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Current Account Dynamics in a Small Open Economy Model of Status Seeking

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Abstract

In this paper we will use a status-preference framework, together with a standard cost of adjustment investment function, to study the dynamics of the small open economy current account balance. We demonstrate that the transitional dynamics of the economy is characterized by two speeds of adjustment: a speed of adjustment arising from status-preference and a speed of adjustment arising from installation costs of investment. This structure implies that the current account balance depends on both speeds of adjustment as well as on the long-run equilibrium. As a consequence, the current account can exhibit non-monotonic behavior in transition to the steady state.

Keywords

Current account, status seeking, relative wealth, open economy dynamics

JEL Classifications

E21, F41

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Contents

1. Introduction

As is well-known, a well-known and problematic characteristic of the small open economy, representative agent model is the fact that consumption and the stock of assets (or debt) do not, under the assumption of perfect capital mobility, converge to "sensible" levels if the domestic rate of time preference differs from the world interest rate. If, for instance, the rate of time preference exceeds the world interest rate, agents then have the incentive to mortgage all their human and nonhuman wealth, with the consequence that consumption converges to zero over time. On the other hand, if the economy "patient" in the sense that its rate of time preference is less than the world interest rate, then it will accumulate wealth to the point that it eventually stops being a small open economy.[1] A solution to this problem proposed in a recent paper by Fisher and Hof (2001) is to assume that representative agents have preferences not only over own consumption, but also over relative wealth. In this formulation, relative wealth serves as a proxy for an individual's status in society.[2] In their Ramsey model of relative wealth, Fisher and Hof (2001) show that the small open economy—given certain restrictions on parameter values—has an interior long-run equilibrium and saddlepoint stable dynamics.[3] The relative wealth specification has also been used by researchers such as Corneo and Jeanne (1997), Rauscher (1997a), and Futagami and Shibata (1998) to model—in a closed economy framework—the effects of status-preferences on variables such as the rate economic growth. The alternative approach of modelling status in a dynamic framework is the relative consumption framework, which has been used by Galí (1994), Persson (1995), Harbaugh (1996), Rauscher (1997b), Grossmann (1998), Ljungqvist and Uhlig (2000), and Fisher and Hof (2000). While the relative consumption approach is an appropriate one for many problems and, furthermore, has a conceptual advantage in the sense that it is easier to "observe" another person's level of consumption, it has the disadvantage that it is incapable (see Fisher and Hof (2001))

[1] This issue is also discussed in Barro and Sala-i-Martin (1995), chapter 3.

[2] See Duesenbery (1949), Scitovsky (1976), Hirsch (1976), Boskin and Sheshinski (1978), Layard (1980), and Frank (1985a,b) for general analyses of the implications of a preference for status in economic decision-making.

[3] See Turnovsky (1997), Fisher (2001), and Fisher and Hof (2001), for extensive discussions of alternative approaches of generating saddlepath transitional dynamics in the small open economy context. In a recent paper Hof and Wirl (2001), using a relative wealth specification of preferences, explore the possibility of multiple equilibria in the small open economy.

of resolving the issue mentioned at the start of this paper: the existence of an interior equilibrium if the rate of time preference is different from the world interest rate. Indeed, the only way to obtain an interior equilibrium in the relative consumption context is to impose equality between the two variables.[4] We believe, therefore, that the relative wealth approach, which we will adopt here, is the most promising one to investigate the role played by status-preference in the evolution of small open economies.

One counter-intuitve property of the small open economy Ramsey model that is not, however, eliminated by the relative wealth approach is the fact that the value of the domestic capital stock—since it is pinned-down by the exogenous world interest rate—does not possess any transitional dynamics. As is well-known, capital stock dynamics in the small open economy context can be restored by assuming that domestic physical investment takes place subject to convex installation costs. This approach has been used by, among others, Brock (1988), Sen and Turnovsky (1989a, 1989b, 1990), and Frenkel, Razin, and Yuen (1996). Using standard restrictions regarding the production function, the capital stock and its shadow price (Tobin's q) then possess saddlepoint stable dynamics. The goal of this paper, then, is to investigate the behavior of overall asset accumulation in a model that features both relative wealth preferences and installation costs of physical investment. To do so, we will use the status-preference framework of Fisher and Hof (2001), together with a standard quadratic installation cost function. After solving for the intertemporal equilibrium of the model, we will show that the small open economy possesses two rates of stable dynamic adjustment: i) a "consumption-side" rate of adjustment that depends on status-preference and ii) a "production-side" rate of adjustment that depends on the characteristics of the installation cost function. It will be case that the dynamics of consumption itself will depend solely on the consumption-side rate of adjustment, while the rate of adjustment of domestic physical capital and its shadow price is a function only of the production-side rate of adjustment. In contrast, we will demonstrate that the evolution of the current account balance—corresponding to the accumulation of net financial assets—will depend on both stable rates of adjustment. Indeed, for given initial stocks of net financial assets and physical capital, the path of the current account need not be

[4] In this case, equilibrium consumption and its shadow value always correspond to their implied steady-state values and, consequently, display no transitional dynamics.

monotonic. In other words, the current account can initially deteriorate, reach a stationary point, and then improve in the transition to steady-state equilibrium. We will show that the factors that determine current account dynamics in our model include the relative sizes of the two speeds of adjustment and the long-run position of the economy, which is a function, in part, of how “strongly” agents value status.

The paper has the following organization. Section 2 describes our model of identical consumer-producers who have preferences over relative wealth and accumulate physical capital subject to installation costs. In this section, we calculate the necessary optimality conditions and derive the resulting macroeconomic equilibrium. This will yield differential equations for consumption (the Euler equation), the current account balance, domestic physical capital, and the shadow price of investment. A crucial feature of this equilibrium is our assumption that all agents make the same choices, i.e., that the equilibrium is a symmetric one. In the next section, section 3, we analyze the steady state of the economy and solve for its linearized dynamics. We can show, due to the economic structure, that the solutions for the capital stock and its shadow value are derived independently from the Euler equation and the current account balance. The solutions for the capital stock and its shadow value are then used to calculate the paths of consumption and net financial assets. The key result of this section is that the adjustment current account balance depends on the speeds of adjustment of both the production and consumption-sectors of the economy. This will imply that the path taken by the current account balance in transition to steady-state equilibrium can be non-monotonic, e.g., the current account can deteriorate prior to its eventual improvement. In part, this is due to the fact that the two rates of stable adjustment exercise opposing influences on the current account. We conclude the paper with brief remarks in section 4.

2. The Model

We start by assuming a small open economy is populated by a large number of identical, infinitely-lived consumer-producers. Without loss of generality, we specify that the population of consumer-producers is constant. As a consumer, we assume that each agent possesses a general instantaneous utility function over own consumption, c, and status, s,

$U = U(c, s)$, that has the following properties

$$U_c > 0, \quad U_s > 0, \quad U_{cc} < 0, \quad U_{ss} \leq 0, \quad U_{cc}U_{ss} - U_{cs}^2 \geq 0, \tag{1a}$$

$$U_{sc}U_c - U_sU_{cc} > 0, \tag{1b}$$

$$\lim_{c \to 0} U_c(c,\, s) = \infty, \qquad \lim_{c \to \infty} U_c(c,\, s) = 0. \tag{1c}$$

According to (1a), the representative agent as a consumer derives positive, though diminishing, marginal utility from own consumption and positive and non-increasing marginal utility from status. In addition, the utility function U jointly concave, according to (1a), in c and s. Condition (1b) imposes normality on preferences. In other words, this means that the marginal rate of substitution of status for consumption, U_s/U_c, depends positively on c. The next condition, condition (1c), describes the limiting behavior of the marginal utility of consumption. As we outlined in the introduction, we specify here that an individual's status—described by the function $s = s(a, A)$—depends on both own net assets (= nonhuman wealth), a, and average net assets of the private sector, A. We assume that the status function is defined for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$. The small open economy's stock of net assets consists of its net stock of loans N owed by the rest of the world and its domestic physical capital stock K. In other words, $A = N + K$. In addition, we assume for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$ that status increases in own wealth, decreases in average wealth, and that the marginal status in own assets is non-increasing, i.e.:

$$s_a > 0, \quad s_A < 0, \quad s_{aa} \leq 0. \tag{2a}$$

Most of the macroeconomic research dealing with the implications of status preference assumes that the status function assume a "ratio" form, i.e., $s(a, A) = \varphi(a/A)$, $\varphi' > 0$, $\varphi'' \leq 0$, where a/A represents relative wealth. A difficulty, however, with this formulation is that it yields anomalous results if negative levels of wealth are allowed. For example, since $s_a = A^{-1}\varphi'$, a negative level of average wealth ($A < 0$) implies that a rise in own wealth *decreases* status. In the same way, the partial derivative of $s(a, A)$ with respect to A, $s_A = -(a/A^2)\varphi'$, means that a rise in average wealth causes status to *improve* (i.e. $s_A > 0$) as long as own wealth is negative ($a < 0$). To eliminate these counter-intuitive cases, we will employ a version of specification of $s(a, A)$ used by Fisher and Hof (2001):

$$s(a, A) \equiv \varphi\left(\frac{a - \bar{a}}{A - \bar{a}}\right), \qquad \bar{a} < 0, \quad \varphi' > 0, \quad \varphi'' \leq 0. \tag{2b}$$

