A SURVEY OF MATHEMATICAL OPTIMIZATION MODELS AND ALGORITHMS FOR DESIGNING AND EXTENDING IRRIGATION AND WASTEWATER NETWORKS

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Forschungsbericht/
Research Memorandum No. 157

July 1980

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Contents

Abstract

This paper presents a state-of-the-art survey of network models and algorithms that can be used as a planning tool in irrigation and wastewater systems. It is shown that the problem of designing or extending such systems basically leads to the same type of mathematical optimization model. The difficulty in solving this model lies mainly in the properties of the objective function. Trying to minimize construction and/or operating costs of a system typically results in a concave cost (objective) function, due to economies of scale. A number of ways to attack such models are discussed and compared, including linear programming, integer programming and specially designed exact and heuristic algorithms. The usefulness of each approach is evaluated in terms of the validity of the model, the computational complexity of the algorithm, the properties of the solution, the availability of software and the capability for sensitivity analysis.

Model builders usually are confronted with a rather annoying decision. When trying to model a complex real system, the question always comes up as to how much complexity should be transferred to the model. If the model is too simple, the solutions of the model might not give reasonable answers to the problem; if the model reflects well the real system, there may be no algorithms at hand to find solutions to the model. Throughout this paper, we shall be dealing with this kind of model building problem in the context of planning water networks.

Planning water networks is a short term for the following set of problems: Consider a region where either an irrigation or a wastewater system should be built or - in case one exists already - should be expanded. Such a system can be described in terms of possible pipes to transport the water, which would be modeled as arcs in a network model. Those pipes would transport water from wells or other water supply facilities to the demand points - like agricultural plants - in the case of an irrigation system. In the case of a wastewater system, the pipes would transport the wastewater from the points where it is produced - towns or houses - to possible sites of wastewater treatment plants. The possible location of wastewater treatment plants, wells and the location of towns and agricultural plants would then be modeled as nodes in a network model. Knowing the demands for water (in the case of an irrigation system) or the production of wastewater, the question remains which pipes and which wastewater treatment plants or wells to build and with what capacity.

Although the decision problem is clear, the formulation of an objective is much less so. Certainly the construction of an irrigation or wastewater network has not only one objective to meet, but a number of them; reliability, rural development as well as political and social considerations being among them. But if the demand for irrigation water or the production of wastewater is part of the constraints of the problem that have to be satisfied, then one major factor in designing a network are certainly the construction and/or operating costs. Although we are fully aware that cost considerations are only one part (although usually a major part) of the decision process it has been proven useful to have models available who can find network designs such that the costs are minimized. The final decision has to be one that takes such a result into account but not necessarily goes along these lines completely, because of other important objectives. Also uncertainties of future developments (e.g. weather, wastewater production, costs) have to be considered and complicate the analysis.

At our present knowledge no multiobjective, stochastic network optimization model is available. But using one of the models presented in this paper one can at least answer particular questions. By making extensive use of sensitivity analysis one can also take some of the inherent uncertainties into account.

In the last 10 years some attempts have been made to use network models for the design of water networks. On the other hand, also some theoretical improvements for solving network optimization problems have been made. In this paper we want to survey the applications, critically discuss the usefulness of some of the models and try to combine applied and methodological aspects of the problem, which appear to be somehow separated in the literature.

Given the above mentioned restrictions for the use of optimization models in water network design, the existing models have different shortcomings and advantages to serve our need. There appear to be five criterions which we find crucial in defining the usefulness of each model: the validity of the model in terms of the real problem; the computational difficulty to solve the model; the properties of the solution of the model; the availability of software to solve the model and the capability for sensitivity analysis with the model in order to consider uncertainties. Therefore in this survey we concentrate on these five criterions.

Although the real world problems to be considered will always remain the same throughout the paper, namely designing and/or extending irrigation and wastewater systems such that construction and/or operating costs are minimized, the presented models are of increasing complexity with decreasing efficiency of the algorithms.

This paper is divided into two parts. The first part deals with models where the cost function of constructing and/or operating the system is linear. In the second part, models with concave cost functions, reflecting the economies of scale in building or operating such systems will be discussed.

1. Models with linear cost functions

Consider the following problem: A regional wastewater system has to be designed. A preparational study has decided which towns to include into this system and where wastewater treatment plants might be located. It is known which wastewater treatment plant locations and which towns can be directly connected with each other by a pipe for transporting the wastewater. Modelling the towns and wastewater treatment plants as nodes and the connecting pipes as arcs, we define a network as $N=(X,A)$, where X is the set of all nodes $j \in X$ and A is the set of all arcs $(i,j) \in A$ with $i,j \in X$. (i,j) thus denotes an arc through which water can flow from node i to node j. Assuming now that the total costs (construction and/or operating costs) of the wastewater treatment plants and of the pipes depend linearly on their capacity, we can formulate the problem of minimizing the costs as

$$\text{minimize} \quad \sum_{(i,j)\in A} c_{ij}\, x_{ij} + \sum_{j \in X} c_{ja}\, x_{ja} \qquad (1)$$

subject to

$$\sum_{\substack{i \\ (i,j)\in A}} \beta_{ij} x_{ij} + \delta_j = \sum_{\substack{\ell \\ (j,\ell)\in A}} x_{j\ell} + x_{ja}$$

$$\text{for all } j\in X \qquad (2)$$

$$0 \le x_{ij} \le u_{ij} \quad \text{for all } (i,j)\in A$$

$$0 \le x_{ja} \le u_{ja} \quad \text{for all } j\in X, \qquad (3)$$

where

$\delta_j \ge 0$ amount of wastewater produced at node j;

