F'/p^2 yD > 0, \tag{6.2c, d}$$

where $D = [U_{xx}(U_{yy} - \eta y^{-2}) - U_{xy}^2][\lambda (F')^2/p^2 - \Lambda Z'] + (U_{yy} - \eta y^{-2})\Lambda\lambda/p^2 > 0$ and $\Lambda = (V'' + \lambda F''/p) < 0$. To guarantee that leisure is a normal good, we impose $[U_{xx}(U_{yy} - \eta y^{-2}) - U_{xy}^2]Z' - U_{xy}\lambda/p > 0$ in (6.1d).

6.2. Expressions for the Long-Run Comparative Statics

Differentiating the steady-system (2.10a)–(2.10f) with respect to η, we derive the following long-run comparative statics expressions discussed in section 3.

$$\frac{\partial \tilde{b}}{\partial \eta} = \frac{-\alpha' \tilde{b}(F')^2}{\tilde{y}F^2\Delta}[\beta U_{xy}/\tilde{p} - Z'U_{xx}] > 0, \tag{6.3a}$$

$$\frac{\partial \tilde{l}}{\partial \eta} = \left(F' \right)^{-1} \frac{\partial \tilde{q}}{\partial \eta} = \frac{-\alpha' F'}{\tilde{y}F\Delta}[\beta U_{xy}/\tilde{p} - Z'U_{xx}] > 0, \tag{6.3b}$$

$$\frac{\partial \tilde{\lambda}}{\partial \eta} = \frac{-\alpha'}{\tilde{y}F\Delta} \left\{ U_{xx}\tilde{p}[Z'\tilde{\Lambda} - \tilde{\lambda}\left(F' \right)^2/\tilde{p}^2] - \tilde{\Lambda}(U_{xy}\beta + \tilde{\lambda}/\tilde{p}) + \rho U_{xy}\tilde{\lambda}\tilde{b}\left(F' \right)^2/(F\tilde{p}) \right\} > 0, \tag{6.3c}$$

$$\frac{\partial \tilde{p}}{\partial \eta} = \frac{-\alpha'}{\tilde{y}F\Delta}[(F')^2(\rho U_{xy}\tilde{b}/F - U_{xx}) - \tilde{\Lambda}] > 0, \tag{6.3d}$$

$$\frac{\partial \tilde{x}}{\partial \eta} = \frac{-\alpha'}{\tilde{y}F\Delta}[-U_{xy}\left(F' \right)^2 (\rho\tilde{b}Z'/F - \beta/\tilde{p}) + Z'\tilde{\Lambda}] \gtrless 0, \tag{6.3e}$$

$$\frac{\partial \tilde{y}}{\partial \eta} = \frac{-\alpha'}{\tilde{y}F\Delta}[U_{xx}(F')^2(\rho\tilde{b}Z'/F - \beta/\tilde{p}) - \beta\tilde{\Lambda}/\tilde{p}] > 0, \tag{6.3f}$$

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where

$$\Delta = \frac{\alpha'}{F} \left\{ U_{xx}\tilde{\lambda}\left(F'\right)^2/\tilde{p} + \tilde{\Lambda}[\tilde{\lambda}/\tilde{p} - Z'(U_{xx}\tilde{p} - U_{xy})] + \beta\tilde{\Lambda}[U_{xy} - 1/\tilde{p}(U_{yy} - \eta\tilde{y}^{-2})] \right\}$$

$$-\alpha'(F')^2/F[\rho\tilde{b}U_{xy}\tilde{\lambda}/\tilde{p}F - \Gamma(\rho\tilde{b}Z'/F - \beta/\tilde{p})] < 0,$$

letting $\tilde{\Lambda} = (V'' + \tilde{\lambda}F''/\tilde{p}) < 0$ and $\tilde{\Gamma} = [U_{xx}(U_{yy} - \eta\tilde{y}^{-2}) - U_{xy}^2] > 0$. To insure $\Delta < 0$, we impose the sufficient condition $(\rho\tilde{b}Z'/F - \beta/\tilde{p}) < 0$, which also guarantees $\partial\tilde{y}/\partial\eta > 0$ in (6.3f).

6.3. Expression for the Trade Balance

Linearizing the expression for the trade balance in terms of the foreign good, given by $TB = Z[p(\lambda, \eta)]/p(\lambda, \eta) - y(\lambda, \eta)$, we obtain:

$$TB = Z(\tilde{p})/\tilde{p} - \tilde{y} - \tilde{p}^{-1}(\tilde{p}y_\lambda - \beta p_\lambda)(\lambda - \tilde{\lambda}). \tag{6.4a}$$

Substituting for (2.14a) and using the fact that $Z(\tilde{p})/\tilde{p} - \tilde{y} = \rho\tilde{b}$, we derive the solution path for the trade balance:

$$TB = \rho\tilde{b} - \frac{(\tilde{p}y_\lambda - \beta p_\lambda)(\theta_{22} - \mu_1)}{\tilde{p}\theta_{21}}(\tilde{b} - b_0)e^{\mu_1 t}. \tag{6.4b}$$

6.4. Expressions for r^b and r^d

(i) For r^b:

Linearizing (2.3) and substituting the expressions (2.14a)–(2.14b), we obtain the following solution path for the bond rate r^b:

$$r^b = \rho - \frac{\alpha'}{F\theta_{21}}[\theta_{21} + \tilde{b}F^{-1}F'l_\lambda(\theta_{22} - \mu_1)](\tilde{b} - b_0)e^{\mu_1 t}. \tag{6.5a}$$

To simplify (6.5a), we substitute for $(\theta_{22} - \mu_1) = \theta_{12}\theta_{21}/(\theta_{11} - \mu_1)$, using the characteristic equation (2.12c):

$$r^b = \rho - \frac{\alpha'}{F(\theta_{11} - \mu_1)}[(\theta_{11} - \mu_1) + \tilde{b}F^{-1}F'l_\lambda\theta_{12}](\tilde{b} - b_0)e^{\mu_1 t}. \tag{6.5b}$$

This expression can be further reduced by substituting into (6.5b) the elements θ_{11}, θ_{12} from the Jacobian matrix **J** of (2.11a). This yields

$$r^b = \rho + \frac{\alpha' \mu_1}{F\left(\theta_{11} - \mu_1\right)}(\tilde{b} - b_0)e^{\mu_1 t}, \tag{6.5c}$$

and is the basis of our discussion regarding the adjustment of r^b in section 2.

(ii) For r^d:

From the definition (2.7), substituting for (6.5c) and for $\dot{\lambda}$ using (2.14a), the expression for the domestic rate of return r^d corresponds to:

$$r^d = r^b + p_\lambda \dot{\lambda}/\tilde{p} = \rho + \mu_1 \left[\frac{\alpha'}{F\left(\theta_{11} - \mu_1\right)} + \frac{p_\lambda\left(\theta_{22} - \mu_1\right)}{\tilde{p}\theta_{21}}\right](\tilde{b} - b_0)e^{\mu_1 t}. \tag{6.6a}$$

Using the fact that $(\theta_{22} - \mu_1)/\theta_{21} = \theta_{12}/(\theta_{11} - \mu_1)$ and substituting for θ_{12}, we can show that (6.6a) simplifies to:

$$r^d = \rho + \frac{\alpha' \mu_1(1 - p_\lambda \tilde{\lambda}/p)}{F\left(\theta_{11} - \mu_1\right)}(\tilde{b} - b_0)e^{\mu_1 t}. \tag{6.6b}$$

We can show that the $(1 - p_\lambda\tilde{\lambda}/p)$ term in (6.6b) equals $-(V'' + \tilde{\lambda}F''/\tilde{p})l_\lambda/F'$, which positive as long as leisure is a normal good, i.e., $l_\lambda > 0$.

6.5. Partial Derivatives of (4.3a)–(4.3d)

Differentiating the instantaneous Pareto-optimal equilibrium (4.2a)–(4.2d) with respect to λ^o and b^o, we calculate the following partial derivatives.

(i) For λ^o:

$$x_\lambda^o = -(D^o)^{-1}\{(U_{yy}/p - U_{xy})(\Lambda^o + \gamma)Z' + \lambda F'/(p^o)^2(U_{xy}F' + U_{yy}\epsilon)\} < 0, \tag{6.7a}$$

$$y_\lambda^o = -(D^o)^{-1}\{[(U_{xx} - U_{xy}/p^o)Z' - \lambda^o/(p^o)^2](\Lambda^o + \gamma) - \lambda^o F'/(p^o)^2(U_{xx}F' - U_{xy}\epsilon)\} < 0, \tag{6.7b}$$

$$p_\lambda^o = (D^o)^{-1}\{(U_{xx}U_{yy} - U_{xy}^2)F'\left(F'/p^o + \epsilon\right) + (U_{yy}/p^o - U_{xy})(\Lambda^o + \gamma)\} > 0, \tag{6.7c}$$

$$l_\lambda^o = (D^o)^{-1}\{(U_{xx}U_{yy} - U_{xy}^2)Z'\left(F'/p^o + \epsilon\right) - \lambda^o/(p^o)^2(U_{xy}F' + U_{yy}\epsilon)\} > 0. \tag{6.7d}$$

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(ii) For b^o:

$$x_b^o = -\phi F' U_{yy} \lambda^o/(p^o)^2 D^o > 0, \qquad y_b^o = \phi F' U_{xy} \lambda^o/(p^o)^2 D^o > 0, \tag{6.8a, b}$$

$$p_b^o = \phi F'(U_{xx}U_{yy}-U_{xy}^2)/D^o > 0, \qquad l_b^o = \phi[(U_{xx}U_{yy}-U_{xy}^2)Z'-U_{yy}\lambda^o/(p^o)^2]/D^o > 0, \tag{6.8c, d}$$

where $D^o = (U_{xx}U_{yy} - U_{xy}^2)[\lambda^o(F')^2/(p^o)^2 - (\Lambda^o + \gamma)Z'] + (\Lambda^o + \gamma)U_{yy}\lambda^o/(p^o)^2 > 0$. Note that we have made the following substitutions into (6.7)–(6.8): $\gamma = -\lambda^o(b^o)^2 F^{-4}\{\alpha''b^o(F')^2 - \alpha'[F''F^2 - 2(F')^2]\} < 0$, $\phi = \lambda F^{-2}F'b^o(2\alpha' + \alpha''b^o/F) > 0$, and $\epsilon = \alpha'(b^o)^2 F'/F^2 > 0$, with $\Lambda^o = (V'' + \lambda^o F''/p^o) < 0$. In (6.7a) and (6.7d) we assume that sufficient conditions for $x_\lambda^o < 0$, $l_\lambda^o > 0$ obtain.