Equation (2b) states that own and average wealth are measured with respect to the "lower bound", or "minimum value", which we denote by $\bar{a}$. Since we do not want to rule-out at the outset the possibility that the economy reaches a long-run equilibrium with a negative stock of net assets, we will follow Fisher and Hof (2001) and specify that $\bar{a}$ takes on a negative value, i.e., $\bar{a} < 0$. The parameter $\bar{a}$ can be interpreted as an indicator of domestic residents' aversion to (or tolerance for) indebtedness. It is easily verified that (2b) satisfies all properties given in (2a) for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$.

To help clarify our subsequent analysis, we will parameterize the instantaneous utility function $U(c, s)$. Specifically, we will employ a version of the parameterization used by Futugami and Shibata (1998)

$$U(c, s) = (1 - \theta)^{-1}\left[\left(c^\eta s^\beta\right)^{1-\theta} - 1\right], \tag{3}$$

where $\eta > 0$, $\beta > 0$, $\theta > 0$, $1 - \eta(1 - \theta) > 0$, $1 - \beta(1 - \theta) \geq 0$, $1 - (\beta + \eta)(1 - \theta) \geq 0$. The parameterization of $U(c, s)$ introduced in (3) requires the extra assumption of $\varphi > 0$ in order to guarantee that $s > 0$.[5] Our assumptions ensure that (3) satisfies the restrictions stated above in conditions (1a-c).

As indicated above, we assume that consumer-producers as savers (borrowers) accumulate domestic physical capital k and lend net financial assets (acquire net debts) n in the international credit market. In a fully integrated world capital market, the rate of return on net loans, or financial assets, equals the exogenous and time-invariant world interest rate r^*. In addition, the representative agent inelastically supplies one unit of labor. Consumer-producers then combine capital and labor to produce per-capita output

[5] The Futagami and Shibata (1998) specification is recovered by setting $\bar{a} = 0$. Equation (3) is also used by Fisher and Hof (2001).

using a standard per-capita production function equal to $\epsilon \cdot f(k)$, where ϵ denotes total factor productivity, and $f(k)$ satisfies the standard neoclassical properties, which are given by $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$, $f(k) \to \infty$ as $k \to \infty$, along with the usual Inada conditions.[6] Accumulation of domestic physical capital, denoted by i, is, however, subject to installation costs that are described by the following constant returns to scale, quadratic function $\Psi(i, k)$:

$$\Psi(i,k) = i\left[1 + \frac{h}{2}\frac{i}{k}\right], \quad h > 0 \tag{4}$$

where the parameter h scales the marginal cost of physical investment. Given these technological and financial market possibilities, we can express the net financial asset and capital accumulation as

$$\dot{n} = \epsilon f(k) + r^* n - i\left[1 + \frac{h}{2}\frac{i}{k}\right] - c, \tag{5a}$$

$$\dot{k} = i. \tag{5b}$$

Employing an infinite horizon, perfect foresight framework, the agent's maximization problem is formulated as follows:

$$\max \int_0^\infty \left\{(1-\theta)^{-1}\left[\left(c^\eta s^\beta\right)^{1-\theta} - 1\right]\right\} e^{-\rho t}\, dt, \quad \rho > 0$$

subject to the flow constraints (5a) and (5b), the initial conditions $n(0) = n_0$ and $k(0) = k_0$, and the No-Ponzi-Game (NPG) condition $\lim_{t\to\infty} a e^{-r^* t} \geq 0$ and where ρ is the exogenous rate of pure time preference and s is given by equation (2b). Regarding time preference, we will assume further that agents are impatient in the sense that rate of time preference ρ exceeds the world interest rate r^*, i.e., $(\rho - r^*) > 0$.[7] In addition, a crucial feature of this optimization problem is that the representative agent takes the time path of average wealth $A = N + K$ as given. In other words, each individual is small enough to neglect his

[6] Subsequently, we will simply call per-capita output, "output."

[7] We will use this assumption below to establish that the dynamic system $(\dot{n}, \dot{c})$ possesses a saddlepoint.

own contribution to the average level of wealth. The current value Hamiltonian for this problem is equal to

$$H(c,n,k,\lambda,q) = (1-\theta)^{-1}\left[\left(c^{\eta}\varphi\left(\frac{n+k-\bar{a}}{N+K-a}\right)^{\beta}\right)^{1-\theta} - 1\right]$$

$$+\lambda\left[\epsilon f(k) + r^{*}n - i\left[1+\frac{h}{2}\frac{i}{k}\right] - c\right] + q'i,$$

where λ is the current shadow value of wealth and $q' = q\lambda$ is the shadow price of domestic capital in terms of foreign assets. The necessary conditions for an interior optimum, $H_c = 0$, $H_i = 0$, $\dot{\lambda} = \rho\lambda - H_n$, and $\dot{q}' = \rho q' - H_k$ are then expressed as:[8]

$$\eta c^{\eta-1}\varphi(\cdot)^{\beta}\left(c^{\eta}\varphi\left(\cdot\right)^{\beta}\right)^{-\theta} = \lambda, \tag{6a}$$

$$i = \frac{(q-1)}{h}k, \tag{6b}$$

$$\dot{\lambda} = \left(\rho - r^{*}\right)\lambda - \beta\left(c^{\eta}\varphi\left(\cdot\right)^{\beta}\right)^{-\theta}\frac{c^{\eta}\varphi(\cdot)^{\beta-1}\varphi'\left(\cdot\right)}{N+K-\bar{a}}, \tag{6c}$$

$$\dot{q}' = \rho q' - \lambda\left[\epsilon f'(k) + \frac{h}{2}\left(\frac{i}{k}\right)^{2}\right] - \beta\left(c^{\eta}\varphi\left(\cdot\right)^{\beta}\right)^{-\theta}\frac{c^{\eta}\varphi(\cdot)^{\beta-1}\varphi'\left(\cdot\right)}{N+K-\bar{a}}. \tag{6d}$$

Using the necessary conditions (6a) and (6c) and the definition $q' = q\lambda$, we can rewrite the necessary condition (6d) for domestic capital as

$$\frac{\epsilon f'(k)}{q} + \frac{(q-1)^{2}}{2hq} + \frac{\beta c}{q\eta(N+K-\bar{a})}\frac{\varphi'\left(\cdot\right)}{\varphi\left(\cdot\right)} + \frac{\dot{q}}{q} = r^{*} + \frac{\beta c}{\eta(N+K-\bar{a})}\frac{\varphi'\left(\cdot\right)}{\varphi\left(\cdot\right)}. \tag{6e}$$

where we have substituted for the optimality condition for investment (6b) to derive the left-hand-side of (6e). Observe, in addition, that condition (6e) describes the equality be-

[8] To conserve space, we set $\varphi\left(\frac{n+k-\bar{a}}{N+K-a}\right) \equiv \varphi\left(\cdot\right)$ and $\varphi'\left(\frac{n+k-\bar{a}}{N+K-a}\right) \equiv \varphi'\left(\cdot\right)$.

tween the rates of return of domestic capital and net loans. The rate of return of domestic capital—the left-hand-side of (6e)—includes four components. The first term is the marginal physical product of capital in terms of the domestic shadow price of capital q. The second term represents reduction in installation costs arising from an additional unit of capital. The third term is the gain in utility—expressed in terms of output—from higher status, generated by an additional unit of capital, while the fourth term, equal to $\dot{q}/q$, represents the capital gain (or loss) from holding physical assets. The rate of return of net financial assets—the right-hand-side of (6e)—is the sum of the exogenous world interest rate and the utility pay-off in terms of output of accumulating "status-enhancing" net financial assets. Note that the status gain of accumulating domestic physical capital is measured in terms of its shadow price q, while that of net financial assets is scaled by the unitary price of output. The assumptions made above in (1a), (2b) and (4) ensure that the Hamiltonian is jointly concave in the control variables c and i and the state variables n and k. This implies that if the limiting transversality conditions $\lim_{t\to\infty} \lambda n e^{-\rho t} = \lim_{t\to\infty} \lambda k e^{-\rho t} = 0$ hold, then the necessary conditions (6a-e) are sufficient for optimality.