$c_{ij} \ge 0$ costs to construct and/or operate the pipe from node i to node j with a flow capacity of $x_{ij} = 1$. Because of the linearity of the costs, the increase of the capacity by $x_{ij} = 1$ will increase the costs by c_{ij}. Depending on the problems, the costs can either reflect the construction costs alone or the maintenance costs alone or some sum of both (if a certain planning horizon is given);

$c_{ja} \ge 0$ costs to construct and/or operate the wastewater treatment plant at node j with a capacity of one unit of water flowing into the plant, namely $x_{ja} = 1$;

$0 < \beta_{ij} \le 1$ percentage of water that is not lost on its way through pipe (i,j), e.g. $\beta_{ij} = 0.9$ means that 90 % of the water flowing through pipe (i,j) is not lost;

x_{ij} flow through pipe (i,j) and equivalently also the capacity of pipe (i,j);

x_{ja} amount of water purified in the wastewater treatment plant at node j and equivalently also the capacity of this plant;

u_{ij} maximum allowed size of pipe (i,j);

u_{ja} maximum allowed size of the wastewater treatment plant at node j.

Model (1) - (3) is called the transshipment problem when all $\beta_{ij}=1$ and generalized network flow problem in case some $\beta_{ij}<1$. The constraints (2) are called conservation of flow equations and simply state that all the incoming flow of wastewater plus the produced wastewater ($=\delta_j$) at a particular node must equal the amount of water purified at node j ($=x_{ja}$) plus the outgoing flow.

Nearly the same model applies in case of the design of an irrigation system. Equations (1) and (3) remain unchanged, but instead of (2) we have to write

$$\sum_{\substack{i \\ (i,j)\in A}} \beta_{ij}\, x_{ij}+x_{ja} = \sum_{\substack{\ell \\ (j,\ell)\in A}} x_{j\ell} + \delta_j \quad \text{for all } j\in X \qquad (2^*)$$

where

$\delta_j \geq 0$ amount of water needed at node j for irrigation;

x_{ja} amount of water produced at node j by wells, rivers or other sources.

Model (1), (2^*), (3) is again a transshipment or a generalized network flow problem.

With the notable exception of the objective function, which is very simple in our case, these models capture all the complexity of the models of AHRENS (1974), RAMOS (1979), JARVIS et al. (1978), JOERES et al. (1974), McCONAGHA and CONVERSE (1973), WANIELISTA and BAUER (1972), and BRILL and NAKAMURA (1978).

However, the advantage of the simple models (1) - (3) and (1), (2^*), (3) are great. Special algorithms exist for the transshipment and for the generalized network flow problem.

G.H. BRADLEY et al. (1977) reported that their FORTRAN-code for transshipment problems solved models with 40.000 arcs and 5.000 nodes in 290 seconds and models with 21.000 arcs and 10.000 nodes in 441 seconds on an IBM360/67. MAURRAS (1972)reported that his FORTRAN-code for the generalized network flow problem solved a model with 3.000 nodes and 8.000 arcs in 500 seconds on a CDC6600 and claimed that his code can handle models with up to 12.000 nodes and 50.000 arcs. Both codes are specializations of the simplex-algorithm simplified by using the special structure of the constraints. In general one can expect that a transshipment problem can be solved in less time than a generalized network flow problem, because of its simpler structure.

Although a general linear programming code can also solve these models, it is much less efficient than the network flow algorithms; experience has shown that the simplex algorithm will take about 50 - 200 times longer to solve the problem. Recent papers, for example ELAM et al. (1977), might lead to even faster codes for the generalized network flow problem. However, the existence of a computer code based on that paper is not yet reported.

A general linear programming code has the advantage that it is readily available as standard software including a very easy handling of sensitivity analysis. Special network flow algorithms are usually available too, although not the algorithms of BRADLEY et al. (1977) or MAURRAS (1972), which can only be obtained from them. Standard network flow codes generally use the out-of-kilter algorithm, which may be less efficient than the later developments.

Therefore, most software that solves network flow problems is superior in all of our criterions, which we mentioned before, except that the validity of a linear cost function as an objective is questionable. It is thus not surprising than no application is reported in the literature using this model. However because the model is so good in terms of the other criterions it is worth mentioning it and - in case the nonlinearities of the cost function can be approximated by a linear cost function - also worth using it.

