Institut für Höhere Studien (IHS), Wien
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Inflation, Growth, and Credit Services

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Institut für Höhere Studien
Stumpergasse 56, A-1060 Wien
Fax: +43/1/599 91-163

Max Gillman
Department of Economics
Central European University
Nador ut. 9
H-1051 Budapest, HUNGARY
Phone: +36/1/327-3227
Fax: +36/1/327-3232
E-mail: Gillman@ceu.hu

Michal Kejak
CERGE-EI
Politickych veznu 7
CZ-111 21 Prague 1, CZECH REPUBLIC
Phone: +420/2/24005-186
Fax: +420/2/2422-7143
E-mail: Michal.Kejak@cerge.cuni.cz
and
Institute for Advanced Studies, Vienna

Ákos Valentinyi
Department of Economics
University of Southampton & CEPR
Highfield
Southampton SO17 1 BJ, UNITED KINGDOM
Phone: +44/1703-595000
Fax: +44/1703-593939
E-mail: A.Valentinyi@soton.ac.uk

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Abstract

The empirical evidence suggests that there is a significant, negative relationship between inflation and economic growth. Conventional monetary growth models, however, predict a significantly smaller growth effect. This paper proposes a monetary growth model with an explicit credit service sector to explain the observed magnitude. Since credit services are assumed costly to produce, the consumers equate the opportunity cost of holding money with the marginal cost of credit. Therefore the technology of the financial sector influences the velocity of money, and consequently, how inflation affects leisure, the time spent accumulating human capital, and the growth rate of output. The calibration shows that the model generates an inflation-growth effect whose magnitude falls in the range found by the empirical studies. Moreover, in contrast to previous works, we are also able to explain an inflation-growth effect that becomes increasingly weak as the inflation rate rises, as the evidence seems to suggest.

Keywords
Economic growth, inflation, costly credit

JEL Classifications
O11, E31

Comments

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Contents

1 Introduction

The empirical research on the relationship between inflation and growth suggests that there is a systematic, significant, negative association between inflation and growth. Although the estimates vary across different studies, it has been found that a 10 percentage point increase in the average inflation rate results in a decrease in the average growth rate of somewhere between 0.2 and 0.7 percentage point, [see Chari, Jones and Manuelli (1996)]. In contrast, existing monetary models of endogenous growth predict a significantly smaller effect of inflation on long run growth. Furthermore, there is also some evidence that the effect of inflation on long run growth is non-linear. More precisely, the marginal effect of inflation on the growth rate appears to be decreasing with the rate of inflation. In contrast, the growth effect of inflation in existing models is either close to linear, or its marginal effect is increasing with inflation. In particular, there is no obvious mechanism in most of these models which would generate the type of non-linearity one can find in the data.

This paper presents a plausible mechanism that ensures that monetary policy and hence inflation affects economic growth, and the predicted effect is consistent with the data. We build an endogenous growth model where human capital accumulation is the engine of growth, and both money and credit services facilitate transactions. The novelty of our approach is that we incorporate a sector which supplies credit services using a labor input. Consequently, the consumer's demand for money and credit services depends on their relative price. As inflation rises, the consumer substitutes away from money to credit and faces an increased cost of credit service production. This induces a lower accumulation rate of human capital, and a stronger substitution towards leisure than in standard monetary models of endogenous growth. We show that this effect becomes weaker as inflation rises explaining the non-linear feature of the growth effect of inflation. For example, a standard cash-in-advance economy lacks any balance of the marginal exchange costs. Given a fixed nominal interest rate for the marginal cost of money, such a balance requires an elastic supply of credit services in the exchange credit models of Aiyagari, Braun and Eckstein (1998), Bogacheva (1999a), and in our model. This added margin is crucial to explain the empirical findings on inflation and growth.

Our model is similar to Gillman and Otto (1998), but extended to endogenous growth as in Lucas (1988) without physical capital. It lacks the cumbersomeness of a store continuum as in Prescott (1987) and Gillman (1993), and the fixity of the credit good as in Schreft (1992) who derives the division into cash and credit goods external to the optimization problem. Aiyagari et al. (1998) also build a banking sector implicitly into a neoclassical growth model to exploit the relative exchange cost feature to explain comovement between the inflation rate and hours worked in banking and inflation. Bansil and Coleman II (1996) use such variable velocity to explain the magnitude of the equity premium in part by having government bonds demanded according to the use of exchange credit, thereby lowering the risk-free real interest.[1]

The rest of the paper is organized as follows. Section 2 reviews the empirical literature on inflation and growth, presents some descriptive statistics for the OECD countries, and confronts the evidence with the theoretical findings. Section 3 outlines the model. Section 4 derives the conditions for the balanced growth path. Section 5 discusses the log-utility case in more detail to obtain some insights from analytical solutions. Section 6 presents the numerical results, and section 7 concludes.