6.6. Expressions for the Deviations of Decentralized Variables from Their Social Optima

Linearizing the decentralized steady-state (2.10a)–(2.10f) about its socially optimal counterpart (4.5a)–(4.5f), we derive the following system in the deviations of $(\tilde{x}, \tilde{y}, \tilde{p}, \tilde{l}, \tilde{\lambda})$ from $(\tilde{x}^o, \tilde{y}^o, \tilde{p}^o, \tilde{l}^o, \tilde{\lambda}^o)$

$$\begin{pmatrix} U_{xx} & U_{xy} & \tilde{\lambda}^o/(\tilde{p}^o)^2 & 0 & -(\tilde{p}^o)^{-1} \\ U_{xy} & U_{yy} - \eta(\tilde{y}^o)^{-2} & 0 & 0 & -1 \\ 0 & 0 & -\tilde{\lambda}^o F'/(\tilde{p}^o)^2 & V'' + \tilde{\lambda}^o F''/\tilde{p}^o & F'/\tilde{p}^o \\ -1 & 0 & -Z' & F' & 0 \\ 0 & -\tilde{p}^o & \beta & \tilde{p}^o \rho F' \tilde{b}^o/F & 0 \end{pmatrix} \begin{pmatrix} \tilde{x} - \tilde{x}^o \\ \tilde{y} - \tilde{y}^o \\ \tilde{p} - \tilde{p}^o \\ \tilde{l} - \tilde{l}^o \\ \tilde{\lambda} - \tilde{\lambda}^o \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -\eta/\tilde{y}^o \\ \lambda^o \alpha'(\tilde{b}^o)^2 F'/F^2 \\ 0 \\ -\Omega \tilde{b}^o \end{pmatrix} \tag{6.9}$$

where we substitute for $(\tilde{b} - \tilde{b}^o) = [F + F'(\tilde{l} - \tilde{l}^o)]\tilde{b}^o/F$ to obtain a five-equation system. Clearly, (6.9) distinguishes between the effects of the relative consumption externality [embodied in the second element of the vector on the right-hand-side of (6.9)] and the borrowing externality

[embodied in the third and fifth elements of the vector on the right-hand-side of (6.9)]. They can, thus, be considered separately. While it is straightforward to solve for the deviations in terms of the relative consumption externality, we do not state the solutions here, since the corresponding expressions differ from the steady-state comparative statics given in (6.3a)–(6.3f) only because the former are multiplied by the status parameter η. Solving then for the deviations in terms of the borrowing externality, we calculate the expressions that are the basis of our discussion in section 4

$$\tilde{l} - \tilde{l}^o = \frac{F'\tilde{b}^o}{\Delta'} \left\{ \frac{\tilde{\lambda}^o \alpha' \tilde{b}^o}{F^2} [\tilde{\lambda}^o/\tilde{p}^o - Z'(U_{xx}\tilde{p}^o - U_{xy}) + \beta(U_{xy} - U_{yy}/\tilde{p}^o)] + \frac{\tilde{\Omega}^o}{\tilde{p}^o}(\tilde{\Gamma}^o Z' - U_{xy}\tilde{\lambda}^o/\tilde{p}^o) \right\} \gtrless 0,$$
(6.10a)

$$\tilde{p} - \tilde{p}^o = \frac{-\tilde{b}^o}{\Delta'} \left\{ \frac{\tilde{\lambda}^o \alpha' \tilde{b}^o (F')^2}{F^2} \left[(\tilde{p}^o U_{xx} - U_{xy}) + \frac{\tilde{p}^o \rho \tilde{b}^o}{F}(U_{yy}/\tilde{p}^o - U_{xy}) \right] \right.$$

$$\left. + \tilde{\Omega}^o[\tilde{\Lambda}^o(U_{yy}/\tilde{p}^o - U_{xy}) + \tilde{\Gamma}^o(F')^2/\tilde{p}^o \right\} \gtrless 0,$$
(6.10b)

$$\tilde{\lambda} - \tilde{\lambda}^o = \frac{\tilde{b}^o}{\Delta'} \left\{ \frac{\tilde{\lambda}^o \alpha' \tilde{b}^o (F')^2}{F^2} \left[(\tilde{\Gamma}^o \beta - U_{xy}\tilde{\lambda}^o/\tilde{p}^o) \right] - \frac{\tilde{p}^o \rho \tilde{b}^o}{F}[\tilde{\Gamma}^o Z' - U_{yy}\tilde{\lambda}^o/(\tilde{p}^o)^2] \right.$$