We next derive the intertemporal macroeconomic equilibrium. In common with the most of the research analyzing the effects of status preference in the representative agent model, we confine our analysis to symmetric equilibria in which identical agents make identical choices. Consequently, $(n+k) = (N+K)$ holds $\forall t \geq 0$. As indicated, $(n+k)$, the net assets, or wealth, of the domestic private sector, consists of physical capital and net financial claims on the rest of the world. Substitution of $(n+k) = (N+K)$ into the optimality conditions (6a-c), (6e) and net financial asset accumulation equation (5a)—where the latter corresponds to the current account balance—results in the following symmetric open economy equilibrium in which the variables (c, i, λ, k, q, n) obey the following relationships

$$\eta c^{\eta-1}\varphi(1)^{\beta}\left(c^{\eta}\varphi\left(1\right)^{\beta}\right)^{-\theta} = \lambda, \tag{7a}$$

$$i = \dot{k} = \frac{(q-1)}{h}k, \tag{7b}$$

$$\dot{\lambda} = (\rho - r^*)\,\lambda - \beta\left(c^{\eta}\varphi\,(1)^{\beta}\right)^{-\theta}\frac{c^{\eta}\varphi(1)^{\beta-1}\varphi'\,(1)}{n+k-\bar{a}}, \tag{7c}$$

$$\dot{q} = r^* q - \epsilon f'(k) - \frac{(q-1)^2}{2h} + \frac{(q-1)\beta c}{\eta(n+k-\bar{a})}\frac{\varphi'(1)}{\varphi(1)} \tag{7d}$$

$$\dot{n} = \epsilon f(k) + r^* n - \frac{\left(q^2-1\right)}{2h}k - c, \tag{7e}$$

as well as the initial conditions $n(0) = n_0$, $k(0) = k_0$ and the transversality conditions $\lim_{t\to\infty}\lambda n e^{-\rho t} = \lim_{t\to\infty}\lambda k e^{-\rho t} = 0$.[9]

We will analyze the properties of the dynamic system (7a-e) in terms of the control variable consumption c, rather than in terms of the shadow of the shadow value of wealth λ. Taking the time derivative of (7a) and substituting the resulting expression into (7c) results, after division by (7a), in the following modified Euler equation

$$\dot{c} = \sigma^e c[r^e(r^*, c, a) - \rho], \tag{8}$$

where the *effective* elasticity of intertemporal substitution σ^e and the *effective* domestic, or internal, rate of return on assets r^e are given, respectively, by

$$\sigma^e \equiv [1-\eta(1-\theta)]^{-1}, \qquad r^e(r^*, c, n, k) \equiv r^* + \frac{(\xi/\eta)\,c}{(n+k-\bar{a})}, \text{ where } \xi \equiv \frac{\beta\varphi'(1)}{\varphi(1)}. \tag{9}$$

As described by Fisher and Hof (2001), the effective rate of return r^e equals the sum of the world interest rate r^* and the marginal rate of substitution of own net assets $a = (n+k)$ for consumption c as perceived by the domestic consumer-producer in a symmetric state in which $(n+k) = (N+K)$ holds (hereafter, symmetric MRS). In this formulation of status-preference, the symmetric MRS of net assets for consumption represents the additional return to saving due to status preference in which the incremental flow of utility from an extra unit of savings is converted into equivalent units of the consumption

[9] An additional implication of the symmetric equilibrium is that the flow of instantaneous utility is independent of the stock of net assets, i.e., $U = U\,(c, \varphi(1))$.

good. Differentiating the expression for $r^e(r^*, c, n, k)$ with respect to c, n, and k, it is straightforward to show that the symmetric MRS, and hence the effective rate of return, is a positive function of consumption and a negative function of net assets.[10] For our purposes, the advantage of the Euler equation (8) modified for status-preference is that consumption (potentially) displays saddlepath dynamics, which implies that the small open economy (potentially) avoids the counter-intuitive transitional dynamics described in the introduction.[11] In particular, the term $\xi \equiv [\beta\varphi'(1)/\varphi(1)]^{-1}$ stated in (9) merits comment. Observe that ξ is a positive function of β, the utility "weight" of status in (3), and of the derivative of $\varphi(1)$, given by $\varphi'(1)$. Following Fisher and Hof (2001), we can treat ξ as a measure of the "importance" of net asset accumulation in the quest for status.[12]

3. Steady State Equilibrium and Macroeconomic Dynamics

Letting $\dot{c} = \dot{q} = \dot{k} = \dot{n} = 0$ in equations (7b), (7d), (7e), and (8), we derive the following steady-state equilibrium:

$$\frac{(\xi/\eta)\,\tilde{c}}{\tilde{n} + \tilde{k} - \bar{a}} = \rho - r^*, \tag{10a}$$

$$\tilde{q} = 1, \tag{10b}$$

$$\epsilon f'(\tilde{k}) = r^*, \tag{10c}$$

[10] See Fisher and Hof (2001) for a detailed analysis of the properties of the effective rate of return based on the general specification of preferences in equations (1)-(2).

[11] The introduction of the effective rate of return implies that the transversality condition can be written as

$$\lim_{t\to\infty}\left\{a(t)\exp\left[-\int_0^t r^e(r^*, c(v), n(v) + k(v))\,dv\right]\right\} = 0.$$

Because we focus on long-run equilibria in which $(n + k)$ assumes finite values, this modified transversality condition implies that the NPG holds with equality.

[12] This is due to the following: i) a higher value of β increases the MRS of status for consumption, i.e., $[U_s(c,s)/U_c(c,s) = (\beta/\eta)(c/s)]$ and ii) the greater is the derivative $\varphi'(1)$, the larger is the marginal status of individual holdings of net assets from the point of view of consumer-producers in symmetric states, i.e., $\left[s_a = \varphi'(1)(a - \bar{a})^{-1}\right]$.

$$\epsilon f(\tilde{k}) - \tilde{c} = -r^* \tilde{n}. \tag{10d}$$

The steady-state equilibrium has the following characteristics. Equation (10a) is the steady-state version of the Euler equation, while (10b) indicates that the steady-state shadow value of domestic capital equals unity. According to (10c), the long-run marginal physical product of capital equals the exogenous world interest rate. Finally, because the current account balance is zero in long-run equilibrium, equation (10d) implies that the steady-state excess of output over consumption corresponds to the negative of net interest income from financial assets. A key characteristic of this long-run equilibrium is that the steady-state values of the "production-side" of the economy, i.e., the values of $\tilde{q}$ and $\tilde{k}$, are determined solely by the steady-state relationships (10b, c) and, consequently, are independent of (10a, d), which we can conveniently term the "consumption-side" of the economy. Solving the steady-state marginal condition (10c) implicitly for $\tilde{k}$, we can express $\tilde{k}$ as a function of total factor productivity ϵ and the world interest rate r^*:

$$\tilde{k} = \tilde{k}(\epsilon, r^*), \qquad \partial\tilde{k}/\partial\epsilon = -f'(\tilde{k})[\epsilon f''(\tilde{k})]^{-1} > 0, \qquad \partial\tilde{k}/\partial r^* = [\epsilon f''(\tilde{k})]^{-1} < 0. \tag{11}$$

