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Conspicuous Consumption, Economic Growth, and Taxation: A Generalization

Walter H. FISHER[*] Franz X. HOF [†]

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Abstract

This paper studies the influence of consumption externalities in the Ramsey model. In contrast to the recent literature, a quite general specification of preferences is used and the concept of the effective intertemporal elasticity of substitution is introduced. We give conditions for the observational equivalence between economies with consumption externalities and externality-free economies. An additional key result is that there exist several types of instantaneous utility functions in which the decentralized solution coincides with the socially planned one in spite of the presence of consumption externalities. The conditions for optimal taxation are also derived.

Keywords: social status, conspicuous consumption, economic growth.

JEL classification: D62, D91, E21, E62.

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1. Introduction

The goal of this paper is to generalize and extend the results of a paper recently published in this *Journal* by Rauscher (1997b), (henceforth Rauscher). Rauscher introduces consumption externalities into an otherwise standard Ramsey model and analyzes the effects of this modification on economic growth and the issue of optimal taxation. He assumes that the instantaneous utility of individuals depends not only on absolute consumption, as in the standard model, but also on relative consumption. This specification expresses two ideas: i) economic agents care about their relative position in society, and ii) social status is determined by relative consumption.[1]

It is usual in this literature to restrict attention to a particular specification of preferences. Rauscher, for instance, assumed that the instantaneous utility is additively separable in own and relative consumption, where its dependence on own consumption is captured by an isoelastic function, while Gali (1994) and Harbaugh (1996) consider other functional forms.[2] In this paper we will generalize and extend Rauscher's results by using specifications of preferences that encompass those employed by these authors. We will, however, follow Rauscher in assuming that agents are identical, in specifying that there is a single consumption good, in neglecting both population growth and the depreciation of physical capital, and in treating labor supply as exogenously given.

Section 2 sets out the model and derives the decentralized solution. We introduce consumption externalities by assuming that preferences depend on the average level of consumption as well as on individual consumption. This approach not only encompasses the relative consumption case in which the utility function can be written as a function of absolute and relative consumption, but also permits alternative interpretations of consumption externalities. We solve the optimizing problem of the representative agent in which individuals take the average level of consumption as given. We then derive the symmetric, decentralized equilibrium and introduce the concept of the decentralized economy's "effective" intertemporal elasticity of substitution. This concept makes it possible to present the decentralized solution in such a way that immediate comparisons with the standard Ramsey model can be made. Proposition 1 then shows that an economy with consumption externalities is observationally equivalent to an externality-free

[1] An alternative branch of this literature models status as determined by relative wealth rather than by relative consumption. This relative wealth approach is employed, for instance, by Corneo and Jeanne (1997), Rauscher (1997a), and Futagami and Shibata (1998).

[2] Gali (1994) uses portfolio and asset pricing models, while Harbaugh (1996) employs a stochastic two-period consumption and savings framework.

economy with a "standard" elasticity of substitution if the effective intertemporal elasticity of substitution equals the standard one. This observational equivalence property implies that usual results for neoclassical and endogenous growth economies remain valid despite the introduction of consumption externalities. The only modification to be made in the usual results is to replace the standard intertemporal elasticity of substitution with the effective one. For instance, proposition 2 shows that if the production function satisfies the usual neoclassical assumptions, then the steady state values of capital and consumption do not depend on the specification of the instantaneous utility function, as in the standard model. Consumption externalities affect only the speed of adjustment through the effective intertemporal elasticity of substitution. Proposition 3 then states necessary conditions for endogenous growth, which involve the properties of both the production function and the effective intertemporal elasticity of substitution. We will also give an illustration in which the relationship between the effective intertemporal elasticity of substitution and the "degree of status-consciousness" is ambiguous, which implies that the relationship between the rate of growth and the degree of status-consciousness is also ambiguous.

In section 3 we will derive the social planner's solution in order to determine whether consumption externalities create distortions in the decentralized economy. Similarly to section 2, we will introduce the concept of the socially planned economy's "effective" intertemporal elasticity of substitution. This will enable us to easily make comparisons between the social and the decentralized solutions. The key result of this section is that there are several quite general types of the instantaneous utility function in which the corresponding decentralized solution coincides with the socially optimal solution in spite of the presence of consumption externalities. According to proposition 4, all utility functions of these types are characterized by the property that the decentralized economy's effective intertemporal elasticity of substitution equals that of the socially planned economy. If this condition is violated, then consumption externalities create distortions in the decentralized equilibrium. In this case the government might try to induce the private sector to attain the social optimum by designing an optimal tax policy.

The issue of optimal taxation is dealt with in section 4. Following Rauscher, we consider both a capital tax and a consumption tax and assume that tax revenues are fully rebated as lump-sum transfers. A consumption tax policy in which the tax rate is constant over time will not affect the time paths of consumption and capital. On the other hand, capital taxation influences consumption and capital formation even if the tax rate is constant. Proposition 6 states the necessary and sufficient conditions for optimal taxation. If the decentralized economy's effective intertemporal elasticity of substitution is less than that of the socially planned economy, then the *effective* preference for smoothing consumption over time is too strong. Consequently, too much consumption is shifted from the future to the present. This misallocation can be avoided either by subsidizing capital or by taxing consumption in such a way that the tax rate falls over time. Both policy measures lower the price of future consumption in terms of present consumption. On the other hand, if the decentralized economy's effective intertemporal elasticity of substitution exceeds that of the socially planned economy, then the *effective* willingness to postpone consumption is too high. Consequently, too much consumption is shifted from the present to the future. In this situation the government must raise the price of future consumption in terms of present consumption by either taxing capital or by taxing consumption in such a way that the tax rate increases over time. These results will be illustrated by two specific examples.

The rest of the paper is organized as follows. Section 5 contains some brief concluding remarks and the paper closes with an mathematical appendix in which the most important proofs and derivations are given.

2. The Model and the Decentralized Solution

We begin by assuming that the economy is populated by a very large number of identical, infinitely-lived individuals. For simplicity, we specify that the population size remains constant over time. As is common practice in the Ramsey model, we restrict attention to the case of perfect foresight. The representative individual wishes to maximize discounted intertemporal utility as represented by $\int_0^\infty e^{-rt} U\left(c\left(t\right), C\left(t\right)\right) dt$, where r is the rate of time preference (which is assumed to be constant) and where U denotes the instantaneous utility function. Following Gali (1994) and Harbaugh (1996), we assume that the instantaneous utility of the representative individual depends not only on own consumption, c, but also on the average consumption in the economy, C. This specification of U allows to introduce consumption externalities into the standard Ramsey model in a simple but still quite general way. A special case arises if U takes the form $U\left(c, C\right) = V\left(c, z\right)$, where $z \equiv c/C$. This specification assumes that instantaneous utility depends both on absolute consumption, c, and relative consumption, z ($\equiv c/C$), where the latter determines the individual's status in the society.[3] We will subsequently refer to this as the "relative consumption case".