$$\left. - \tilde{\Omega}^o[\tilde{\Lambda}^o[\tilde{\Gamma}^o Z' - U_{yy}\tilde{\lambda}^o/\tilde{p}^o] - \tilde{\lambda}^o(F')^2\tilde{\Gamma}^o/(\tilde{p}^o)^2 \right\} \gtrless 0,$$
(6.10c)

$$\tilde{x} - \tilde{x}^o = \frac{\tilde{b}^o}{\Delta'} \left\{ \frac{\tilde{\lambda}^o \alpha' \tilde{b}^o (F')^2}{F^2} \left[(U_{yy} - \tilde{p}^o U_{xy})(\rho \tilde{b}^o Z'/F - \beta/\tilde{p}^o) + \tilde{\lambda}^o/(\tilde{p}^o)^2 \right] \right.$$

$$\left. - \tilde{\Omega}^o[Z'\tilde{\Lambda}^o(U_{yy}/\tilde{p}^o - U_{xy}) + U_{xy}\tilde{\lambda}^o F'/(\tilde{p}^o)^2] \right\} < 0,$$
(6.10d)

$$\tilde{y} - \tilde{y}^o = \frac{-\tilde{b}^o}{\Delta'} \left\{ \frac{\tilde{\lambda}^o \alpha' \tilde{b}^o (F')^2}{F^2} \left[(U_{xy}/\tilde{p}^o - U_{xx})(\rho \tilde{b}^o Z'/F - \beta/\tilde{p}^o) + \tilde{\lambda}^o \rho \tilde{b}^o/(F\tilde{p}^o) \right] \right.$$

$$\left. - \tilde{\Omega}^o \left[\tilde{\Lambda}^o[\tilde{\lambda}^o/(\tilde{p}^o)^2 - Z'(U_{xx} - U_{xy}/\tilde{p}^o)] + U_{xx}\tilde{\lambda}^o(F')^2/(\tilde{p}^o)^2 \right] \right\} \gtrless 0,$$
(6.10e)

where

$$\Delta' = U_{xx}\tilde{\lambda}^o (F')^2 /\tilde{p}^o + \tilde{\Lambda}^o \left[\tilde{\lambda}^o/\tilde{p}^o - Z'(U_{xx}\tilde{p}^o - U_{xy}) + \beta(U_{xy} - U_{yy}/\tilde{p}^o) \right]$$

$$- (F')^2 \left[\rho \tilde{b}^o U_{xy}\tilde{\lambda}^o/(\tilde{p}^o F) - \tilde{\Gamma}^o(\rho \tilde{b}^o Z'/F - \beta/\tilde{p}^o) \right] < 0,$$

and letting $\tilde{\Lambda}^o = (V'' + \tilde{\lambda}^o F''/\tilde{p}^o) < 0$, $\tilde{\Gamma}^o = (U_{xx}U_{yy} - U_{xy}^2) > 0$, and $\tilde{\Omega}^o = -\tilde{p}^o(\rho + \alpha'\tilde{b}^o/F) < 0$. To guarantee $\Delta' < 0$ and $(\tilde{x} - \tilde{x}^o) < 0$, we impose the sufficient condition $(\rho \tilde{b}^o Z'/F - \beta/\tilde{p}^o) < 0$ used above in (6.3a)–(6.3f). Observe that we set $\eta \equiv 0$ in (6.10a)–(6.10e).

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6.7. Solution for the Trade Balance in the Social Optimum

Using (4.3b, c) the trade balance in terms of the foreign good in the social optimum is given by $TB^o = Z[p(\lambda^o, b^o)]/p(\lambda^o, b^o) - y(\lambda^o, b^o)$. Linearizing this expression and substituting for the solution path (4.8), we obtain:

$$TB^o = \rho \tilde{b}^o - \left\{ \frac{(\tilde{p}y_\lambda^o - \beta p_\lambda^o)(\theta_{22}^o - \mu_1^o)}{\tilde{p}^o \theta_{21}^o} - \frac{(\tilde{p}y_b - \beta^o p_b^o)\theta_{21}}{\tilde{p}^o} \right\} (\tilde{b}^o - b_0^o)e^{\mu_1^o t}. \qquad (6.11a)$$

Using the fact that $(\theta_{22}^o - \mu_1^o)/\theta_{21}^o = \theta_{12}^o/\theta_{11}^o - \mu_1^o$ and substituting for θ_{11}^o and θ_{12}^o from (4.6b), we can show that (6.11a) reduces to:

$$TB^o = \rho\tilde{b}^o + (\theta_{11}^o - \mu_1^o)^{-1}\{\det \mathbf{J}^o - (\tilde{\lambda}^o/F^2)(2\alpha' + \alpha''b^o/F)F'l_\lambda^o \tilde{b}^o \rho - \mu_1(\tilde{p}y_b^o - \beta^o p_b^o)/\tilde{p}^o\}(\tilde{b}^o - b_0^o)e^{\mu_1^o t}.$$
$$(6.11b)$$

Since for (local) saddlepoint equilibria $\det \mathbf{J}^o < 0$, a sufficient condition for the term in $\{\cdot\}$ brackets to be positive is $[\tilde{p}y_b^o - \beta^o p_b^o]/\tilde{p}^o < 0$, which holds as long as $(U_{xx}U_{yy} - U_{xy}^2) - U_{xy}\lambda/p^2 > 0$. Differentiating (6.11a) with respect to time, it is clear that the trade balance improves along a saddlepath in which $b_0^o < \tilde{b}^o$.