According to (11), the steady-state domestic capital stock is a positive function of total factor productivity and a negative function of the world interest rate. Substituting (11) into (10a) and (10d), we derive the two-equation system that determines the steady-state values of consumption and net financial assets

$$\frac{(\xi/\eta)\,\tilde{c}}{\tilde{n} + \tilde{k}(\epsilon, r^*) - \bar{a}} = \rho - r^*, \tag{12a}$$

$$\epsilon f(\tilde{k}(\epsilon, r^*)) - \tilde{c} = -r^* \tilde{n}. \tag{12b}$$

which can then be jointly solved for $\tilde{c}$ and $\tilde{n}$:

$$\tilde{c} = \frac{(\rho - r^*)\,r^*}{\rho - [1 + (\xi/\eta)]r^*} \left[\frac{\epsilon f\,(k\,(\epsilon, r^*))}{r^*} - \left(\tilde{k}(\epsilon, r^*) - \bar{a}\right) \right], \tag{13a}$$

$$\tilde{n} = \frac{1}{\rho - [1 + (\xi/\eta)]r^*} \left[(\xi/\eta)\, \epsilon f\left(k\left(\epsilon, r^*\right)\right) - (\rho - r^*) \left(\tilde{k}(\epsilon, r^*) - \bar{a} \right) \right]. \tag{13b}$$

Using (11) and (13b), we can also determine the expression for the steady state excess of domestic net assets over its lower bound, i.e., $\left[\left(\tilde{n} + \tilde{k}\right) - \bar{a}\right] = (\tilde{a} - \bar{a})$:

$$\left[\left(\tilde{n} + \tilde{k}\right) - \bar{a}\right] = (\tilde{a} - \bar{a}) = \frac{(\xi/\eta)\, r^*}{\rho - [1 + (\xi/\eta)]r^*} \left[\frac{\epsilon f\left(k\left(\epsilon, r^*\right)\right)}{r^*} - \left(\tilde{k}(\epsilon, r^*) - \bar{a} \right) \right]. \tag{13c}$$

We will restrict our attention to long-run equilibria with positive levels of consumption and a positive excess of domestic net assets over its lower bound, i.e., $\tilde{c} > 0$ and $\left[\left(\tilde{n} + \tilde{k}\right) - \bar{a}\right] = (\tilde{a} - \bar{a}) > 0$. This requires that we impose on the solutions of (13a) and (13c), the following restrictions:

$$\rho - [1 + (\xi/\eta)]r^* > 0, \qquad \frac{\epsilon f\left(\tilde{k}\left(\epsilon, r^*\right)\right)}{r^*} - \left(\tilde{k}(\epsilon, r^*) - \bar{a} \right) > 0. \tag{14}$$

The first condition of (14) imposes—in effect—an upper bound on the status-preference term ξ for given values of ρ, r^*, and η. The second condition of (14) states that the present discounted value of output exceeds the difference between the steady-state capital stock and the parameter $\bar{a}$. The conditions in (14) permit, nevertheless, $\tilde{n}$ or $\tilde{a}$ to assume positive or negative values.[13] What is the effect of an increase in ξ on the steady-state values of $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$? It is straightforward to show that higher values of ξ—corresponding to a higher degree of status-consciousness—lead to higher values of $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$. We can also demonstrate that an improvement in total factor productivity ϵ increases $\tilde{c}$, $\tilde{n}$, and $(\tilde{a} - \bar{a})$, while a rise in the world interest rate r^* has ambiguous effects on the values of these steady state variables.

The next step is to linearize the differential equations for $\dot{c}$, $\dot{q}$, $\dot{k}$, and $\dot{n}$ about the

[13]Obviously, if the signs of both conditions in (14) are reversed, then $\tilde{c}$ and $(\tilde{a} - \bar{a})$ are still positive. In this case, however, the solution to the homogenous system $(\dot{c}, \dot{n})$ in equation (20) below does not possess saddlepoint dynamics.

steady-state equilibrium described above. This procedure yields the following relationships

$$\dot{c} = \frac{\sigma^e\,(\xi/\eta)\,\tilde{c}}{\tilde{n}+\tilde{k}-\bar{a}}(c-\tilde{c}) - \frac{\sigma^e\,(\xi/\eta)\,\tilde{c}^2}{\left(\tilde{n}+\tilde{k}-\bar{a}\right)^2}(k-\tilde{k}) - \frac{\sigma^e\,(\xi/\eta)\,\tilde{c}^2}{\left(\tilde{n}+\tilde{k}-\bar{a}\right)^2}(n-\tilde{n}),$$

$$= \sigma^e\,(\rho - r^*)\,(c-\tilde{c}) - \sigma^e\,(\rho-r^*)^2\,(\xi/\eta)^{-1}\left[(k-\tilde{k}) + (n-\tilde{n})\right], \tag{15a}$$

$$\dot{q} = \left[r^* + \frac{(\xi/\eta)\,\tilde{c}}{\tilde{n}+\tilde{k}-\bar{a}}\right](q-1) - \epsilon f''\left(\tilde{k}(\epsilon, r^*)\right)(k-\tilde{k})$$

$$= \rho\,(q-1) - \epsilon f''\left(\tilde{k}(\epsilon, r^*)\right)(k-\tilde{k}), \tag{15b}$$

$$\dot{k} = h^{-1}\tilde{k}(q-1), \tag{15c}$$

$$\dot{n} = -(c-\tilde{c}) - h^{-1}\tilde{k}(q-1) + r^*(k-\tilde{k}) + r^*(n-\tilde{n}). \tag{15d}$$

where we have used the fact from (13a, c) that $\tilde{c}/(\tilde{a}-\bar{a}) = (\xi/\eta)^{-1}(\rho - r^*)$ to derive the second equalities of (15a) and (15b). Observe that the differential equations (15b) and (15c) constitute an two-equation dynamic system in $\left(\dot{q}, \dot{k}\right)$ that can be solved independently of (15a) and (15d), the differential equations for consumption and the current account balance.[14] Re-expressing (15b) and (15c) as a matrix system yields:

$$\begin{pmatrix} \dot{q} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} \rho & -\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right) \\ h^{-1}\tilde{k} & 0 \end{pmatrix} \begin{pmatrix} q-1 \\ k-\tilde{k} \end{pmatrix}. \tag{16}$$

The dynamic and stability properties of (16) can be determined by examining its characteristic equation, which is equal to

$$0 = -(\rho-\mu)\,\mu + h^{-1}\tilde{k}\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right) = \mu^2 - \mathrm{tr}\,(\mathbf{J})\,\mu + \det(\mathbf{J}), \tag{17}$$

[14] This will not be the case if the employment decision is endogenous. In that case the evolution of capital stock and its shadow value does depend on the Euler equation and the current account balance.

and where the trace and determinant of the Jacobean matrix $\mathbf{J}$ in (16) are given by:

$$\operatorname{tr}(\mathbf{J}) = \mu_1 + \mu_2 = \rho > 0, \quad \det(\mathbf{J}) = \mu_1\mu_2 = h^{-1}\tilde{k}\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right) < 0.$$

Since $\det(\mathbf{J}) < 0$, the steady state equilibrium $\left(\tilde{k}, 1\right)$ described a saddlepoint, such that stable and unstable eigenvalues have the following properties: $\mu_1 < 0$, $\mu_2 > 0$, $|\mu_1| < \mu_2$. Employing standard techniques, we can then calculate the following stable solution for the saddlepath dynamics of the domestic capital stock and its shadow value

$$q - 1 = \mu_1 h\tilde{k}^{-1}(k - \tilde{k}) = \frac{\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right)}{\rho - \mu_1}(k - \tilde{k}), \tag{18a}$$

$$k = \tilde{k} - \left(\tilde{k} - k_0\right)e^{\mu_1 t}, \tag{18b}$$

where the domestic capital stock adjusts from a given initial value, k_0.[15] The dynamics of this system is illustrated in Figure 1, which depicts the $\dot{q} = 0$ locus, the $\dot{k} = 0$ locus (equal to $\tilde{q} = 1$), the stable saddlepath XX, and the steady-state equilibrium $\left(\tilde{k}, 1\right)$ corresponding to point D. Because the slope of the $\dot{q} = 0$ locus is given by $(dq/dk)\,|_{\dot{q}=0} = \left(\epsilon f''\left(\tilde{k}\right)/\rho\right) < 0$, it is clear that:

$$\left.\frac{dq}{dk}\right|_{XX} = \frac{\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right)}{\rho - \mu_1} > \left.\frac{dq}{dk}\right|_{\dot{q}=0} = \frac{\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right)}{\rho}.$$

In other words, the $\dot{q} = 0$ locus is "steeper" that the saddlepath XX.[16] Since the stable saddlepath XX is negatively sloped, the capital stock and its shadow value move in opposite directions, i.e., sgn $(\dot{q})$ =sgn $\left(\dot{k}\right)$. In Figure 1 we have also indicated at point B the initial position of the production-side of the economy as well as the arrows determining the direction of movement in the phase plane. Since $q(t) > 1$ between points B

[15] See Turnovsky (2000) for a recent exposition of these methods. Note that we are implicitly imposing the transversality condition for physical capital in order to eliminate the part of the solution that contains the unstable eigenvalue μ_2. We follow the same procedure below in solving the $(\dot{c}, \dot{n})$ system.