The above models do not apply to the expansion of a wastewater or an irrigation system. This is so, because we must now differentiate between the capacity of a unit (e.g. pipe, wastewater treatment plant, well) and the flow through the unit. While in the above described models the size of a unit would never be greater than the flow, this will not necessarily be true if some units already exist. The model with linear costs for expanding a water network is formulated as:

$$\text{minimize} \quad \sum_{(i,j)\varepsilon A} c_{ij}\, y_{ij} + \sum_{j\varepsilon X} c_{ja}\, y_{ja} \qquad (4)$$

subject to (2) for a wastewater problem or (2*) for an irrigation problem and

$$0 \leq y_{ij} \text{ and } 0 \leq x_{ij} \leq q_{ij} + y_{ij} \quad \text{for all } (i,j)\varepsilon A$$

$$0 \leq y_{ja} \text{ and } 0 \leq x_{ja} \leq q_{ja} + y_{ja} \quad \text{for all } j\varepsilon X \qquad (5)$$

where

q_{ij} existing size of pipe (i,j);

q_{ja} existing size of a wastewater treatment plant or a well at node j;

y_{ij} expansion of pipe (i,j), for example by building an additional pipe from node i to j;

y_{ja} expansion of a wastewater treatment plant or a well at node j;

x_{ij} flow from node i to j;

x_{ja} amount of purified wastewater (or of produced irrigation water) at node j.

All other coefficients remain the same as in (1), (2) or (2^*) respectively. The way the cost function (4) is defined, it is assumed, that additional costs arise only from the additional capacities that have to be built and operated, and that the costs of the original network for maintenance and operating remain the same, no matter in what way the network is expanded. It is easy to see that at the optimal solution of (4), (5) together with (2) or (2^*) it must hold that

$$y_{ij} = \max (0, x_{ij} - q_{ij}) \text{ and } y_{ja} = \max (0, x_{ja} - q_{ja}).$$

Therefore, if $q_{ij} = 0$ we can set $y_{ij} = x_{ij}$ and thus reduce the size of the problem. In general one has to use a standard linear programming code to solve this problem. However, if $\beta_{ij} = 1$ for all $(i,j) \varepsilon A$ a special algorithm can be used. This algorithm is described in MANDL (1979a). In the worst case this algorithm requires the solution of $\sum_{j \varepsilon X} \delta_j$ different shortest path problems, each requiring of the order $O(n^2)$ operations, where n is the number of nodes in the set X. The only available computer code reported is by MANDL (1979b), but experience of the performance of this code for large networks is lacking.

All the models discussed so far have in common that the algorithms find the globally optimal solution. On the other hand the assumption that costs are linearly depending on the size is certainly a great simplification of reality. However, if, in fact, it turns out that all decision variables are much greater than zero in the optimal solution (a not too likely event) the linear cost model is still reasonable. This is so, because many real world cost functions with economies of scale become near to linear for larger quantities like the function given in Fig. 1.

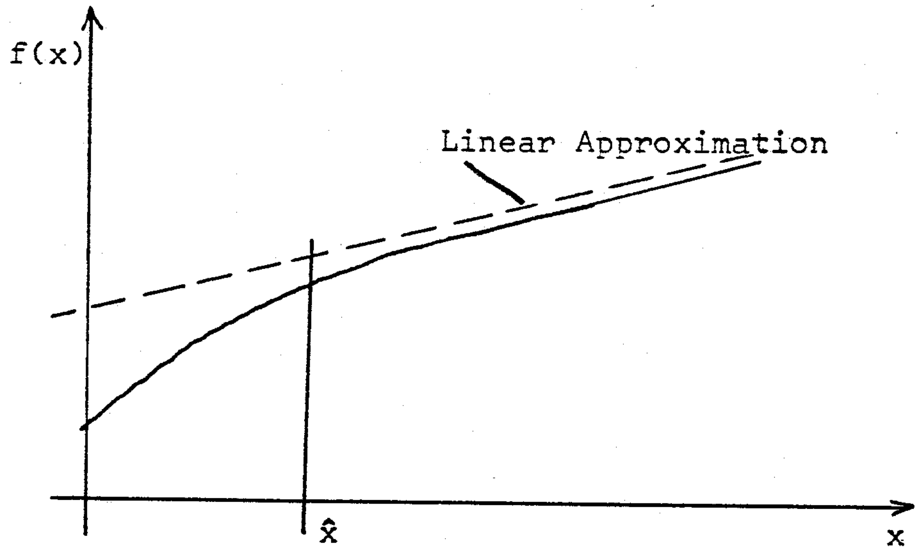

Fig. 1: Concave Function and Linear Approximation

In cases where some decision variables are close to zero (less than $\hat{x}$ in Fig. 1), the decision to construct a unit, based on the linear cost model, may be incorrect.