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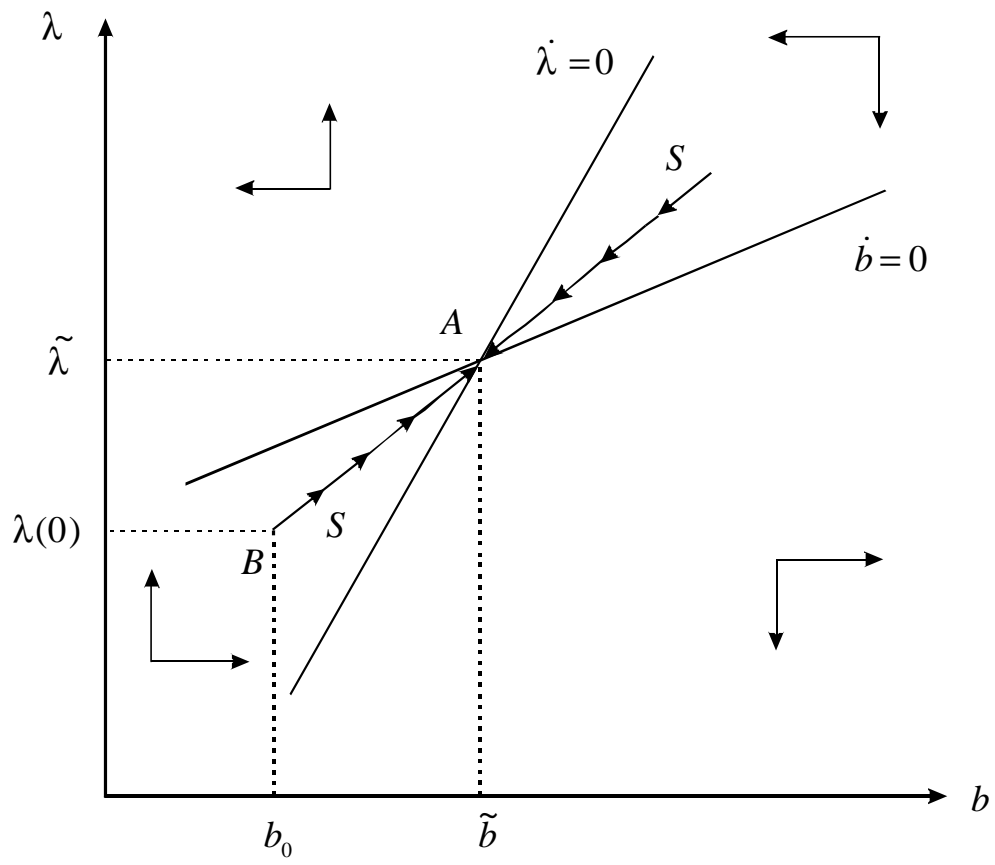
Figure 1: Phase Diagram

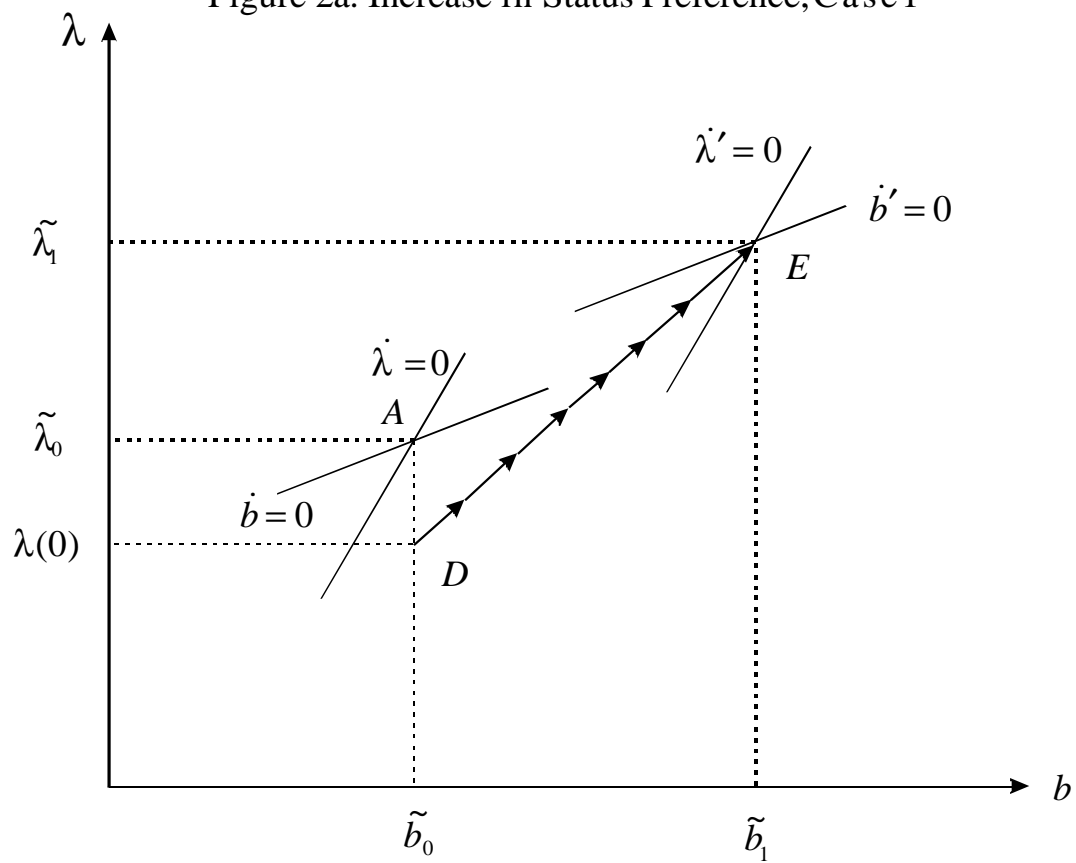Figure 2a: Increase in Status Preference, Case 1