[16] The slope of $\dot{q} = 0$ is unambiguously negative, since it is evaluated in the steady-state equilibrium in which $\tilde{q} = 1$ and $\left[(\xi/\eta)\,\tilde{c}/\left(\tilde{n} + \tilde{k} - \bar{a}\right)\right] = \rho - r^*$.

and D, physical investment is positive, $\dot{k}(t) > 0$, during the entire transition to long-run equilibrium. Solving the characteristic equation (17) for stable eigenvalue μ_1, we obtain:

$$\mu_1 = \frac{1}{2}\left\{\mathrm{tr}(\mathbf{J}) - \sqrt{[\mathrm{tr}(\mathbf{J})]^2 + 4\left|\det(\mathbf{J})\right|}\right\} = \frac{1}{2}\left\{\rho - \sqrt{\rho^2 - 4h^{-1}\tilde{k}\epsilon f''\left(\tilde{k}(\epsilon, r^*)\right)}\right\}. \tag{19a}$$

This expression reveals that speed of stable adjustment of the production-side of the economy—equal to $|\mu_1|$—is a function of the rate of time preference ρ and of the curvature properties of the installation cost function $\Psi(i, k)$ and the production function $f(k)$, in addition to total factor productivity ϵ.[17] It is, furthermore, independent of the parameters of the instantaneous utility function described above in equation (3). By direct calculation, we can show the following relationships between $|\mu_1|$ and ρ, ϵ, $f''\left(\tilde{k}\right)$, and h obtains:

$$\frac{\partial|\mu_1|}{\partial\rho} \gtreqless 0, \quad \frac{\partial|\mu_1|}{\partial\epsilon} > 0, \quad \frac{\partial|\mu_1|}{\partial\left(f''\left(\tilde{k}\right)\right)} < 0, \quad \frac{\partial|\mu_1|}{\partial h} < 0. \tag{19b}$$

The conditions in (19b) imply: i) an ambiguous relationship between the rate of time preference ρ and $|\mu_1|$, ii) a positive relationship between total factor productivity ϵ and $|\mu_1|$, iii) a negative relationship between the curvature of the production function $f''\left(\tilde{k}\right)$ and $|\mu_1|$, and iv) a negative relationship between the installation cost function parameter h and $|\mu_1|$.

Our next step is to use the solutions (18a) and (18b) from the production-side of the model to determine the paths of consumption and the current account. Substituting (18a) and (18b) into the linearized expressions for $\dot{c}$ and $\dot{n}$ in (15a, d), we obtain the following nonhomogenous differential equation system in $(\dot{c}, \dot{n})$

$$\begin{pmatrix} \dot{c} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} \\ -1 & r^* \end{pmatrix}\begin{pmatrix} c - \tilde{c} \\ n - \tilde{n} \end{pmatrix} + \begin{pmatrix} -d_{12} \\ -(r^* - \mu_1) \end{pmatrix}\left(\tilde{k} - k_0\right)e^{\mu_1 t}, \tag{20}$$

where

$$d_{11} = \sigma^e(\rho - r^*) > 0, \quad d_{12} = -\sigma^e(\rho - r^*)^2(\xi/\eta)^{-1} < 0.$$

[17]In particular, note that the term $h^{-1}\tilde{k}$ corresponds to $\left[\partial^2\Psi(i,k)/\partial i^2\right]^{-1}$.

The complete solution of (20) is the sum of the general solution of the homogenous system and a particular solution of the nonhomogenous part of (20). Employing the same methods used to solve (16), it is easy to show that the stable solution to the homogenous part of (20) can be expressed as

$$c = \tilde{c} + (r^* - \omega_1) A_1 e^{\omega_1 t}, \tag{21a}$$

$$n = \tilde{n} + A_1 e^{\omega_1 t}, \tag{21b}$$

where $\omega_1 < 0$ is the stable eigenvalue, to be solved below, of the Jacobian $\mathbf{G}$ of homogenous part of (20) and A_1 is an arbitrary constant to be determined. From the Jacobian matrix $\mathbf{G}$, it is straightforward to show that the characteristic equation of $\mathbf{G}$—identical to the one derived by Fisher and Hof (2001) in their model without capital stock dynamics—is equal to

$$0 = (d_{11} - \omega)(r^* - \omega) + d_{12} = \omega^2 - \mathrm{tr}(\mathbf{G})\,\omega + \det(\mathbf{G}), \tag{22}$$

where the trace and determinant of the Jacobian of (20) correspond to

$$\mathrm{tr}(\mathbf{G}) = \omega_1 + \omega_2 = d_{11} + r^* = \sigma^e (\rho - r^*) + r^* > 0,$$

$$\det(\mathbf{G}) = \omega_1 \omega_2 = d_{11} r^* + d_{12} = -\sigma^e (\rho - r^*) (\xi/\eta)^{-1} [\rho - [1 + (\xi/\eta)] r^*] < 0.$$

The expressions for tr($\mathbf{G}$) and det($\mathbf{G}$) imply that the eigenvalues of $\mathbf{G}$, given our assumptions, obey the following relationships: $\omega_1 < 0$, $\omega_2 > 0$, $|\omega_1| < \omega_2$. Note that the condition $\rho - r^* > 0$ is sufficient for tr($\mathbf{G}$) > 0. In contrast, det($\mathbf{G}$) < 0 is ensured only if both $\rho - r^* > 0$ and the first condition in (14), equal to $[\rho - [1 + (\xi/\eta)] r^* > 0]$, hold. If det($\mathbf{G}$) < 0, then the homogenous part of the solution of (20) displays saddlepoint dynamics. Using the characteristic equation (22), we can write the expression for the stable eigenvalue ω_1 as:

$$\omega_1 = \frac{1}{2}\left\{ \mathrm{tr}(\mathbf{G}) - \sqrt{[\mathrm{tr}(\mathbf{G})]^2 + 4\,|\det(\mathbf{G})|} \right\}$$

$$= \frac{1}{2}\left\{\sigma^{e}\left(\rho-r^{*}\right)+r^{*}-\sqrt{\left[\sigma^{e}\left(\rho-r^{*}\right)+r^{*}\right]^{2}+4\sigma^{e}\left(\rho-r^{*}\right)\left(\xi/\eta\right)^{-1}\left[\rho-\left[1+\left(\xi/\eta\right)\right]r^{*}\right]}\right\}. \tag{23}$$

Clearly, the speed of stable adjustment $|\omega_1|$ of the homogenous system depends on the value of the world interest rate r^* and preference parameters such as the rate of time discount ρ, the effective intertemporal elasticity of substitution $\sigma^e = [1 - \eta(1-\theta)]^{-1}$, and the term ξ that measures the importance of relative wealth for status-conscious consumers. Observe, in addition, that $|\omega_1|$ depends neither on the characteristics of the production function nor of the installation cost function. Regarding the ξ, Fisher and Hof (2001) have shown that a higher value of ξ, reflecting a greater importance of status, unambiguously lowers the stable speed of adjustment $|\omega_1|$. Concerning the other parameters, it is straightforward to show that the the relationships between $|\omega_1|$ and, respectively, ρ, r^*, and σ^e, are ambiguous.