2. Models with Concave Cost Functions

A more realistic formulation is obtained by using costs that are concave with size. This results in the model

$$\text{minimize} \quad \sum_{(i,j)\varepsilon A} f_{ij}(x_{ij}) + \sum_{j\varepsilon X} f_{ja}(x_{ja}) \qquad (6)$$

subject to (2) or (2^*) and (3), where $f_{ij}(x_{ij})$ and $f_{ja}(x_{ja})$ are concave functions (not necessarily continuous or differentiable) like the example shown in Fig. 1.

While the general model (6) subject to (2) or (2^*) and (3) seems to be a very realistic model, its use is limited because either only locally optimal solutions can be found or, when a globally optimal solution is available, rapidly increasing computation time prohibits the use of this model for large networks. We shall therefore first discuss simplified versions of this general model for which special algorithms have been developed recently.

Consider a model which consists of the minimization of (6) subject to $x_{ij} \geq 0$ for all $(i,j) \varepsilon A$ and $x_{ja} \geq 0$ for all $j \varepsilon X$ and (2) or (2^*) with $\beta_{ij}=1$ for all $(i,j) \varepsilon A$ and where the network $N=(X,A)$ is a tree, e.g. a network with a structure shown in Fig. 2.

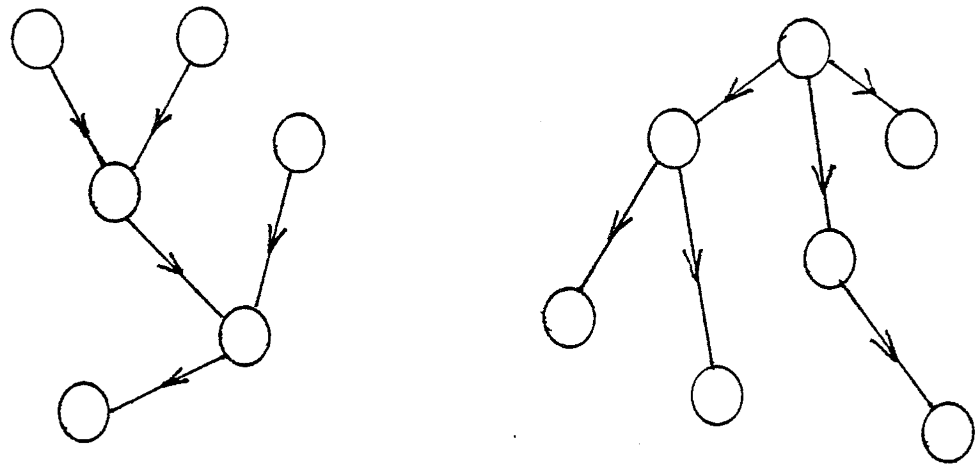

Fig. 2: Trees

Compared to the general model (6) subject to (2) or (2^*) and (3), we assume here that no losses of flow occur, that there are no capacity constraints of type (3) in the system and that the possible network $N=(X,A)$ is of a special structure. In this case POLYMERIS (1978) developed an algorithm which finds the global optimum to this problem. Furthermore, the computational complexity depends on the structure of the tree. If the tree is of the type shown in Fig. 3, then the algorithm needs $O(n^2)$ operations where n is the number of nodes in the network. If the tree is of the type shown in Fig. 4, then the algorithm needs $O(2^n)$ operations.

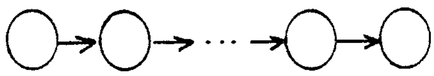

Figure 3: Computationally Simple Tree

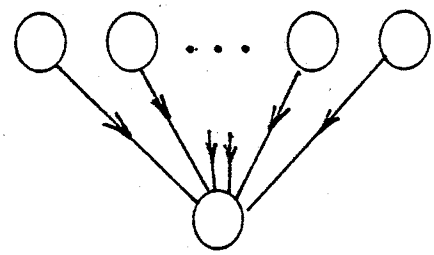

Fig. 4: Computationally Difficult Tree

In case the network is a simple tree (as given in Fig. 3) CONVERSE (1972) and also WHITLACH and ReVELLE (1976) dealt with this problem, which typically arises if a wastewater network is designed along a river. The solution procedure of CONVERSE (1972) is the same as the one by POLYMERIS (1978) applied to this special case - a type of dynamic programming algorithm. WHITLACH and ReVELLE (1976) present a heuristic algorithm for an extended problem in which already existing wastewater treatment plants can be considered and also certain requirements for the water quality can be taken into account. Both additional problems cannot be solved by POLYMERIS (1978) or CONVERSE (1972). Computationally , the two different approaches are comparable, but sensitivity analysis can be performed more easily using the method of WHITLACH and ReVELLE (1976). Computer codes for the above mentioned algorithms are only available from the authors. The algorithm of POLYMERIS (1978) was also implemented by MANDL (1979b).