Gali (1994), Harbaugh (1996), and Rauscher use the simplifying assumption that the instantaneous utility functions U and V take particular functional forms. The

[3] In contrast to Frank (1985), we do not distinguish between positional and non-positional goods in determining status.

specifications used by Gali and Harbaugh will be described below, [see (20)]. In Rauscher the following additively separable preferences are assumed:[4]

$$V(c,z) = \frac{1}{1-1/\beta}\left(c^{1-1/\beta}-1\right) + s(z), \quad \beta > 0, \quad s' > 0, \quad s'' < 0. \tag{1}$$

One of the main objectives of Rauscher's paper is to compare economies in which conspicuous consumption plays a role to economies without status competition, [see propositions 1, 2, and 3 in Rauscher]. In this context he considers externality-free economies in which the standard instantaneous utility function $u(c)$ equals $u(c) = (1-1/\beta)^{-1}\left(c^{1-1/\beta}-1\right)$. In contrast, we will consider quite general specifications of U (resp. V) and u. This will make it possible to both generalize existing results and to derive new insights into the role of consumption externalities. Nevertheless, in order to ensure that the individual's optimization problem is well-behaved, we must impose several restrictions on preferences. First, we assume that U exhibits positive and diminishing marginal utility of own consumption c:

$$U_c(c,C) > 0, \quad U_{cc}(c,C) < 0. \tag{2}$$

In terms of the marginal utility of average consumption we allow for both $U_C < 0$, "negative spillovers", and $U_C > 0$, "positive spillovers". In the latter case we can think of private consumption as possessing some public good features. The second order partial derivative U_{cC} may be of either sign.[5] In a symmetric equilibrium in which identical individuals choose identical consumption levels, the instantaneous utility of each individual is given by $\tilde{U}(c) \equiv U(c,c)$. We assume further that $\tilde{U}(c)$ is increasing in c and strictly concave:

$$\tilde{U}'(c) = U_c(c,c) + U_C(c,c) > 0, \tag{3}$$

$$\tilde{U}''(c) = U_{cc}(c,c) + 2U_{cC}(c,c) + U_{CC}(c,c) < 0. \tag{4}$$

Condition (3), $\tilde{U}'(c) > 0$, ensures that the instantaneous utility of each individual rises if the economy moves from one symmetric situation to another in which each individual has a higher consumption level. Condition (4) implies that the marginal utility $\tilde{U}'(c)$ diminishes as c increases. Obviously, (3) is never violated in the case of positive spillovers, $U_C > 0$. If, however, $U_C < 0$, (3) requires the negative spillover effect caused by the increase in average consumption to be more than offset by the positive effect resulting from the rise in own consumption. With

[4] In Rauscher the paramter β is replaced by σ. We will use σ below to denote general expressions for the elasticity of substitution.

[5] For a discussion of the role of U_{cC} at the individual level, see Hof (1999a).

respect to the relative consumption case, the corresponding curvature conditions on V are

$$V_c(c,z) > 0, \quad V_{cc}(c,z) < 0, \quad V_z(c,z) > 0, \tag{5}$$

$$V_{cc}(c,z) + 2c^{-1}zV_{cz}(c,z) + c^{-2}z^2V_{zz}(c,z) < 0. \tag{6}$$

Condition (5) implies that the marginal utility of V with respect to both absolute consumption c and relative consumption z is positive. In addition, the marginal utility of absolute consumption decreases as c increases. Assumption (6) corresponds to the relative consumption version of (4).

Turning to the flow budget constraint of the representative consumer-producer, we follow Rauscher in assuming that physical capital does not depreciate. Abstracting from a public sector, the flow budget constraint equals

$$\dot{k} = f(k) - c, \tag{7}$$

where k and $f(k)$ denote the representative individual's capital stock and flow of output, respectively. We assume that the production function f exhibits the following standard properties: $f'(k) > 0$, $f''(k) \leq 0$, $f(0) = 0$, and $f(k) \rightarrow \infty$ as $k \rightarrow \infty$. We will discuss below the properties required for endogenous growth.

The representative individual chooses $c(t)$ to maximize discounted intertemporal utility, given by $\int_0^\infty e^{-rt}U(c(t), C(t))\,dt$, subject to the flow budget constraint (7), and the initial condition $k(0) = k_0 > 0$. A crucial feature of this optimization problem is that the representative individual takes $C(t)$ as given when choosing $c(t)$. In other words, each individual is small enough to neglect his own contribution to the average consumption level in the economy. We solve the optimization problem by applying optimal control theory.[6] The current-value Hamiltonian is given by $H = U(c,C) + \lambda[f(k) - c]$, where the costate variable λ denotes the current shadow price of capital. The necessary optimality conditions, $H_c = 0$ and $\dot{\lambda} = r\lambda - H_k$, can be written as

$$U_c(c,C) - \lambda = 0, \tag{8}$$

$$\dot{\lambda} = -\lambda \cdot [f'(k) - r], \tag{9}$$

with the transversality condition given by

$$\lim_{t\rightarrow\infty} e^{-rt}\lambda k = 0. \tag{10}$$

[6] Following Barro and Sala-i-Martin (1995), we will restrict our attention to cases in which attainable utility is bounded and both consumption and capital are strictly positive along the optimal paths.

The assumptions made so far ensure that if $(c(t), k(t))$ satisfies the transversality condition (10) – in addition to the flow budget constraint (7), the initial condition $k(0) = k_0$, the necessary optimality conditions (8) and (9) – then it is an optimal path (a proof is available from the authors on request).

In a symmetric macroeconomic equilibrium identical individuals make identical choices, so that $c = C$ and $z \equiv c/C = 1$. It will be useful to present this (symmetric) decentralized solution in such a way that comparisons with the standard Ramsey model can be made at a glance. Recall that in the standard model overall utility of the representative consumer-producer is given by $\int_0^\infty e^{-rt} u(c(t))\, dt$, where $u(c)$ is the standard instantaneous utility function with $u' > 0$, $u'' < 0$. It is well known that in the absence of any government intervention the dynamic evolution of c and k is governed by the capital accumulation equation, $\dot{k} = f(k) - c$, and the usual Euler equation

$$\frac{\dot{c}}{c} = \sigma(c)\left[f'(k) - r\right], \qquad \sigma(c) \equiv -\frac{u'(c)}{cu''(c)}, \tag{11}$$

where $\sigma(c)$ denotes the standard intertemporal elasticity of substitution. The initial condition on the capital stock is $k(0) = k_0$, while the transversality condition can be written as

$$\lim_{t\to\infty} \left\{ k(t) \cdot \exp\left[-\int_0^t f'(k(v))\, dv \right] \right\} = 0. \tag{12}$$

The decentralized solution of the model with consumption externalities can be represented analogously.[7] While the capital accumulation equation, the initial condition, and the transversality condition are identical to their standard versions, the standard Euler equation is replaced with

$$\frac{\dot{c}}{c} = \sigma^{ed}(c)\left[f'(k) - r\right], \tag{13}$$

where

$$\sigma^{ed}(c) \equiv -\frac{U_c(c,c)}{c\left[U_{cc}(c,c) + U_{cC}(c,c)\right]} \tag{14}$$

is the decentralized economy's *effective* intertemporal elasticity of substitution (the superscripts e and d stand for "effective" and "decentralized", respectively). In the relative consumption case, in which $U(c,C) = V(c, c/C)$, (14) becomes

$$\sigma^{ed}(c) = -\frac{V_c(c,1) + c^{-1}V_z(c,1)}{c\left[V_{cc}(c,1) + c^{-1}V_{cz}(c,1) - c^{-2}V_z(c,1)\right]}. \tag{15}$$

[7] For a proof of the following results see the appendix, subsection C, in which symmetric macroeconomic equilibria with taxes and transfers are analyzed. The results for the decentralized economy without a public sector are simply obtained by setting taxes and transfers to zero.