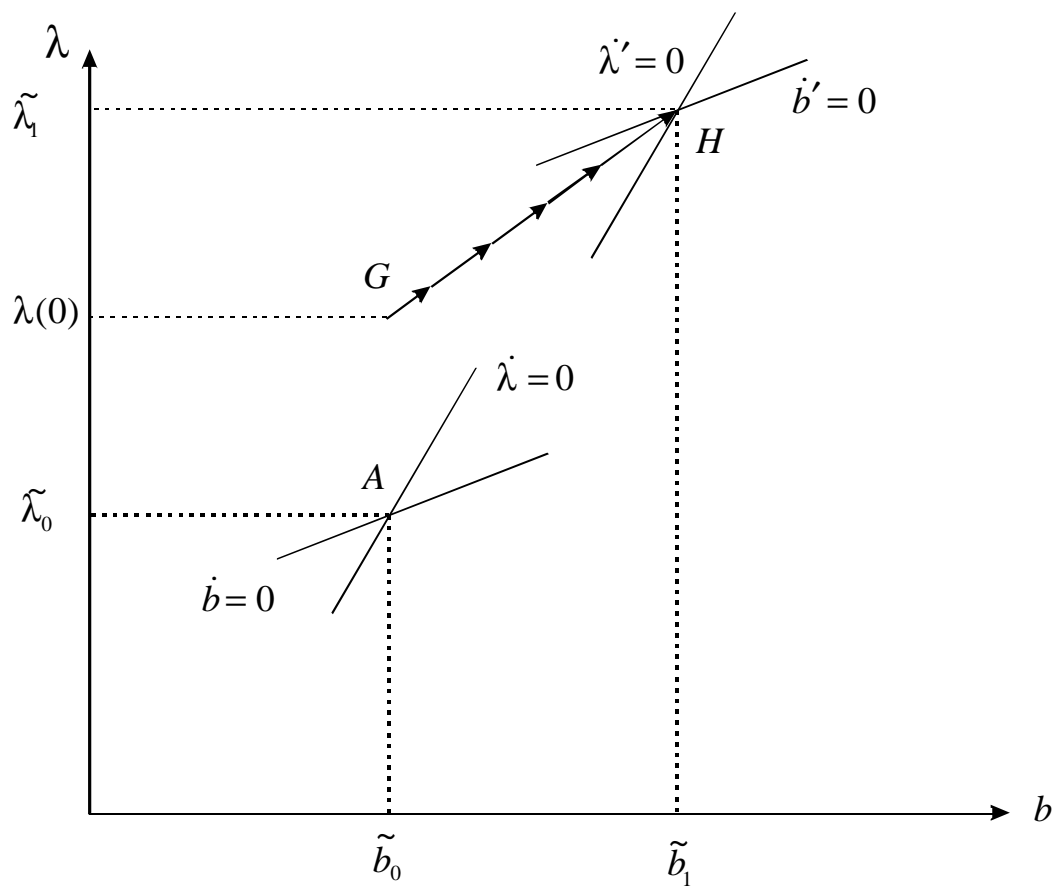Figure 2b: Increase in Status Preference, Case 2
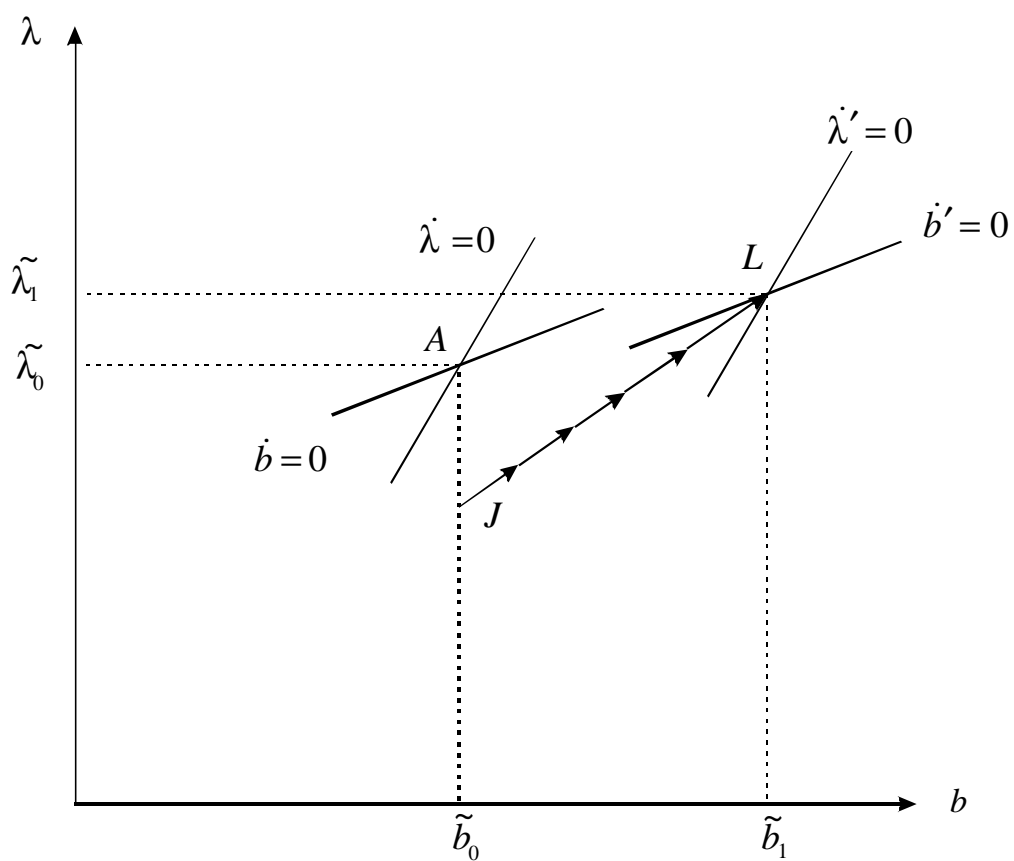
Figure 2c: Increase in Status Preference, Case 3