When the underlying network N=(X,A) is arbitrary but still no losses are allowed ($\beta_{ij} = 1$) and no capacity constraints are imposed, a number of authors have suggested solution methods.McCONHAGA and CONVERSE (1973) propose a heuristic algorithm, which is fast but does not necessarily find the optimal solution.

Two algorithms were published by GALLO et al. (1979 and 1980). The algorithm of GALLO et al. (1979) uses the fact that an optimal solution will be a basic feasible solution of (2) or

(2^*) (together with the non-negativity constraint). In GALLO's algorithm, all adjacent basic feasible solutions of a given basic feasible solution are examined and if no better solution is among those, the algorithm stops. Otherwise the search is repeated from the better basic feasible solution. Each iteration of the algorithm requires $O(n^3)$ operations (n being the number of nodes), while for the number of necessary iterations no good upper bound exists (it is the same upper bound as for the simplex-algorithm, namely $\frac{(m+n)!}{m!(m+n)!}$, where m is the number of arcs in the network). The solution found by this algorithm is locally optimal, because it is at least as good as any adjacent basic feasible solution. Other algorithms which find solutions to this problem (e.g. separable programming, nonlinear programming) will also find a locally optimal solution, but one which is only locally optimal in an ε-neighbourhood, $\varepsilon>0$, of the solution. The latter concept of optimality is certainly weaker than the one obtained by GALLO et al. (1979).

The only available computer code for this algorithm is from the authors. They reported on having solved problems up to 48 nodes and 174 arcs within 58 seconds on an IBM370/168 computer.

Another algorithm with similar properties of the solution than the ones mentioned above was published by WALKER (1976). This algorithm can be applied if the functions of the objective function (6) are of the type

$$f_{ij}(x_{ij}) = c_{ij}x_{ij} + k_{ij}s_{ij},$$

where $s_{ij} = 0$ if $x_{ij} = 0$

and $s_{ij} = 1$ if $x_{ij} > 0$.

This so-called fixed charge problem assumes a linear cost function but with fixed costs if a wastewater treatment plant, well or pipe is to be built. WALKER's algorithm consists of two parts. In a first step, a locally optimal solution is found with the same properties as the solution obtained by GALLO's algorithm (e.g. all adjacent basic solutions are worse in terms of the objective). A heuristic search procedure then tries to obtain better local optima. This means that WALKER's algorithm for the fixed charge model will produce a solution which is at least as good as that found by GALLO's algorithm applied to the same model. For about the same size of problem (14 nodes and 30-50 arcs) WALKER has reported a solution time of 7.6 seconds on an IBM360/65, and GALLO has reported a solution time (with a general concave cost function) of one second on an IBM370/168. Although there is no clear indication for it, one would expect the WALKER algorithm, because it is especially designed for this type of problems, to perform better for the fixed charge problem than GALLO's algorithm.

An algorithm which finds the global optimum was given by GALLO et al. (1980). It is a specially designed branch-and-bound algorithm. The largest test problem with 34 nodes and 122 arcs required 3 minutes CPU time on an IBM370/168.

Comparing the four different approaches by McCONHAGA and CONVERSE (1973), WALKER (1976) and GALLO et al. (1979 and 1980) one can state that there is little difference as far as the validity of the model is concerned, because they solve the same model except for WALKER (1976). Computer codes are only available from the authors. While GALLO et al. (1980) finds the global optimum, WALKER (1976) and GALLO et al. (1979) find at least a local optimum, only a feasible solution can be guaranteed by McCONHAGA and CONVERSE (1973). However, in terms of computational efficiency the order is reversed. McCONHAGA's and CONVERSE's algorithm is fast while the algorithm of GALLO et al. (1980) can , but not necessarily must be fast. Therefore, the heuristic algorithm is also very suitable for sensitivity analysis.

For the general model of (6) subject to (2) or (2^*) and (3) different approaches have been used. For the design of an irrigation network RAMOS (1978) used separable programming, the idea of which is to approximate a function of one variable by a sequence of linear segments as given in Fig. 5.