Next, we specify that the particular solution of (20) corresponds to the following set of equations

$$c = \tilde{c} + E_1 e^{\mu_1 t}, \tag{24a}$$

$$n = \tilde{n} + F_1 e^{\mu_1 t}, \tag{24b}$$

where E_1 and F_1 are given constants to be determined and μ_1 is, as before, the stable eigenvalue derived above from the production-side of the economy. Substitution of the particular solutions (24a) and (24b) into the nonhomogenous dynamic system (20), yields the following expressions for the constants E_1 and F_1

$$E_1 = 0, \quad F_1 = \left(\tilde{k} - k_0\right),$$

which implies that the particular solutions (24a, b) for consumption and net financial assets become:

$$c = \tilde{c}, \quad n = \tilde{n} + \left(\tilde{k} - k_0\right) e^{\mu_1 t}. \tag{25a, b}$$

Combining the solutions (21a) and (21b) of the homogenous system with the particular solutions (25a) and (25b), we obtain the following general solution of the nonhomogenous system (20):

$$c = \tilde{c} + (r - \omega_1) A_1 e^{\omega_1 t} \tag{26a}$$

$$n = \tilde{n} + A_1 e^{\omega_1 t} + \left(\tilde{k} - k_0\right) e^{\mu_1 t}. \tag{26b}$$

The arbitrary constant A_1 is then determined using the assumption that the stock of net financial assets adjusts from its initial value, i.e., $n(0) = n_0$. Using the general solution (26b) for net financial assets, this implies $A_1 = -\left[(\tilde{n} - n_0) + \left(\tilde{k} - k_0\right)\right] = -(\tilde{a} - a_0)$. Consequently, the complete solutions for consumption and net financial assets equal:

$$c = \tilde{c} - (r^* - \omega_1)\left[(\tilde{n} - n_0) + \left(\tilde{k} - k_0\right)\right] e^{\omega_1 t}, \tag{27a}$$

$$n = \tilde{n} - \left[(\tilde{n} - n_0) + \left(\tilde{k} - k_0\right)\right] e^{\omega_1 t} + \left(\tilde{k} - k_0\right) e^{\mu_1 t}. \tag{27b}$$

To review our results thus far, we have showed that the small open economy with a status-preference for relative wealth and installation costs of adjusting physical capital possesses two distinct stable speeds of adjustment, $|\mu_1|$ and $|\omega_1|$. The dynamics of the capital stock and its shadow value depend solely on $|\mu_1|$, while that of consumption depends solely on $|\omega_1|$. Inspection of the solution for net financial assets in (27b) shows, however, that it depends on both $|\mu_1|$ and $|\omega_1|$. Taking the time derivative of equation (27b)—which converts it into an expression for the current account balance—we obtain:

$$\dot{n}(t) = -\omega_1\left[(\tilde{n} - n_0) + \left(\tilde{k} - k_0\right)\right] e^{\omega_1 t} + \mu_1\left(\tilde{k} - k_0\right) e^{\mu_1 t}. \tag{28}$$

Assuming that the initial conditions for net financial assets and physical capital are below their steady-state values, i.e., $(\tilde{n} - n_0) > 0$, $\left(\tilde{k} - k_0\right) > 0$, the expression for $\dot{n}(t)$ in (28) reveals that the two speeds of adjustment work in opposite directions.[18] In other

[18] In terms of an exogenous experiment, we can think of this as corresponding to the case in which the economy responds to a permanent total factor productivity shock—starting from an initial condition

words, the part of the current account dynamics that depends on $|\omega_1|$ is associated with an *improvement* in the current account ($\dot{n}(t) > 0$), while the part of the current account dynamics that depends on $|\mu_1|$ is associated with a *deterioration* in the current account ($\dot{n}(t) < 0$). Clearly, then, the dynamics of the current account balance depend, in part, on the relative sizes of $|\mu_1|$ and $|\omega_1|$ and, consequently, on the parameters of the production and consumption-sides of the small open economy.

To investigate in more detail the conditions under the influence of either $|\omega_1|$ or $|\mu_1|$ dominates, we must determine whether there exists a time $t^* > 0$ such that $\dot{n}(t^*) = 0$. Solving (28) for t^*, we obtain

$$t^* = \frac{\ln\left[\frac{-\omega_1(\tilde{n}-n_0)}{(\omega_1-\mu_1)\left(\tilde{k}-k_0\right)}\right]}{(\mu_1-\omega_1)} \tag{29a}$$

It is then straightforward to show from (29a) that the necessary conditions for $t^* > 0$ are given by:

$$t^* > 0 \quad \Longleftrightarrow \quad 0 < \left[\frac{-\omega_1(\tilde{n}-n_0)}{(\omega_1-\mu_1)\left(\tilde{k}-k_0\right)}\right] < 1, \quad (\omega_1-\mu_1) < 0. \tag{29b}$$

The latter condition is, of course, equivalent to $|\mu_1| > |\omega_1|$, i.e., that the speed of adjustment of the production-side of the economy exceeds the speed of adjustment of the consumption-side of the economy. Observe, in addition, that the first condition for the existence of $t^* > 0$ depends not only on the relative sizes of $|\mu_1|$ and $|\omega_1|$, but also on the ratio of changes in the long-run stocks of assets, $\left[(\tilde{n}-n_0)/\left(\tilde{k}-k_0\right)\right]$. If, on the other hand, the conditions in (29b) do not hold, then the path of $n(t)$ will not reach a stationary value during the transition to steady-state equilibrium. In this case, the current account improves ($\dot{n}(t) > 0$) during the entire phase of adjustment starting from $n(0) = n_0$, $k(0) = k_0$.

The question also arises how agents' status-consciousness influences the conditions in (29b) and, consequently, the behavior of the current account. Consider, for instance, the case of an increase in status-consciousness, i.e., a rise in ξ. As indicated above, an increase

(n_0, k_0)—by increasing $\tilde{n}$ and $\tilde{k}$. Obviously, the initial value of consumption, $c(0)$, will be chosen consistent with the economy's transversality conditions stated above.

in ξ lowers $|\omega_1|$, which, according to (28), puts downward pressure on the current account balance in the short-run. On the other hand, higher values of ξ also lead to greater long-run stocks of net international financial assets $\tilde{n}$, which, in turn, requires the accumulation of current account surpluses. This implies that a greater degree of status-consciousness will have an ambiguous effect on the conditions in (29b) and, therefore, on the dynamics of the current account, given the parameters $(\rho,\, r^*,\, \sigma^e,\, \epsilon,\, h)$ and the curvature properties of the functions $\{f(k),\, \Psi(i,k)\}$. Due to the relative complexity of these relationships, a numerical analysis would provide a potential avenue of gaining greater intuition regarding the effect of greater ξ on the current account balance. We will leave this exercise for future work.

We can illustrate these two cases by means of phase diagrams.[19] Figure 2a illustrates in the case in which the two conditions in (29b) hold, while Figure 2b depicts the case in which they do not.[20] As illustrated in Figure 2a, the current account at $t = 0$ deteriorates, i.e., $\dot{n}(0) < 0$. This decline in the current account balance reflects the relatively rapid adjustment of the physical capital stock compared to that of the stock of net financial assets. We can identify two reasons for this response. One is the fact that the speed of adjustment of the production-side of the economy is greater that of the consumption-side, $|\mu_1| > |\omega_1|$. Intuitively, this implies a relatively large initial "build-up" of adjustment costs of investment, which places downward pressure on the current account balance. The other key factor is that the long-run increase in the capital stock in this case exceeds a critical level determined by the first condition in (29b).[21] After $t = 0$, the economy proceeds along the locus LMN in its transition to the steady-state equilibrium at point N. Observe that consumption continuously rises along the locus LMN. This reflects the on-going increase in output that is due to the rising stock of capital. At point M, determined by solution for $\dot{n}(t^*) = 0$, where t^* is given (29a), the stationary, and minimum, value of $n(t^*)$ is reached.

[19] In Figures 2 we assume positive values for n_0 and $\tilde{n}$. As discussed above, the possibility that these stocks assume negative values is not excluded, however.

[20] If only the second condition in (29b) holds, while the first condition is violated, then the current account balance is positive ($\dot{n}(t) > 0$) during the entire adjustment phase.