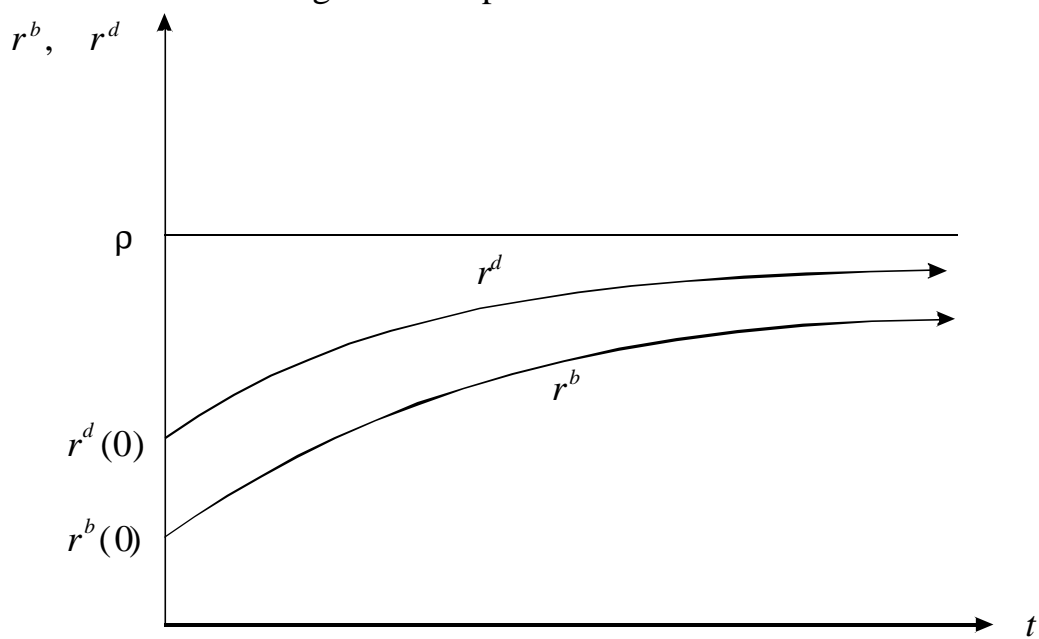## Figure 3: Response Paths of Interest Rates