2 Inflation and Growth: Evidence and Theory

There is a large body of literature which investigates the empirical relationship between inflation and growth. In their well-known paper Körmendi and Meguire (1985) report a negative relationship between inflation and economic growth in a cross section of countries. De Gregorio(1992, 1993) finds, in a panel using 6-year average data, that a reduction

[1] Bogacheva (1999b) uses the approach to explain 9-12 month forward exchange rates better than random walk models.

in the level of the inflation rate by 17 percentage point yields a 0.4 percentage point increase in the growth rate of output. Similarly, Fischer(1991, 1993) estimates with cross-sectional and panel data that a 10 percentage point increase in the inflation rate decreases the growth rate by between 0.3 and 0.4 percentage point.[2] Using various instrumental variables on a panel of ten year averages, Barro(1996, 1997) concludes that a 10 percentage point increase in the inflation rate lowers the economic growth by 0.2 to 0.3 percentage point.[3] The negative effect of inflation on growth also appears to be robust to choices of alternative policy indicators.

There is also evidence that the growth effect of inflation weakens at higher inflation rate. Fischer (1993) divides countries into three groups according to their average inflation rate, between 0 and 15 percent, 15 and 40 percent, and above 40 percent, he finds an increase in the inflation rate by 10 percentage point associated with decreases in the growth rate of 1.3 percentage point, 0.75 percentage point, and 0.19 percentage point. Also Barro (1997) and Bruno and Easterly (1998) both report that countries with annual inflation above 40% grow significantly lower than countries with inflation rates below 40%. This can also be viewed as an indication that the effect of inflation on growth is non-linear.

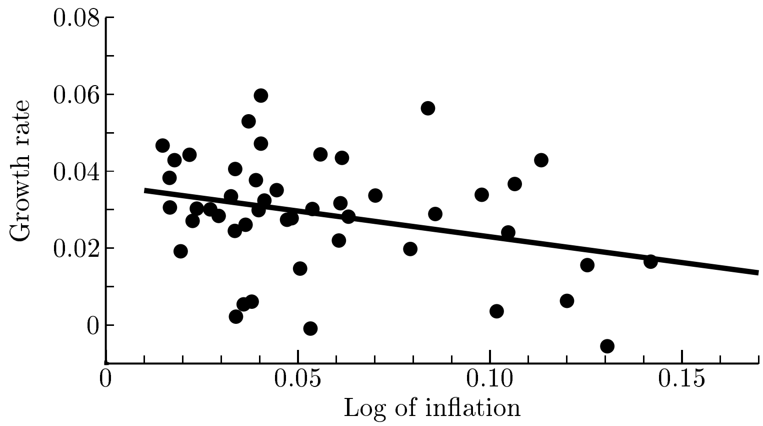
We also calculated some simple descriptive statistics about the relationship between inflation and growth for the 24 OECD countries for the period 1951-1997.[4] Figure 1 plots the average log of inflation against the average growth rate where averages are taken for each year separately across all countries. The line with a slope of -0.134 indicates a negative relationship between inflation and growth. Moreover, since we regressed the log of inflation on growth, the obtained relationship is non-linear. It should also be mentioned that the coefficient is significant for log of inflation while it is not significant for inflation implying that the relationship between inflation and growth is more likely to be non-linear

[2] This result is very close to those of Roubini and Sala-i-Martin (1992).

[3] See also Andrés, Domenech and Molinas (1996), Ghosh and Phillips (1998), and Gylfason and Herbertson (1996) for results.

[4] The source of the data is IMF International Financial Statistics.