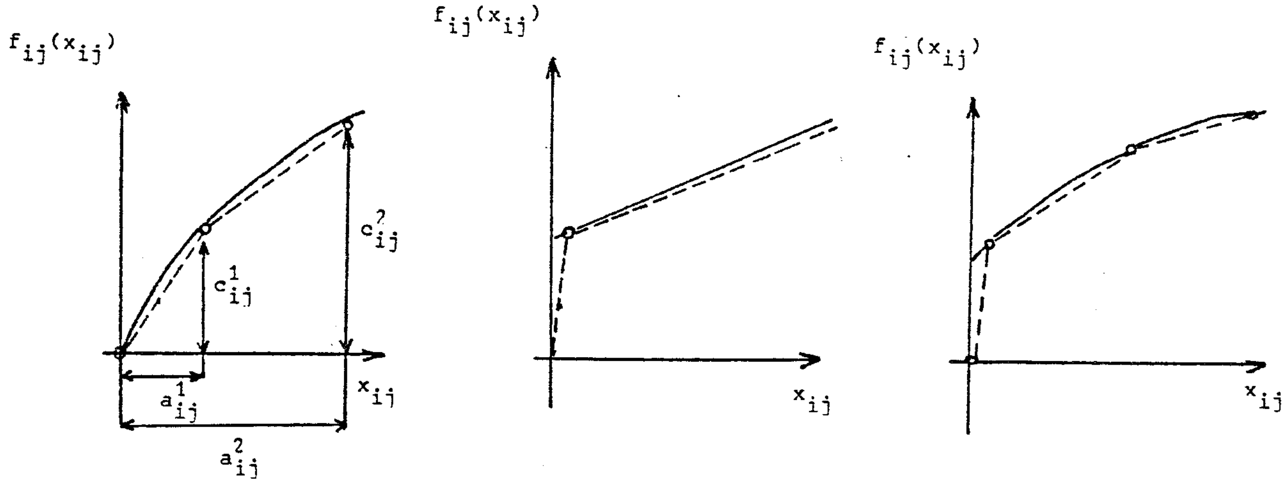

Fig. 5: Linear Approximations for Different Concave Functions

The resulting model can be stated as

$$\text{minimize} \quad \sum_{(i,j)\varepsilon A} \sum_{k\varepsilon K_{ij}} c_{ij}^{k} z_{ij}^{k} + \sum_{j\varepsilon X} \sum_{k\varepsilon K_{ja}} c_{ja}^{k} z_{ja}^{k} \qquad (7)$$

subject to (2) or (2*), (3) and

$$x_{ij} = \sum_{k\varepsilon K_{ij}} a_{ij}^{k} z_{ij}^{k} \qquad \text{for all } (i,j)\varepsilon A$$

$$x_{ja} = \sum_{k\varepsilon K_{ja}} a_{ja}^{k} z_{ja}^{k} \qquad \text{for all } j\varepsilon X$$

$$\sum_{k\varepsilon K_{ij}} z_{ij}^{k} = 1 \qquad \text{for all } (i,j)\varepsilon A \qquad (8)$$

$$\sum_{k\varepsilon K_{ja}} z_{ja}^{k} = 1 \qquad \text{for all } j\varepsilon X$$

$$z_{ij}^{k} \geq 0 \qquad \text{for all } (i,j)\varepsilon A \text{ and } k\varepsilon K_{ij}$$

$$z_{ja}^{k} \geq 0 \qquad \text{for all } j\varepsilon X \text{ and } k\varepsilon K_{ja}$$

The new parameters a_{ij}^k and c_{ij}^k can be computed from the concave objective function $f_{ij}(x_{ij})$, which was introduced in (6). It must hold that $f_{ij}(a_{ij}^k)=c_{ij}^k$, where a_{ij}^k is a point where the linear approximation is equal to the concave function (the graphic interpretation can be seen in Fig. 5). Choosing values a_{ij}^k, such that a good linear approximation is achieved, is part of the modelling process. The new variables z_{ij}^k are artificial variables which represent the variable x_{ij} defined in model (1) - (3).

Solving this model with the separable programming mode of a linear programming code bares no great difficulties as these codes are usually available. It has been shown by MANDL (1977) that the maximum number of operations per simplex-step is of the order $O(n^2 + n \cdot m)$, where n is the number of constraints and m the number of variables. Because the size of model (7) differs from (1) in the number of variables and not in the number of constraints, the computation time per simplex-step only increases linearly in the number of variables. However, the number of simplex-steps required for finding an optimal solution will in general be higher for (7) than for (1). This does not really restrict the application of separable programming, because commercial simplex codes can solve very large problems. Also, sensitivity analysis is a well developed feature of these codes. The major shortcoming of (7) is, however, the fact that only a locally optimal solution can be guaranteed. The separable programming algorithm will terminate at a basic feasible solution if all adjacent basic feasible solutions have a greater value of the objective function.

It is worth noting in this context that the set of basic feasible solutions of (6) subject to (2) or (2^*) and (3) is a subset of the set of basic feasible solutions of (7) subject to (2) or (2^*), (3) and (8). Therefore any locally optimal solution (compared to adjacent basic feasible solutions) of (6) subject to (2) or (2^*) and (3) will also be a local optimum of the separable programming problem (7) subject to (2) or (2^*), (3) and (8), but the reverse need not be true.