Figure4a:SocialEquilibrium, Case 1

λ^o
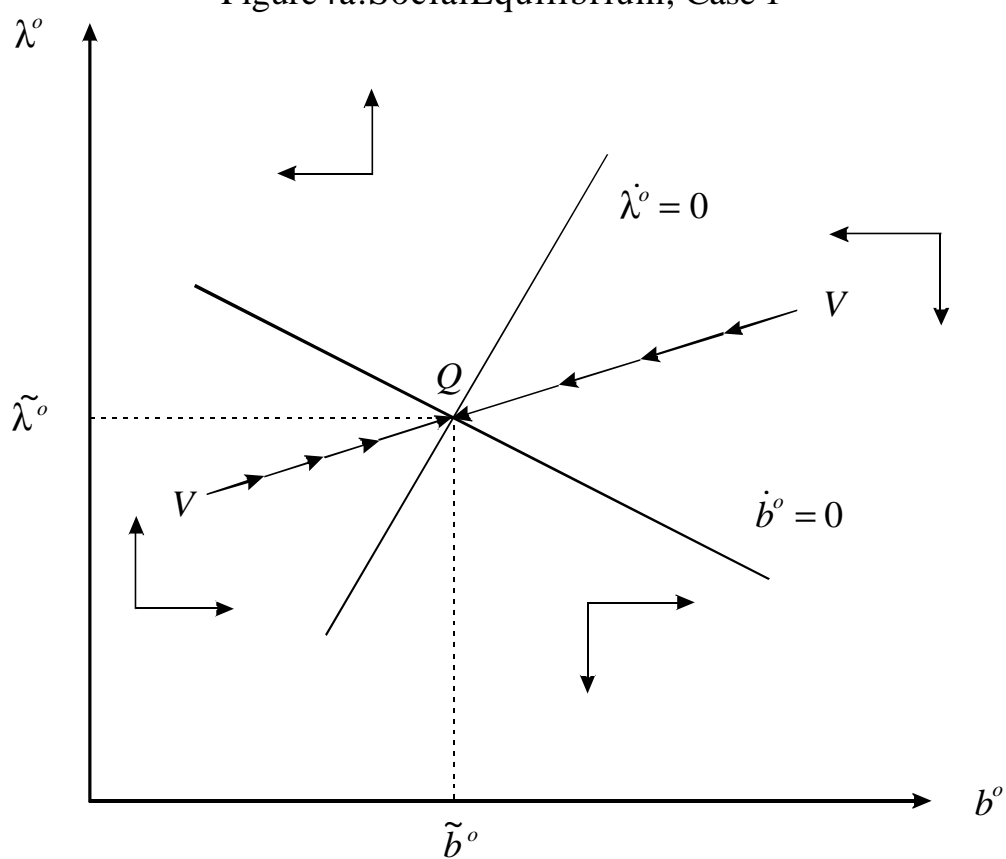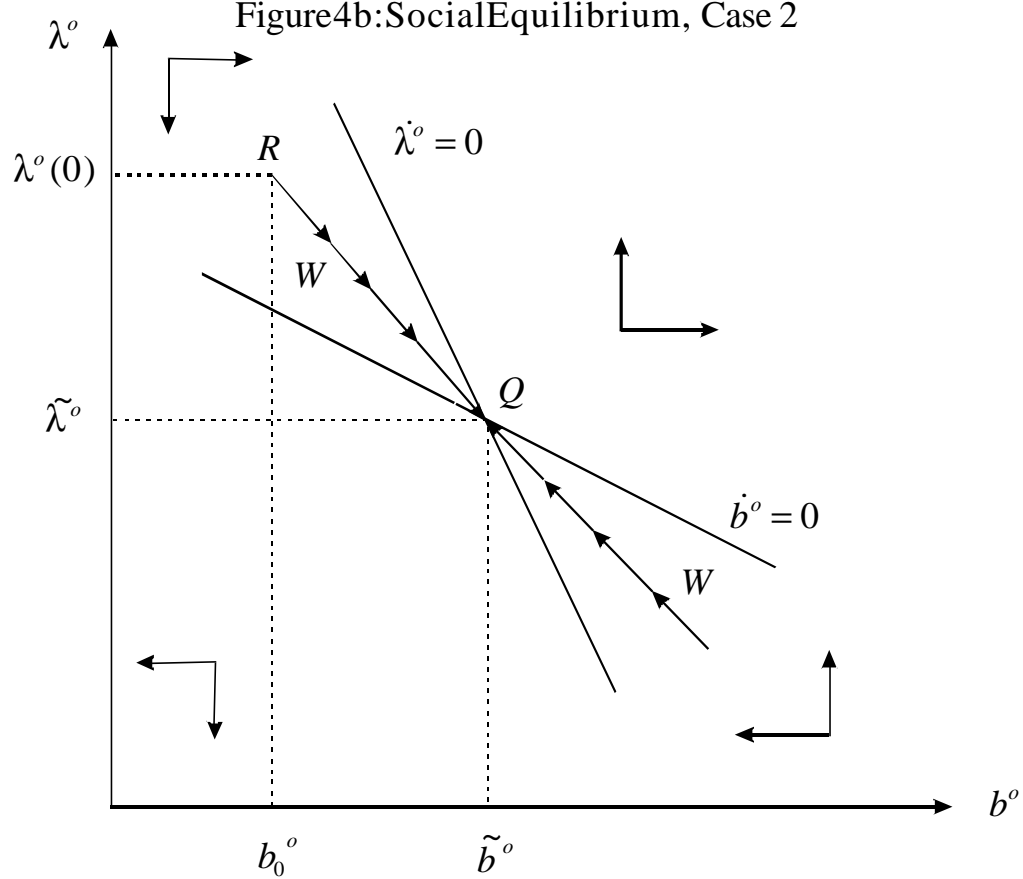
$\dot{\lambda}^o = 0$

V

$\tilde{\lambda}^o$

Q

V

$\dot{b}^o = 0$

b^o

$\tilde{b}^o$

Figure4b:SocialEquilibrium, Case 2

λ^o

$\dot{\lambda}^o = 0$

$\lambda^o(0)$

R

W

$\tilde{\lambda}^o$

Q

$\dot{b}^o = 0$

W

b^o

b_0^o

$\tilde{b}^o$