In the following we will assume that

$$U_{cc}(c,c) + U_{cC}(c,c) < 0, \tag{16}$$

in order to guarantee that $\sigma^{ed}(c) > 0$. In the relative consumption case (16) becomes

$$V_{cc}(c,1) + c^{-1}V_{cz}(c,1) - c^{-2}V_z(c,1) < 0. \tag{17}$$

A detailed economic interpretation of (16) is given in Hof (1999a). Loosely speaking, this condition ensures that individuals do not "overreact" to changes in average consumption and that equilibrium consumption still depends negatively on λ, as in the standard model. With these results, we can state the following proposition.

Proposition 1. ***(Observational equivalence)****: Consider two decentralized economies – abstracting from a public sector – which differ only with respect to the instantaneous utility function. Assume that one of these economies exhibits consumption externalities so that the instantaneous utility function of the representative agent is given by $U(c,C)$, while the other economy is externality-free so that the instantaneous utility function can be written as $u(c)$. Let decentralized solutions exist for both economies.*

(a) If $\sigma^{ed}(c) = \sigma(c)$, then the two economies are observationally equivalent in the sense that their decentralized solutions are identical.

(b) $\sigma^{ed}(c) = \sigma(c)$ holds if the instantaneous utility functions $U(c,C)$ and $u(c)$ satisfy the condition that

$$u'(c) = \chi \cdot U_c(c,c), \tag{18}$$

where χ is an arbitrary positive constant. In the relative consumption case this condition is written as

$$u'(c) = \chi \cdot \Big[V_c(c,1) + c^{-1}V_z(c,1)\Big]. \tag{19}$$

The proof of part a is obvious from the results described above, since both economies have the same initial condition, $k(0) = k_0$, the same transversality condition, (12), and the same capital accumulation equation, $\dot{k} = f(k) - c$. Hence, their decentralized solutions are identical if and only if their Euler equations are identical. From (11) and (13), it follows that this is the case if $\sigma^{ed}(c) = \sigma(c)$. For the proof of part b see the appendix, subsection A.

We will next provide two illustrations of the property of observational equivalence. First, if the utility function $V(c,z)$ takes the form used by Rauscher, then $V_c(c,1) = c^{-1/\beta}$ and $V_z(c,1) = s'(1)$. Substituting these expressions into (19), we obtain $u'(c) = \chi\left[c^{-1/\beta} + c^{-1}s'(1)\right]$. Integration of this condition with respect to c yields that the economy with consumption externalities studied by Rauscher is observationally equivalent to any externality-free economy in which the instantaneous utility function takes the form

$$u(c) = \frac{\chi}{1-1/\beta}\left(c^{1-1/\beta} - 1\right) + \chi s'(1)\ln c + \kappa, \qquad \chi > 0,$$

where κ is an arbitrary constant. Next, we let preferences assume a functional form that encompasses the specifications used by Harbaugh (1996) and Gali (1994):[8]

$$U(c,C) = \frac{1}{1-\alpha}\left(cC^{-\gamma}\right)^{1-\alpha}, \quad \alpha > 0,\ \gamma < 1,\ \gamma + \alpha(1-\gamma) > 0. \tag{20}$$

By substituting $U_c(c,c) = c^{-\gamma-\alpha(1-\gamma)}$ into (18) and integrating the resulting expression with respect to c, we obtain the following externality-free utility function that is equivalent to the Harbaugh-Gali type:

$$u(c) = \frac{\chi}{1-\theta}\left(c^{1-\theta} - 1\right) + \kappa, \qquad \theta = \gamma + \alpha(1-\gamma), \quad \chi > 0.$$

The property of observational equivalence implies that some of the standard results of the Ramsey model can be readily applied to the modified model with consumption externalities. We state here two propositions that describe the properties of the dynamic equilibria in the neoclassical and endogenous growth cases, (the proofs are available from the authors on request).

Proposition 2. *If the production function satisfies the conditions*

$$f'' < 0 \qquad \text{and} \qquad \lim_{k\to\infty} f'(k) < r < \lim_{k\to 0} f'(k),$$

then the decentralized solution exhibits the following properties:

[8] Technically, we should write (20) as $U(c,C) = (1-\alpha)^{-1}\left[\left(cC^{-\gamma}\right)^{1-\alpha} - 1\right]$ in order to include the (logarithmic) case in which $\alpha = 1$. There are negative (resp. positive) consumption spillovers if γ is positive (resp. negative). Harbaugh's specification, which rules out positive consumption spillovers, is obtained with $0 < \gamma < 1$. Gali's specification takes the form $U(c,C) = (1-\alpha)^{-1}c^{1-\alpha}C^{\eta\alpha}$, with $\alpha > 0$ and $\eta < 1$. This functional form is recovered from (20) by setting $\gamma = -(1-\alpha)^{-1}\eta\alpha$. In the context of our model we must impose an additional restriction, namely $(1-\alpha)^{-1}\eta\alpha > -1$, in order to ensure that $\gamma < 1$ holds as required by (20).

(a) *The economy does not generate endogenous, steady state growth. The steady state* (k^*, c^*)*, uniquely determined by* $c^* = f(k^*)$ *and* $f'(k^*) = r$*, is a saddlepoint, with the corresponding stable arm satisfying the transversality condition. The specification of the instantaneous utility function* U *only influences the transitional dynamics but not the steady state* (k^*, c^*).

(b) *In the neighborhood of the steady state, the optimal paths of* k *and* c *can be approximated by*

$$k(t) = k^* + (k_0 - k^*) e^{\mu_1 t}, \qquad c(t) = c^* + (k_0 - k^*) \mu_2 e^{\mu_1 t},$$

where $\mu_1 < 0$ *and* $\mu_2 > 0$ *denote the roots of the characteristic polynomial such that* $\mu_1 + \mu_2 = r$ *and* $\mu_1 \mu_2 = c^* \sigma^{ed}(c^*) f''(k^*) < 0$*. The difference* $(k(t) - k^*)$ *declines at the rate* $|\mu_1|$*, which, in turn, depends positively on* $\sigma^{ed}(c^*)$*. The stable arm,* $c = c^* + \mu_2 (k - k^*)$*, is positively sloped in the* (k, c) *plane, where its slope, given by* μ_2*, increases with* $\sigma^{ed}(c^*)$.

Part a of our proposition 2 shows that Rauscher's proposition 1, (which says that if the production function $f(k)$ satisfies the standard neoclassical properties, then conspicuous consumption does not affect the long-run steady state), is robust with respect to the specification of the instantaneous utility function V. The main message of part b is that the higher is the effective intertemporal elasticity of substitution, the lower is the level of consumption at $t = 0$, while the higher is the speed of convergence towards the steady state. The next proposition states necessary conditions for the occurrence of long-run endogenous growth and describes how it depends on the effective intertemporal elasticity of substitution.