If the globally optimal solution is more important than computational efficiency, the approximation of (6) subject to (2) or (2^*) and (3) as an integer optimization problem will be appropriate. This model, again a so-called fixed charge network model (but more general than the one already discussed), can be formulated as follows:

$$\text{minimize} \quad \sum_{(i,j)\varepsilon A} \sum_{k\varepsilon K_{ij}} (b_{ij}^k y_{ij}^k + c_{ij}^k z_{ij}^k) +$$

$$+ \sum_{i\varepsilon X} \sum_{k\varepsilon K_{ja}} (b_{ja}^k y_{ja}^k + c_{ja}^k z_{ja}^k) \qquad (9)$$

subject to (2) or (2^*), (3) and

$$x_{ij} = \sum_{k\varepsilon K_{ij}} y_{ij}^k \qquad \text{for all } (i,j)\varepsilon A$$

$$x_{ja} = \sum_{k\varepsilon K_{ja}} y_{ja}^k \qquad \text{for all } j\varepsilon X \qquad (10)$$

$$y_{ij}^k \leq M z_{ij}^k \qquad \text{for all } (i,j)\varepsilon A \text{ and } k\varepsilon K_{ij}$$

$$y_{ja}^k \leq M z_{ja}^k \qquad \text{for all } j\varepsilon X \text{ and } k\varepsilon K_{ja} \qquad (11)$$

$$\sum_{k\varepsilon K_{ij}} z_{ij}^k \leq 1 \qquad \text{for all } (i,j)\varepsilon A$$

$$\sum_{k\varepsilon K_{ja}} z_{ja}^k \leq 1 \qquad \text{for all } j\varepsilon X \qquad (12)$$

$$z_{ij}^k \quad \varepsilon\{0,1\} \qquad \text{for all } (i,j)\varepsilon A \text{ and } k\varepsilon K_{ij}$$

$$z_{ja}^{k} \in \{0,1\} \qquad \text{for all } j\in X \text{ and } k\in K_{ja} \quad (13)$$

Where M is an arbitrary number, M >> 1.

In Fig. 6 it can be seen how this approximation is derived for one variable.