[21] ¿From (29b), this condition is satisfied if

$$\left(\tilde{k} - k_0\right) > \frac{-\omega_1}{(\omega_1 - \mu_1)}\left(\tilde{n} - n_0\right), \text{ where } (\omega_1 - \mu_1) > 0.$$

Thereafter, for $t > t^*$, both the stock of net financial assets and consumption increase toward their long-run values $(\tilde{c}, \tilde{n})$.[22] This reflects the relative reduction in the rate of physical capital accumulation compared to the rate of net financial asset accumulation as $t \to \infty$. Observe in Figure 2a that the slope of the LMN locus approaches $(r^* - \omega_1) > 0$ as the path of (c, n) approaches point N. Using the solutions in (27a, b), this was derived by calculating the following ratio:

$$\frac{(c - \tilde{c})}{(n - \tilde{n})} = \frac{-(r^* - \omega_1)\left[(\tilde{n} - n_0) + \left(\tilde{k} - k_0\right)\right]}{-\left[(\tilde{n} - n_0) + \left(\tilde{k} - k_0\right)\right] + \left(\tilde{k} - k_0\right) e^{(\mu_1 - \omega_1)t}}. \tag{30}$$

Clearly, since $|\mu_1| > |\omega_1|$ in this case, we have:

$$\text{slope } \frac{(c - \tilde{c})}{(n - \tilde{n})} \to (r^* - \omega_1), \text{ as } t \to \infty.$$

We turn next to Figure 2b, which illustrates the case in which the conditions in (29b) do not hold simultaneously. The adjustment of net financial assets and consumption in Figure 2b is depicted by the locus PQ. Along PQ, the current account balance is always positive and agents enjoy increasing levels of consumption, i.e., $\dot{n}(t) > 0$, $\dot{c}(t) > 0$. While physical capital does accumulate in this case, its positive effect on the domestic demand for goods and services is insufficient to cause the current account balance to initially deteriorate during the transition to the new steady state at point Q. We graph in Figure 2b the case in which the slope of the PQ locus approaches zero as (c, n) approach their

[22]In deriving Figures 2a and 2b, we used the fact that slope of the (n, c) locus is equal to

$$\frac{dc}{dn} = \frac{dc/dt}{dn/dt} = \frac{\dot{c}(t)}{\dot{n}(t)}$$

while the curvature of the locus is given by

$$\frac{d^2c}{dn^2} = \frac{\frac{d^2c}{dt^2}\frac{dt}{dn}\frac{dn}{dt} - \frac{dc}{dt}\frac{d^2n}{dt^2}\frac{dt}{dn}}{(dn/dt)^2} = \frac{\ddot{c}(t)\,\dot{n}(t) - \dot{c}(t)\,\ddot{n}(t)}{\dddot{n}(t)}.$$

long-run values at point Q. Recalling equation (30), this holds if $|\omega_1| > |\mu_1|$, i.e.,[23]

$$\text{slope } \frac{(c-\tilde{c})}{(n-\tilde{n})} \rightarrow 0, \text{ as } t \rightarrow \infty.$$

Before we leave this section, it is important to discuss a key difference between Figure 1, the phase diagram for the production-side of the economy, and Figures 2a and 2b that describe the dynamics of the consumption-side of the economy. In particular, note that we have illustrated in Figures 2 neither the $\dot{n} = 0$ and $\dot{c} = 0$ loci nor the arrows of motion of the (n, c) phase plane. Recalling the expressions for $\dot{n}(t)$ and $\dot{c}(t)$ in equations (7e) and (8), observe that they both depend on the value of the capital stock k at time t. Since, however, the capital stock is a dynamic variable that evolves continuously, the positions of the $\dot{n} = 0$ and $\dot{c} = 0$ loci—as well as the directional arrows in the phase plane—are also constantly shifting through time. Consequently, we do not draw them in Figures 2a and 2b.

4. Conclusions

As is well-known, the basic small open economy Ramsey model has two problematic characteristics: i) the lack of an interior equilibrium if the rate of time preference differs from the world interest rate and ii) no investment dynamics if the marginal physical product of capital is always equal to the exogenous and time invariant world interest rate. Fisher and Hof (2001) use a model of status-preference, with status a function of relative wealth, to solve the first problem. Nevertheless, the domestic capital stock in their model possesses no transitional dynamics. The purpose of this paper is to combine the representation of preferences given in Fisher and Hof (2001) with a standard adjustment cost specification of physical capital accumulation. We found that small open economy transitional dynamics is a function of two speeds of adjustment: one arising from the status-consciousness and the other derived from the process of accumulating physical assets. In particular, we determined that the dynamics of the economy "de-couples" into a consumption-side and a production-side. We showed, nevertheless, that the current account balance de-

[23]If the second condition of (29b) is satisfied, i.e., $|\mu_1| > |\omega_1|$, while the first condition is violated, then then slope of locus PQ approaches $(r^* - \omega_1)$ as $t \rightarrow \infty$.

pends on both speeds of adjustment. In fact, the two speeds of adjustment tend to have opposing short-run effects on current account dynamics. A larger production-side speed of adjustment leads initially to current account deficits, while a larger consumption-side speed of adjustment results in current account surpluses in the short-run. Consequently, the current account balance can exhibit non-monotonic paths. For instance, the current account balance can initially deteriorate before improving in transition to the steady-state equilibrium.

In analyzing in greater detail the dynamics of the current account, we derived the conditions in which it reaches a stationary value *prior* to steady-state equilibrium. These conditions depend on the speeds of adjustment as well as on the long-run values of net financial assets and physical capital. Because the degree of status-consciousness affects both the long-run equilibrium and the speed of adjustment of the consumption-side of the economy, its relationship to the evolution of the current account is complex and ambiguous. In future work we will conduct a numerical analysis to acquire further intuition regarding the relationship between the degree of status-consciousness and the current account balance.