Proposition 3. *Let the decentralized solution exist and assume that it exhibits endogenous steady-state growth in the sense that* $\lim_{t\to\infty} \left(\dot{k}(t)/k(t)\right) > 0$ *and* $\lim_{t\to\infty} (c(t)/k(t)) > 0$*. It is then the case that*

$$\lim_{c\to\infty} \sigma^{ed}(c) > 0, \tag{21}$$

$$\left[1 - \frac{1}{\lim_{c\to\infty} \sigma^{ed}(c)}\right] \lim_{k\to\infty} f'(k) < r < \lim_{k\to\infty} f'(k) \tag{22}$$

holds and that the transversality condition (12) is satisfied. Moreover, it is true that

$$\lim_{t\to\infty} \left(\frac{\dot{k}(t)}{k(t)}\right) = \lim_{t\to\infty} \left(\frac{\dot{c}(t)}{c(t)}\right) = \lim_{c\to\infty} \sigma^{ed}(c) \cdot \left[\lim_{k\to\infty} f'(k) - r\right]. \tag{23}$$

The case of endogenous growth will be illustrated by considering the model in which the instantaneous utility function is of the generalized Harbaugh-Gali type given in (20), and the production function is of the AK-type, i.e., $f(k) = ak$. We also assume that $a(1-\alpha)(1-\gamma) < r < a$ holds. Since the effective intertemporal elasticity of substitution $\sigma^{ed}(c)$ in this case is given by

$$\sigma^{ed}(c) = \frac{1}{\gamma + \alpha(1-\gamma)} > 0, \tag{24}$$

it is easily verified that the conditions (21) and (22) in proposition 3 are satisfied. The decentralized solution also exhibits the property that the growth rates of c and k are constant over time and equal to:

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{a-r}{\gamma + (1-\gamma)\alpha}. \tag{25}$$

In addition, the transversality condition holds if and only if the initial level of consumption, $c(0)$, is chosen according to:

$$c(0) = \left[\frac{r - a(1-\alpha)(1-\gamma)}{\gamma + (1-\gamma)\alpha}\right] k_0 > 0. \tag{26}$$

Differentiating the right-hand sides of (24), (25), and (26) with respect to γ, we obtain the following results. If $\alpha > 1$, then both $\sigma^{ed}(c)$ and the common growth rate of c and k depend positively on γ, while the initial level of consumption, $c(0)$, depends negatively upon γ. If $\alpha < 1$, then the opposite results hold. If we follow Harbaugh in restricting attention to $0 < \gamma < 1$, then (20) is compatible with a relative consumption interpretation since we can rewrite (20) as $U(c, C) = V(c, c/C)$ with $V(c, z) \equiv (1-\alpha)^{-1}\left(c^{1-\gamma} z^{\gamma}\right)^{1-\alpha}$. The parameter γ can be interpreted as measure of the "degree of status-consciousness". The case in which $\gamma = 0$ corresponds to a situation in which status does not matter at all, while $\gamma \to 1$ describes the limiting case in which status as measured by relative consumption is the only thing the consumer values. Thus, our results imply that if the instantaneous utility function is of the Harbaugh type, then the relationship between the decentralized growth rate and the degree of status-consciousness is ambiguous. A similar ambiguity is described by Rauscher in his proposition 2. He compares a status-seeking society in which $V(c, z)$ takes the form given by (1) to a society without status-seeking behavior in which $u(c)$ equals $u(c) = (1 - 1/\beta)^{-1}\left(c^{1-1/\beta} - 1\right)$ and finds that if the production function is of the AK-type, then an economy which is populated by status-seeking agents grows faster (resp. slower) if β is less (resp. greater) than unity.

Proposition 3 of our paper can be further used to illustrate that Rauscher's proposition 3 – *"Along an endogenous growth path, the long-term growth rate of an economy with status competition will not be less than that of an economy without status competition."* – is *not* robust with respect to the specification of the preferences. From (23) and its analogue for the externality-free economy, (which is simply obtained by replacing $\sigma^{ed}(c)$ by $\sigma(c)$), it follows that the long-term growth rate of an economy with consumption externalities will not be less than that of an externality-free economy if and only if $\lim_{c\to\infty} \sigma^{ed}(c) \geq \lim_{c\to\infty} \sigma(c)$.

3. The Social Planner's Solution

From a normative point of view the relevant question is not whether economies with consumption spillovers grow fast or slowly, but whether they grow too fast or too slowly compared to the socially optimal growth rate. To see whether the existence of consumption spillovers leads to social nonoptimality, we follow the usual practice of comparing the decentralized solution to the results from a social planner's problem. Suppose that there exists a benevolent social planner who dictates the choices of consumption over time and who seeks to maximize the welfare of the representative individual. Since individuals are identical, we assume that the social planner assigns the same consumption level to each agent so that $c = C$ holds. Consequently, the instantaneous utility of each individual is given by $\tilde{U}(c) \equiv U(c, c)$. The social planner's optimization problem is then to choose $c(t)$ so as to maximize $\int_0^\infty e^{-rt}\tilde{U}(c)\,dt$, subject to the economy's resource constraint, $\dot{k} = f(k) - c$, and the initial condition $k(0) = k_0$. The necessary optimality conditions are given by

$$\tilde{U}'(c) = U_c(c, c) + U_C(c, c) = \lambda \tag{27}$$

and $\dot{\lambda} = -\lambda \cdot [f'(k) - r]$, where the latter condition is identical to (9). The transversality condition is given by (10). Using (27) and $\dot{\lambda} = -\lambda \cdot [f'(k) - r]$, we obtain the following Euler equation for consumption

$$\frac{\dot{c}}{c} = \sigma^{es}(c)\left[f'(k) - r\right], \tag{28}$$

where

$$\sigma^{es}(c) = -\frac{\tilde{U}'(c)}{c\tilde{U}''(c)} = -\frac{U_c(c, c) + U_C(c, c)}{c\left[U_{cc}(c, c) + 2U_{cC}(c, c) + U_{CC}(c, c)\right]} > 0 \tag{29}$$

will be called the socially planned economy's *effective* intertemporal elasticity of substitution (the superscript "s" stands for "social"). From the conditions (3) and (4), it follows that $\sigma^{es}(c) > 0$. In the relative consumption case equation (29) simplifies to

$$\sigma^{es}(c) = -\frac{V_c(c,1)}{cV_{cc}(c,1)} > 0. \tag{30}$$

The capital accumulation equation, the initial condition, and the transversality condition are identical to the versions that apply to the decentralized economy, i.e., they are given by $\dot{k} = f(k) - c$, $k(0) = k_0$, and (12). The Euler equations for consumption for the decentralized and socially planned economies, (13) and (28), respectively, are identical if and only if the effective intertemporal elasticities of substitution, $\sigma^{ed}(c)$ and $\sigma^{es}(c)$, are also identical. This leads immediately to the proposition 4.

Proposition 4. ***(Social optimality of the decentralized solution)***

(a) Assume that both the decentralized solution and the socially planned solution exist. If the corresponding effective intertemporal elasticities of substitution coincide, i.e., $\sigma^{ed}(c) = \sigma^{es}(c)$, then the decentralized solution coincides with the socially planned solution in spite of the existence of consumption spillovers.[9]

(b) The condition $\sigma^{ed}(c) = \sigma^{es}(c)$ is equivalent to

$$\frac{|U_C(c,c)|}{U_c(c,c)} = \delta, \tag{31}$$

where δ is an arbitrary positive constant. If there are negative spillovers, then assumption (3) requires that $0 < \delta < 1$.

(c) In the relative consumption case the condition $\sigma^{ed}(c) = \sigma^{es}(c)$ is equivalent to

$$\frac{V_z(c,1)}{cV_c(c,1)} = \eta, \tag{32}$$

where η is an arbitrary positive constant.