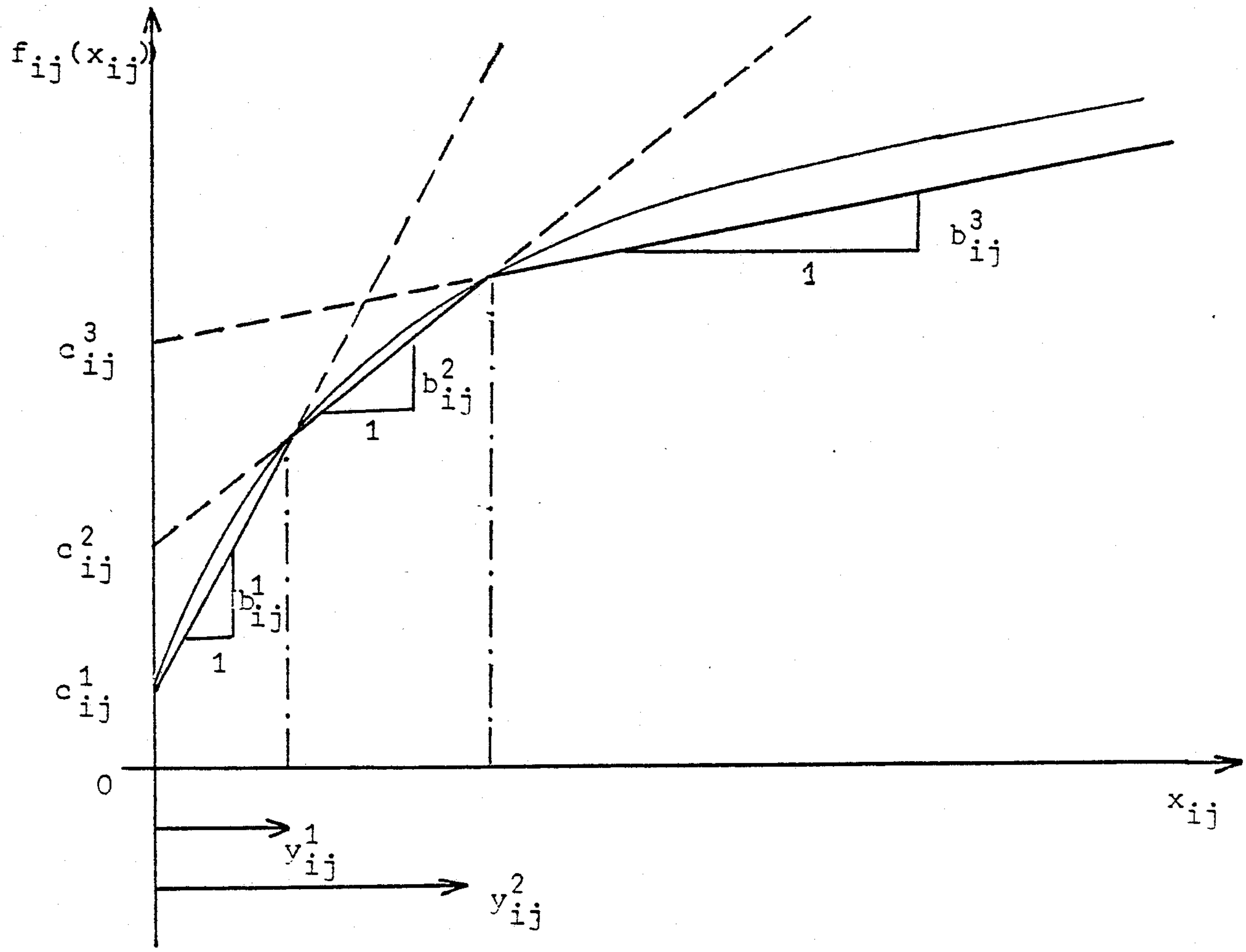

Fig. 6: Fixed-Charge Approximation to Concave Cost Function

The equations (11) and (12) insure that only one of the linear approximations may be used at a time. This model has attracted considerable attention. JARVIS et al. (1978) have used this model in the design of a regional wastewater system. The same model was also studied by BRILL and NAKAMURA (1978) and by JOERES et al. (1974). JOERES et al. (1974), who were the first to discuss this model, used a standard integer programming code on a Univac 1108. The model they solved had 81 constraints, 49 continuous and 47 integer variables. An optimal solution was found after 14.4 minutes CPU time. JARVIS et al. (1978) who used a special branch-and-bound algorithm by RARDIN and UNGER (1976) reported on solving the model with up to 100 integer variables within 6 minutes CPU time on a Univac 1108. As the objective of BRILL's and NAKAMURA's (1978) approach is to produce a variety of good (close to optimality) solutions for further inspection, the latter method is not comparable to the two other ones in terms of computational efficiency.

Let us now sum up the different aspects of the four presented approaches to solve the general model (6) subject to (2) or (2^*) and (3).

- The validity of the model is the same for all approaches, because they all approximate the concave objective function (6) by a piecewise linear function.
- In terms of computational efficiency the use of separable programming - the approach taken by RAMOS (1978) - is the only possibility for large networks. At present it is unlikely that an integer programming approach will give results for networks with more than 200 nodes. The examples discussed by JARVIS et al. (1978) consisted of not more than 50 nodes.
- Sensitivity analysis is also less expensive using separable programming instead of branch-and-bound techniques.
- Computer codes for both separable as well as integer programming are usually available on larger computers. The special codes developed by JARVIS et al. (1978) and by BRILL and NAKAMURA (1978) are only available from them.
- The main advantage of the use of integer programming is to find the globally optimal solution. Although empirical evidence is missing, the author would not be surprised if local and global optimum differ substantially in practical problems.

Let us finally discuss some extensions of the models presented in this paper. One direction, similar to the work by WHITLACH and ReVELLE (1976), is to explicitely consider water quality constraints in the design of a wastewater system as was done by PINGRY and SHAFTEL (1979). They try to minimize the construction and operating costs of the system. The chosen solution procedure is a heuristic search algorithm.

Much more attention has been given to an extension of the irrigation network design problem. In water distribution networks not only the demand for water at a particular node is of importance but also the pressure (head) of the water must be above a certain lower limit at each demand point. ALPEROVITS and SHAMIR (1977), BHAVE (1978), CENEDESE and MELE (1978), DEB (1976) and JACOBY (1968) have all dealt with this problem. Because of the complexity of the problem, it is divided into two parts. First the structure of the network is decided. This means that it is decided which pipes to build, but no decision is made concerning the diameters (capacities) of the pipes. Most of the mentioned works already assume that this first part of the problem is solved. The second part consists in deciding on the diameters of each pipe to meet the pressure requirements with minimum costs.

3. Summary and Conclusions

A number of models useful in the design or expansion of wastewater or irrigation systems have been discussed. It turns out that the user has to make a tradeoff between computational efficiency (and thus the size of network he can analyze), quality of the solution (global versus local optimum) and validity of the model. No model presently satisfies all of these criteria equally well. In case of a large network we suggest that one should always first try the approach with a linearized cost function together with extensive sensitivity analysis to find out how sensitive the solution is to different approximations of the real (presumably concave) cost function. If the solution remains fairly stable and all capacities are greater than zero, then this suggests that the solution found is quite robust (however, not necessarily the optimal solution to the problem). If the network is of smaller size (e.g. not more than 200 nodes) one should try an integer programming approach as suggested by JOERES et al. (1974), JARVIS et al. (1978) and BRILL and NAKAMURA (1978). If the network, however, is of a special structure - namely a tree - then the algorithms of POLYMERIS (1978) or CONVERSE (1972) will certainly be more efficient than a general integer programming code. All these models do not consider water quality constraints explicitely and optimize the network design in terms of construction and/or operating costs only. Even so, these models can contribute substantially to the decision problem of designing water network systems.

Acknowledgements

The author appreciates very much the encouragement to write this paper and the many fruitful suggestions to improve it by David H. MARKS, Massachusetts Institute of Technology. This work has been made possible by a grant from Max Kade Foundation, Inc., New York. An early discussion with F. RAMOS, M.I.T., made the author first think about some questions raised in this paper. Also the constructive criticism by two anonymous referees were of substantial help to improve the paper.

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