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Figure 1

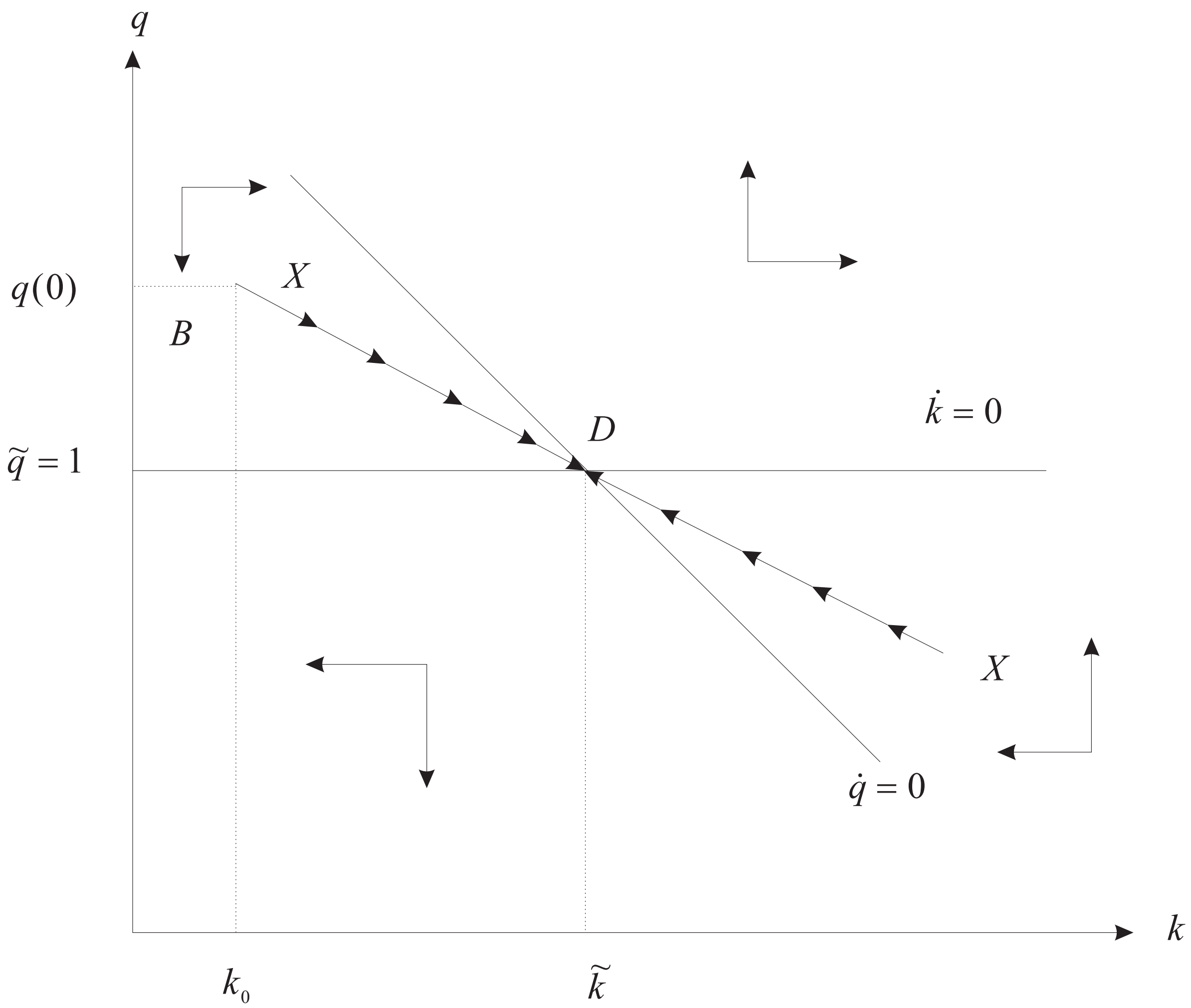


Figure 2a

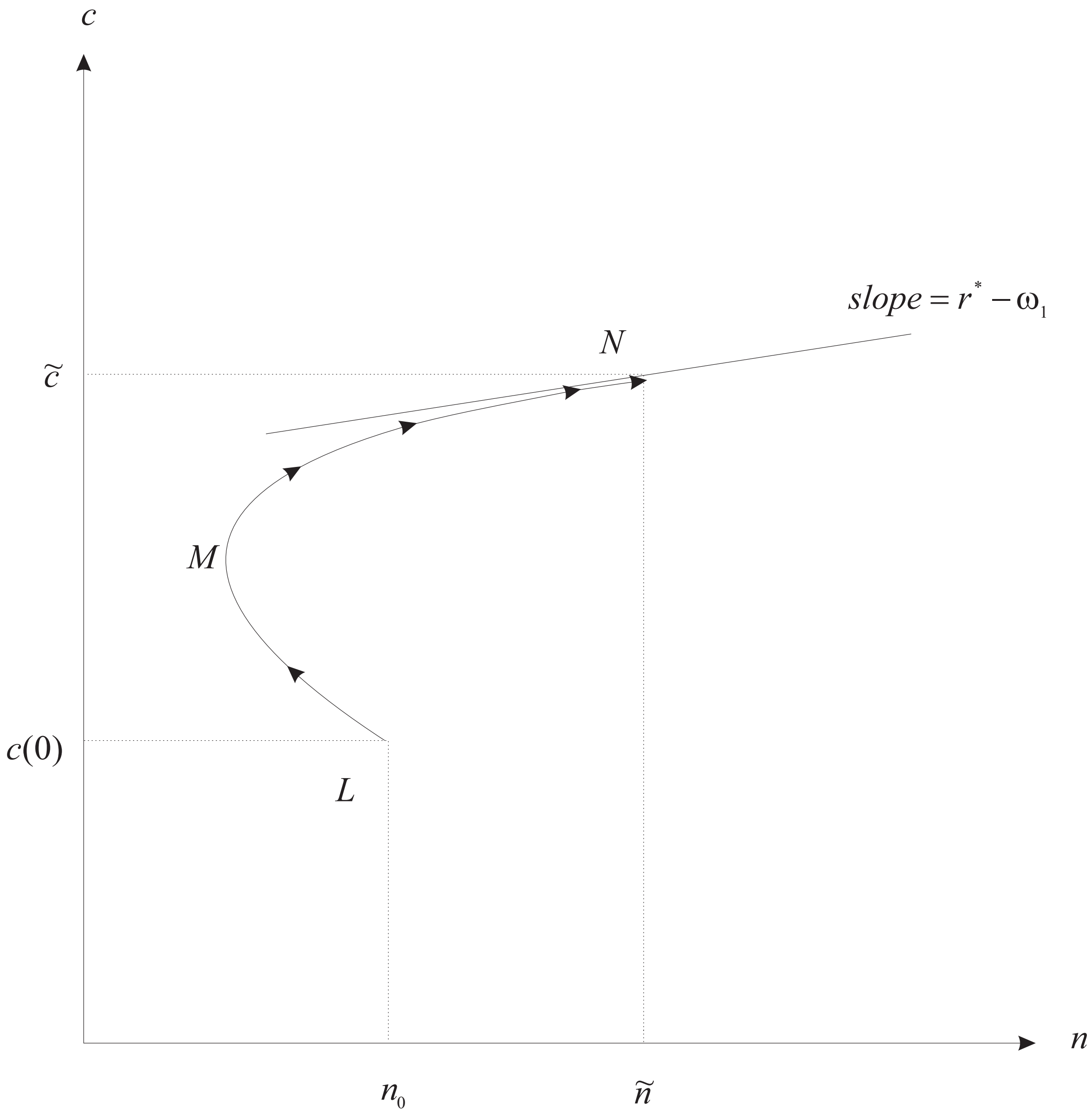
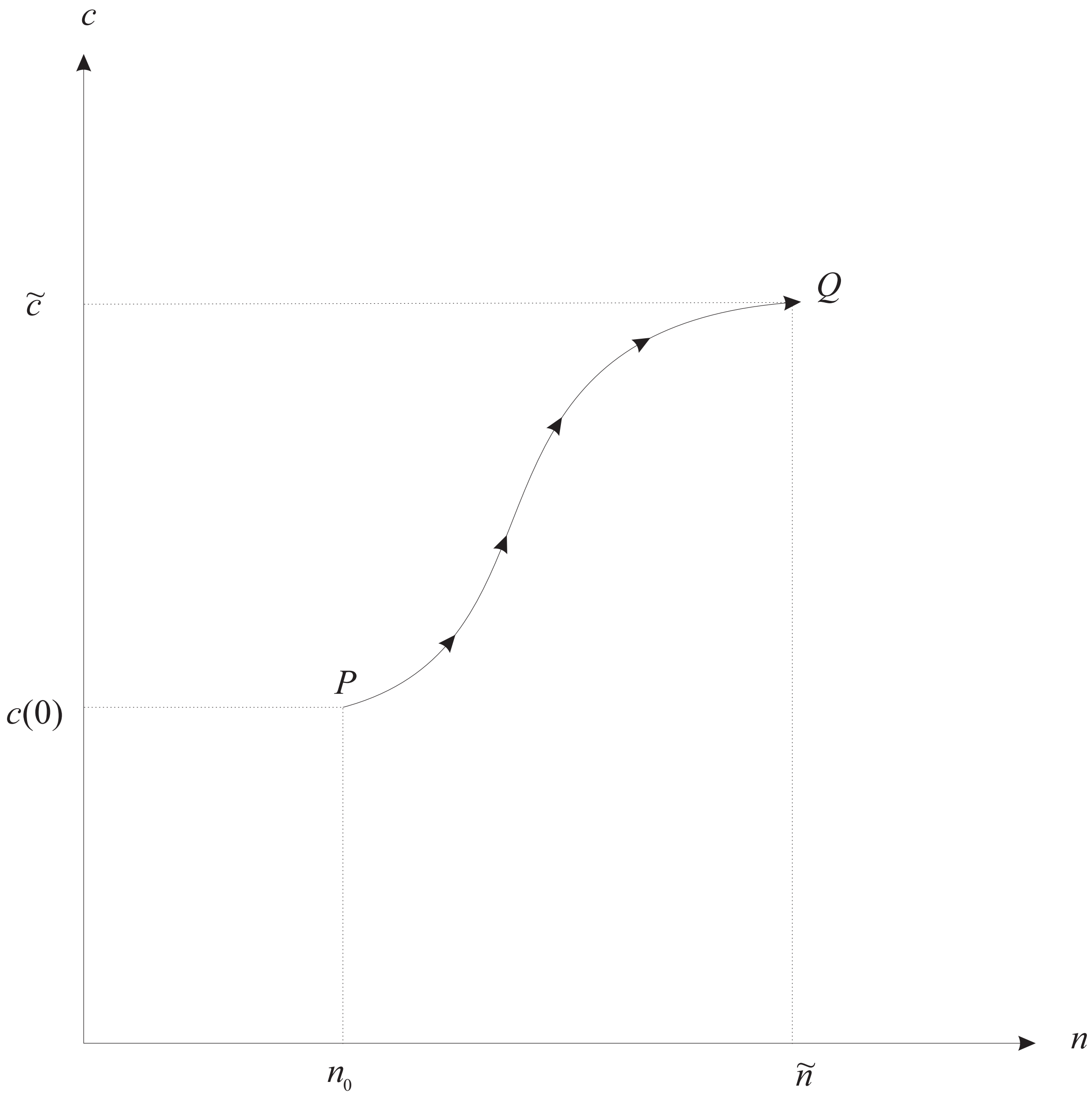


Figure 2b



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