[9] If $\sigma^{ed}(c) = \sigma^{es}(c)$, then the decentralized solution and the socially planned solution will have the same rate of growth in the shadow price of capital, $\dot{\lambda}(t)/\lambda(t)$. The time paths of $\lambda(t)$ will, however, deviate. If there are negative consumption externalities, then the shadow price of capital in the decentralized economy exceeds that of the socially planned economy at any time t. If there are positive consumption externalities, then the opposite is the case.

Part a is evident from the considerations indicated above. For the remainder of proposition 4, see the proof in the appendix, subsection B. The conditions given in part b of proposition 4 can be interpreted graphically. If the iso-utility curves, $U(c, C) =$ constant, are plotted in the (C, c) plane, then their slope is given by $-U_C(c, C)/U_c(c, C)$. Obviously, the iso-utility curves are positively (resp. negatively) sloped in the presence of negative (resp. positive) consumption spillovers. From part b of proposition 4 follows that $\sigma^{ed}(c) = \sigma^{es}(c)$ holds for all c if and only if the slope of the iso-utility curves is constant along the 45 degree line. Moreover, condition (3) requires that this constant slope be less than unity in the presence of negative spillovers.

The next proposition shows that if the decentralized solution is socially optimal, then this property is robust with respect to transformations of the underlying instantaneous utility function.

Proposition 5. ***(Transformations and social optimality):*** *Let $G(\cdot)$ denote a transformation function and assume that the decentralized solutions corresponding to $U(c, C)$ and $G[U(c, C)]$ exist.*

(a) *If $U(c, C)$ satisfies the conditions (2), (3), (4), and (16), then the property that the transformation G is strictly increasing and concave, $G' > 0$ and $G'' \leq 0$, is also sufficient for $G[U(c, C)]$ to satisfy these four conditions.*

(b) *Assume that $G' > 0$ and $G'' \leq 0$. If the decentralized solution corresponding to $U(c, C)$ is socially optimal, then the decentralized solution corresponding to $G[U(c, C)]$ is socially optimal as well.*

The proof of proposition 5 is available on request from the authors. We will next show, by giving six examples, that there exist several specifications of the instantaneous utility function in which the decentralized solution coincides with the socially planned solution.[10]

Example 1: If $U(c, C) = V(c, c/C)$ and V takes the form used by Rauscher, equation (1), then $V_z(c, 1)/[cV_c(c, 1)] = s'(1)/c^{1-1/\beta} > 0$. The condition for social optimality given in part c of proposition 4, (32), requires that the right hand side of this equation is independent of c. Obviously, this is fulfilled if and only if $\beta = 1$ which implies that $\sigma^{es}(c) = \sigma^{ed}(c) = 1$.

[10] Hof (1999b) has shown, however, that if labor supply is endogenously determined, then the decentralized solution always departs from its Pareto optimal counterpart in the relative consumption case.

Example 2: If $U(c, C)$ is of the generalized Harbaugh-Gali type, equation (20), then $|U_C(c, c)| / U_c(c, c) = |\gamma|$. If there are negative consumption spillovers, i.e., if γ is positive, then $|\gamma| = \gamma$, where $\gamma < 1$ holds by assumption. Thus, the conditions for social optimality given in part b of proposition 4 are satisfied for all admissible values of α and γ. Alternatively, we can show that $\sigma^{es}(c) = \sigma^{ed}(c)$, where $\sigma^{ed}(c)$ is given by (24). This result implies that a decentralized economy with Harbaugh-Gali preferences may either grow faster or more slowly, depending on the values of α and γ, but that it will never grow too fast or too slowly.

The remaining four examples employ general types of preferences that can be represented as transformations of underlying additively separable or multiplicatively separable utility functions with consumption externalities. For convenience, we restrict our attention to transformation functions G that are strictly increasing and concave.

Example 3 (General type I): Assume that U takes the form

$$U(c, C) = G(g(c) - \gamma g(C) + \chi), \quad \gamma < 1,$$

where γ and χ are constants. In order to ensure that U is well-behaved, i.e., that U satisfies the conditions (2), (3), (4), and (16), we must introduce additional restrictions on g. Sufficient (but not necessary) restrictions are given by $g' > 0$ and $g'' < 0$. Note that this specification yields positive consumption spillovers if and only if γ is negative. Since $|U_C(c, c)| / U_c(c, c) = |\gamma|$, where $|\gamma|$ is less than unity in the case of negative consumption spillovers, the conditions for social optimality given in part b of proposition 4 are satisfied.

Example 4 (General type II): If U takes the form

$$U(c, C) = G\left(\chi \cdot g(c) [g(C)]^{-\gamma}\right), \quad \gamma < 1,$$

then the additional assumptions that $\chi > 0$, $\gamma > 0$, $g(x) > 0$ for $x > 0$, $g' > 0$, and $g'' \leq 0$ are sufficient (but not necessary) for U to be well-behaved. It is easily verified that $|U_C(c, c)| / U_c(c, c) = |\gamma|$ holds, where $|\gamma|$ is less than unity in the case of negative consumption spillovers. Hence, the conditions for social optimality given in part b of proposition 4 are satisfied.

Example 5 (General type III): Assume that V takes the form

$$V(c, z) = G(\chi + \gamma \ln c + s(z)), \quad \gamma > 0, \quad s' > 0,$$

where χ and γ are constants. In order to ensure that V is well-behaved, i.e., that V satisfies the conditions (5), (6), and (17), we introduce an additional condition on s. A sufficient (but not necessary) restriction is given by $s'' \leq 0$. The condition for social optimality, (32), is satisfied since $V_z(c,1) / [cV_c(c,1)] = \gamma^{-1}s'(1) > 0$.

Example 6 (General type IV): If V takes the form

$$V(c,z) = G(\chi c^{\gamma} s(z)), \quad \gamma > 0,\ s(z) > 0,\ s' > 0,$$

where χ and γ are constants, the following additional restrictions are sufficient (but not necessary) for V to be well-behaved: $\chi > 0$, $\gamma \leq 1$ (with strict inequality if $G'' = 0$ holds), $s'' \leq 0$, and $(1-\gamma)\gamma s(z) - 2\gamma z s'(z) - z^2 s''(z) > 0$. The condition for social optimality, (32), is satisfied since $V_z(c,1) / [cV_c(c,1)] = \gamma^{-1}s'(1)/s(1) > 0$.

4. Optimal Taxation in the Decentralized Economy

If consumption spillovers create distortions in the decentralized economy, then a welfare maximizing government would induce the private sector to reach the social optimum by designing an optimal tax policy. We now introduce a tax on consumption, a tax on capital, and lump-sum transfers. Under these assumptions, the flow budget constraint of the representative consumer-producer is

$$\dot{k} = f(k) - (1+\tau_c)c - \tau_k k + T, \tag{33}$$

where τ_c, τ_k, and T denote the tax rate on consumption, the tax rate on capital, and lump-sum transfers, respectively. The representative individual chooses $c(t)$ in order to maximize overall utility, $\int_0^{\infty} e^{-rt} U(c(t), C(t))\, dt$, subject to (33). A crucial feature of this optimization problem is that the representative individual takes not only $C(t)$, but also $\tau_c(t)$, $\tau_k(t)$, and $T(t)$ as given when choosing $c(t)$. The current value Hamiltonian of this optimization problem is given by $H = U(c,C) + \lambda[f(k) - (1+\tau_c)c - \tau_k k + T]$. The necessary optimality conditions, $H_c = 0$ and $\dot{\lambda} = r\lambda - H_k$, become

$$U_c(c,C) - \lambda \cdot (1+\tau_c) = 0, \tag{34}$$

$$\dot{\lambda} = -\lambda \cdot [f'(k) - \tau_k - r]. \tag{35}$$

The transversality condition is given, as before, by (10). If $(c(t), k(t))$ satisfies (33), (34), (35), (10), and the initial condition $k(0) = k_0$, then it is an optimal path. In the following we restrict attention to a symmetric macroeconomic equilibrium in which tax revenues are fully rebated.[11] In such a situation we have $c = C$

[11] The proofs of the following results are given in the appendix, subsection C.

and $\tau_c c + \tau_k k = T$. Under these conditions, the dynamic evolution of c and k is governed by the capital accumulation equation, $\dot{k} = f(k) - c$, and the tax-adjusted Euler equation for consumption

$$\frac{\dot{c}}{c} = \sigma^{ed}(c)\left[f'(k) - r - \left(\tau_k + \frac{\dot{\tau}_c}{1+\tau_c}\right)\right] \tag{36}$$

with the transversality condition now equal to

$$\lim_{t\to\infty}\left\{k(t)\cdot\exp\left[-\int_0^t \left[f'(k(v)) - \tau_k(v)\right]dv\right]\right\} = 0 \tag{37}$$

Note that if the consumption tax rate τ_c is constant over time, then the time paths of $c(t)$ and $k(t)$ are independent of τ_c. Such a tax policy only influences the level of the shadow price of capital, $\lambda(t)$, but leaves its rate of change, $\dot{\lambda}(t)/\lambda(t)$, unaffected. Loosely speaking, the reason for this ineffectiveness result is that a constant tax rate does not affect the price of future consumption in terms of present consumption.[12] The crucial question of this section is whether the social optimum can be attained in a decentralized economy by optimally choosing the time paths of the tax rates $\tau_k(t)$ and $\tau_c(t)$. In the following, it will be convenient to denote the decentralized solution and the socially optimal solution of (c, k) by (k^d, c^d) and (k^s, c^s), respectively. Proposition 6 gives necessary and sufficient conditions for optimal capital and consumption taxation.

Proposition 6. ***(Optimal tax policy)****: Assume that the socially planned solution $(k^s(t), c^s(t))$ exists.*

(a) *(Necessary conditions): If there exist time paths of $\tau_k(t)$ and $\tau_c(t)$ such that the decentralized solution $(k^d(t), c^d(t))$ is identical with the socially planned solution $(k^s(t), c^s(t))$, then*

$$\tau_k(t) + \frac{\dot{\tau}_c(t)}{1+\tau_c(t)} = \left[\frac{\sigma^{ed}(c^s(t)) - \sigma^{es}(c^s(t))}{\sigma^{ed}(c^s(t))}\right]\left[f'(k^s(t)) - r\right]. \tag{38}$$

(b) *(Sufficient conditions): If the time paths of $\tau_k(t)$ and $\tau_c(t)$ satisfy (38) and the time paths of $\tau_k(t)$ and $k^s(t)$ satisfy the tax-adjusted transversality condition of the decentralized economy (37), then decentralized solution exists and $(k^d(t), c^d(t)) = (k^s(t), c^s(t))$.*

[12] In the endogenous employment case, Hof (1999b) has shown that a constant consumption tax does influence the economy's equilibrium dynamics and can be used as an optimal policy tool.

For the proof see the appendix, subsection D. The qualitative implications of the optimal tax policy (38) can be interpreted as follows. If $\sigma^{ed} < \sigma^{es}$, then the *effective* preference for smoothing consumption over time is too strong. Consequently, too much consumption is shifted from the future to the present. This misallocation can be avoided either by subsidizing capital or by taxing consumption so that the tax rate falls over time (i.e., $\dot{\tau}_c < 0$). Both policy measures lower the price of future consumption in terms of present consumption. On the other hand, if $\sigma^{ed} > \sigma^{es}$, then the *effective* willingness to postpone consumption is too high. Consequently, too much consumption is shifted from the present to the future. In this situation a welfare maximizing government must raise the price of future consumption in terms of present consumption, either through the taxation of capital or through the taxation of consumption where the tax rate increases over time. We can next illustrate proposition 6 by two examples.

We consider first the case in which $V(c, z)$ is given by (1), the production function is of the AK-type, $f(k) = ak$, and the condition $a - (a/\beta) < r < a$ holds. Rauscher's specification implies that

$$\sigma^{es}(c) = \beta, \qquad \sigma^{ed}(c) = \left[\frac{c^{1-1/\beta} + s'(1)}{c^{1-1/\beta} + \beta s'(1)}\right] \sigma^{es}(c). \tag{39}$$

The socially planned solution is given by $c^s(t) = [\beta r + a(1-\beta)] k_0 e^{\beta(a-r)t}$ and $k^s(t) = k_0 e^{\beta(a-r)t}$. The assumption that $a - (a/\beta) < r$ ensures that (c^s, k^s) satisfies the transversality condition for the socially planned economy, while the assumption that $r < a$ ensures that the growth rates of per capita consumption and per capita capital are positive. The necessary condition for optimal taxation (38) then becomes

$$\tau_k(t) + \frac{\dot{\tau}_c(t)}{1 + \tau_c(t)} = \frac{(1-\beta) s'(1)(a-r)}{[(\beta r + a(1-\beta)) k_0]^{(\beta-1)/\beta} e^{-(1-\beta)(a-r)t} + s'(1)}. \tag{40}$$

We will now consider the case in which the government only imposes a capital tax, so that $\tau_c = 0$. It can be shown that the time paths of $\tau_k(t)$ and $k^s(t)$ satisfy the transversality condition of the decentralized economy.[13] Hence, from part b of proposition 6, it follows that the social optimum can be attained in the decentralized economy by setting $\tau_c(t) = 0$ and choosing $\tau_k(t)$ according to (40). This optimal capital tax has the following properties:

$$\operatorname{sgn}(\tau_k) = \operatorname{sgn}(1-\beta), \quad \dot{\tau}_k(t) > 0,$$

[13] A proof is available from the authors on request.

$$\lim_{t\to\infty}\tau_k(t)=\begin{cases}(1-\beta)(a-r)>0 & \text{if } \beta<1,\\ 0 & \text{if } \beta>1.\end{cases}$$

If $\beta<1$, then capital should be taxed, where the optimal tax rate increases over time and converges to a finite level. If, on the other hand, $\beta>1$, then capital should be subsidized, where the optimal rate of subsidy declines over time and converges to zero. What is the economic interpretation of this optimal tax policy?

From (39) follows that if $\beta<1$, then

$$\sigma^{ed}(c)>\beta=\sigma^{es}(c),\qquad \frac{\partial\sigma^{ed}(c)}{\partial c}>0,\qquad \lim_{c\to\infty}\sigma^{ed}(c)=1$$

Since $\sigma^{ed}(c)>\sigma^{es}(c)$, the *effective* willingness to postpone consumption is too high. In the absence of any government intervention we have

$$c^d(0)<c^s(0),\qquad \frac{\dot{c}^d(t)}{c^d(t)}=\sigma^{ed}\left(c^d(t)\right)(a-r)>\beta(a-r)=\frac{\dot{c}^s(t)}{c^s(t)}>0.$$

In other words, in the decentralized economy the initial level of consumption is too low compared to the socially optimal level, while the rate of growth in per capita consumption is too high. Moreover, the difference between the growth rates increases over time and converges to $(1-\beta)(a-r)>0$ as $t\to\infty$. Hence, in order to attain the social optimum, capital has to be taxed, where the tax rate is rising over time and converging to a finite level.

On the other hand, if $\beta>1$, then

$$\sigma^{ed}(c)<\beta=\sigma^{es}(c),\qquad \frac{\partial\sigma^{ed}(c)}{\partial c}>0,\qquad \lim_{c\to\infty}\sigma^{ed}(c)=\beta.$$

Since $\sigma^{ed}<\sigma^{es}$, the *effective* preference for smoothing consumption over time is too strong. If $\tau_k=0$, then

$$c^d(0)>c^s(0),\qquad 0<\frac{\dot{c}^d(t)}{c^d(t)}=\sigma^{ed}\left(c^d(t)\right)(a-r)<\beta(a-r)=\frac{\dot{c}^s(t)}{c^s(t)}.$$

In this case the initial level of consumption is too high in the decentralized economy, while the rate of growth in per capita consumption is too low at any time t. However, the difference between the growth rates decreases over time (in absolute value) and converges to zero as $t\to\infty$. Hence, in order to attain the social optimum capital has to be subsidized, although the rate of subsidy declines over time and converges to zero in the limit.

For the sake of completeness, we also consider the case in which the government imposes a tax only on consumption. The expression for optimal taxation, (40), and the condition $1 + \tau_c(t) > 0$ are satisfied if and only if

$$\tau_c(t) = B \cdot \left\{ [(\beta r + a(1-\beta)) k_0]^{(\beta-1)/\beta} + s'(1) e^{(1-\beta)(a-r)t} \right\} - 1,$$

where B is an arbitrary positive constant, which implies that the optimal time path of the consumption tax rate is not uniquely determined. If $\beta > 1$, then $\dot{\tau}_c(t) < 0$ and $\lim_{t\to\infty} \tau_c(t) < \infty$. On the other hand, if $\beta < 1$, then $\dot{\tau}_c(t) > 0$ and $\tau_c(t) \to \infty$ as $t \to \infty$, (in contrast to the optimal capital tax rate which converges to a finite value).

Finally, consider the case in which $V(c, z)$ takes the form

$$V(c, z) = \frac{1}{1-\theta} \left[(c + \delta z)^{1-\theta} - 1 \right], \qquad \theta > 0, \quad \delta > 0,$$

the production function is of the AK-type, $f(k) = ak$, and $a - a\theta < r < a$ holds. This specification of the utility function implies that

$$\sigma^{es}(c) = \frac{c+\delta}{\theta c}, \qquad \sigma^{ed}(c) = \frac{\theta c}{\theta c + \delta} \sigma^{es}(c) < \sigma^{es}(c).$$

From (38), it is obvious that capital has to be subsidized, since with these preferences the effective decentralized elasticity of substitution is always less than its socially optimal counterpart.[14] For this reason, the *sign* of the optimal capital tax rate – unlike in the previous example – does *not* depend on the parameters of the instantaneous utility function.

5. Concluding Remarks

Rauscher introduced conspicuous consumption into an otherwise standard Ramsey model with exogenous labor supply in order to study the effects of the quest for status – as measured by relative consumption – on economic growth and optimal tax policy. In this paper we generalized Rauscher's results by using general specifications of the instantaneous utility function that encompass the functional forms employed by him and other recent authors. The key concept we used in this paper is that of the "effective" intertemporal elasticity of substitution. It enabled us to give conditions for the observational equivalence between economies with

[14] The explicit solutions for $k^s(t)$, $c^s(t)$, and $\tau_k(t)$ in this case are given in Hof (1999a).

consumption externalities that arise from status-seeking behavior and externality-free economies. Using this concept, we also showed that there exist several types of the instantaneous utility function in which the decentralized solution coincides with the socially planned one in spite of the presence of consumption externalities. Under these types of preferences, decentralized economies populated by status-seeking agents may grow faster or more slowly, but they will never grow too fast or too slowly compared to the social optimum. Dealing with the issue of optimal taxation, we considered both a capital tax and a consumption tax. We showed, with respect to the issue of optimal capital taxation, that whether capital should be optimally taxed or subsidized depends on whether the effective intertemporal elasticity of substitution is larger or smaller than its socially optimal counterpart.

Appendix

A. Proof of Proposition 1 – Part b

Using (11) and (13), the condition that $\sigma(c) = \sigma^{ed}(c)$ can be written as

$$-\frac{u'(c)}{cu''(c)} = -\frac{U_c(c,c)}{c\left[U_{cc}(c,c) + U_{cC}(c,c)\right]}.$$

Using the definitions $\varphi(c) \equiv \ln u'(c)$ and $\phi(c) \equiv \ln U_c(c,c)$, this condition can be rewritten as $\varphi'(c) = \phi'(c)$. Integration with respect to c yields $\varphi(c) = \phi(c) + \kappa$ where κ is an arbitrary constant. Substituting the definitions of ϕ and φ we obtain $\ln u'(c) = \ln U_c(c,c) + \kappa$ which is equivalent to $u'(c) = \chi U_c(c,c)$, where $\chi \equiv e^{\kappa}$ is an arbitrary positive constant. Hence, $\sigma(c) = \sigma^{ed}(c)$ is equivalent to (18).

In the relative consumption case in which U takes the form $U(c,C) = V(c, c/C)$, we have $U_c(c,C) = V_c(c, c/C) + C^{-1}V_z(c, c/C)$ and $U_c(c,c) = V_c(c,1) + c^{-1}V_z(c,1)$. Substituting the latter expression into (18) we obtain (19).

B. Proof of Proposition 4 – Part b and part c

Proof of (b): From (14) and (29) follows that $\sigma^{ed}(c) = \sigma^{es}(c)$ holds if and only if

$$-\frac{U_c(c,c)}{c\left[U_{cc}(c,c) + U_{cC}(c,c)\right]} = -\frac{U_c(c,c) + U_C(c,c)}{c\left[U_{cc}(c,c) + 2U_{cC}(c,c) + U_{CC}(c,c)\right]}.$$

It is easily verified that this condition is equivalent to

$$\frac{U_{Cc}(c,c) + U_{CC}(c,c)}{U_C(c,c)} = \frac{U_{cc}(c,c) + U_{cC}(c,c)}{U_c(c,c)}.$$

Using the definitions $\phi(c) \equiv |U_C(c,c)|$ and $\varphi(c) \equiv U_c(c,c)$, this condition can be rewritten as $\phi'(c)/\phi(c) = \varphi'(c)/\varphi(c)$. Taking into account that $\phi(c) > 0$ and $\varphi(c) > 0$, the latter condition can be rewritten as $d[\ln\phi(c)]/dc = d[\ln\varphi(c)]/dc$. Integration with respect to c yields $\ln\phi(c) = \ln\varphi(c) + \kappa = \ln(\delta\varphi(c))$, with $\delta \equiv e^{\kappa} > 0$, where κ is an arbitrary constant implying that δ is an arbitrary positive constant. Obviously, the latter condition is equivalent to $\phi(c) = \delta\varphi(c)$, which can be rewritten as $|U_C(c,c)|/U_c(c,c) = \delta$. Clearly, this condition is identical to (31).

Next, we show that if there are negative consumption externalities, (i.e., $U_C < 0$), then $\delta < 1$. If $U_C(c,c) < 0$, then condition (3) can be written as $U_c(c,c) - |U_C(c,c)| > 0$, which implies that $\delta = |U_C(c,c)|/U_c(c,c) < 1$.

Proof of (c): In the relative consumption case in which U takes the form $U(c,C) = V(c,c/C)$, we have $U_c(c,c) = V_c(c,1) + c^{-1}V_z(c,1)$ and $U_C(c,c) = -c^{-1}V_z(c,1) < 0$. Substituting these expressions into the general condition for social optimality given by (31) we obtain

$$\frac{|U_C(c,c)|}{U_c(c,c)} = \frac{c^{-1}V_z(c,1)}{V_c(c,1) + c^{-1}V_z(c,1)} = \delta,$$

where δ is an arbitrary positive constant with $\delta < 1$ (due to the fact that there are negative consumption externalities, i.e., $U_C < 0$). Rearranging, we obtain $V_z(c,1)/[cV_c(c,1)] = \eta$, where $\eta \equiv \delta/(1-\delta)$ is an arbitrary positive constant (due to $0 < \delta < 1$). Obviously, this condition is identical to (32).

C. Symmetric Equilibria with Government Intervention

In a symmetric macroeconomic equilibrium identical individuals choose identical consumption levels, implying that $c = C$ holds. By assumption, the government runs a balanced budget, i.e., $\tau_c c + \tau_k k = T$. Substitution of $\tau_c c + \tau_k k = T$ into the flow budget constraint of the representative consumer-producer (33) yields the capital accumulation equation $\dot{k} = f(k) - c$. Substituting $c = C$ into the necessary optimality condition for consumption (34) we obtain $U_c(c,c) = (1+\tau_c)\lambda$ and $\ln\lambda = \ln U_c(c,c) - \ln(1+\tau_c)$. Differentiating the latter condition with respect to time t we get

$$\frac{\dot{\lambda}}{\lambda} = \frac{[U_{cc}(c,c) + U_{cC}(c,c)]\,\dot{c}}{U_c(c,c)} - \frac{\dot{\tau}_c}{1+\tau_c}.$$

Substituting this result into the second necessary optimality condition (35), rearranging and using the definition of $\sigma^{ed}(c)$ given by (14), we obtain the Euler equation (36):

$$\frac{\dot{c}}{c} = \sigma^{ed}(c)\left[f'(k) - r - \left(\tau_k + \frac{\dot{\tau}_c}{1+\tau_c}\right)\right].$$

Finally, we show that the transversality condition is given by (37). Integration of (35) with respect to time yields

$$\lambda(t) = e^{rt}\lambda(0)\exp\left[-\int_0^t [f'(k(v)) - \tau_k(v)]\,dv\right],$$

where $\lambda(0) = [1+\tau_c(0)]^{-1} U_c(c(0), c(0))$. Substituting these results into the transversality condition (10), and taking into account that $U_c > 0$ and $1+\tau_c > 0$ by assumption, it is obvious that the transversality condition can be written as

$$\lim_{t\to\infty}\left\{k(t)\cdot\exp\left[-\int_0^t [f'(k(v)) - \tau_k(v)]\,dv\right]\right\} = 0,$$

which is identical with (37).

D. Proof of Proposition 6

In the following we assume that the socially planned solution $(k^s(t), c^s(t))$ exists.

Proof of (a): Assume that there exist time paths of $\tau_k(t)$ and $\tau_c(t)$ which ensure that $k^d(t) = k^s(t)$ and $c^d(t) = c^s(t)$. If this is the case, then $\dot{k}^d(t) = \dot{k}^s(t)$ and $\dot{c}^d(t) = \dot{c}^s(t)$ also obtains. In the decentralized economy the rate of growth of per capita consumption is given by [see (36)]

$$\frac{\dot{c}^d(t)}{c^d(t)} = \sigma^{ed}\left(c^d(t)\right)\left[f'\left(k^d(t)\right) - r - \left(\tau_k(t) + \frac{\dot{\tau}_c(t)}{1+\tau_c(t)}\right)\right]. \tag{D1}$$

Similarly, in the socially planned economy the growth rate of consumption is given by [see (28)]

$$\dot{c}^s(t)/c^s(t) = \sigma^{es}(c^s(t))\left[f'(k^s(t)) - r\right]. \tag{D2}$$

Substituting $k^d(t) = k^s(t)$, $c^d(t) = c^s(t)$, and $\dot{c}^d(t) = \dot{c}^s(t)$ into (D1) we obtain

$$\frac{\dot{c}^s(t)}{c^s(t)} = \sigma^{ed}(c^s(t))\left[f'(k^s(t)) - r - \left(\tau_k(t) + \frac{\dot{\tau}_c(t)}{1+\tau_c(t)}\right)\right]. \tag{D3}$$

From (D2) and (D3) it follows that

$$\tau_k(t) + \frac{\dot{\tau}_c(t)}{1+\tau_c(t)} = \left(\frac{\sigma^{ed}(c^s(t)) - \sigma^{es}(c^s(t))}{\sigma^{ed}(c^s(t))}\right)\left[f'(k^s(t)) - r\right],$$

which is identical with (38).

Proof of (b): Assume that the time paths of $\tau_k(t)$ and $\tau_c(t)$ satisfy (38). Substituting (38) into (36), we obtain

$$\frac{\dot{c}}{c} = \sigma^{ed}(c)\left[f'(k) - r - \left(\frac{\sigma^{ed}(c^s(t)) - \sigma^{es}(c^s(t))}{\sigma^{ed}(c^s(t))}\right)[f'(k^s(t)) - r]\right]. \quad \text{(D4)}$$

Hence, the decentralized solution (c^d, k^d) is determined by the differential equation (D4), the differential equation $\dot{k} = f(k) - c$, the initial condition $k(0) = k_0$, and the transversality condition (37). In the following we will show that the socially planned solution (c^s, k^s) satisfies the same differential equations, the same initial condition, and the same transversality condition, which implies that $(c^d, k^d) = (c^s, k^s)$.

Obviously, (c^s, k^s) satisfies the differential equation $\dot{k} = f(k) - c$ and the initial condition $k(0) = k_0$. It also satisfies the transversality condition of the decentralized economy, since in part (b) it is explicitly assumed that the time paths of $\tau_k(t)$ and $k^s(t)$ satisfy (37). Since (c^s, k^s) satisfies the differential equation (28) by definition, we have $\dot{c}^s(t)/c^s(t) = \sigma^{es}(c^s(t))[f'(k^s(t)) - r]$. Simple transformations of this expression yield

$$\frac{\dot{c}^s(t)}{c^s(t)} = \sigma^{ed}(c^s(t))\left[f'(k^s(t)) - r - \left(\frac{\sigma^{ed}(c^s(t)) - \sigma^{es}(c^s(t))}{\sigma^{ed}(c^s(t))}\right)[f'(k^s(t)) - r]\right],$$

which shows that (c^s, k^s) also satisfies the differential equation (D4).

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