Figure 1: Inflation and growth in the OECD countries



Note: Each point represents the OECD average for a given year

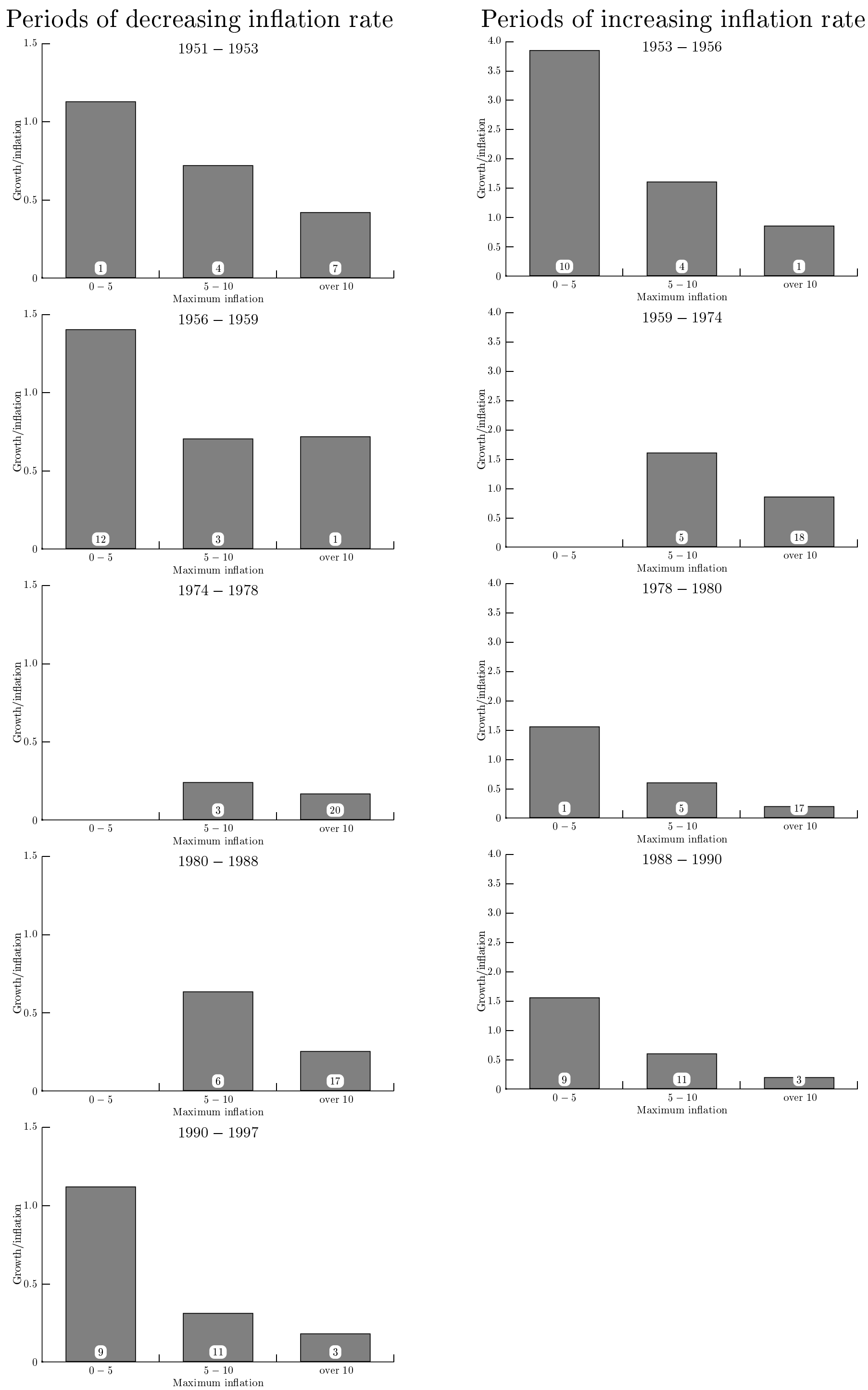
than linear.

The next group of statistics indicates that the non-linearity of the growth effect of inflation is robust. We divide the sample period into sub-periods according to whether the average inflation in the OECD countries was increasing or decreasing.[5] In addition, the countries are grouped into three categories by the average of the maximum inflation reached by each country over the sub-period. Figure 2 displays the value of the average growth/average inflation where the average is taken across countries and across time within each sub-period.[6] The average growth rate per unit of average inflation tends to fall as inflation rises to the next category, and this fall usually occurs with a decreasing magnitude. This indicates both that inflation-growth relationship is negative and that the relationship is weaker at a higher rate of inflation. It is also important to note that the correlation between inflation and growth is -0.30 for all increasing periods of inflation, and it is -0.23 for all decreasing periods. This suggests that the inflation-growth relationship tends to be stronger in periods of rising inflation.

[5] A sub-period is defined by years of a nearly monotonically rising or declining average OECD inflation rate. In choosing sub-periods, we allow a one year reverse-direction change in the average OECD inflation rate of less than 1% to be included in a sub-period.

[6] The results are robust for a range of alternative specifications how sub-periods are built and averages are taken.

Figure 2: Inflation, growth and non-linearity in the OECD countries



Note: The numbers in each column indicate the number of observations.

The empirical literature reviewed above and our descriptive statistics indicate the negative relationship between inflation and growth. There are many monetary growth models which are able to replicate the effect *qualitatively* [see for example De Gregorio(1992, 1993), and Roubini and Sala-i-Martin (1992)]. However, these models fail to generate the same effect *quantitatively*. In particular, they generate substantially weaker growth effect of inflation. For example, Gomme (1993), with a stochastic two-sector endogenous growth model with elastic labor supply, human capital, and money, reports a 10.5 percentage point per quarter increase in inflation lowers the growth rate by 0.2 percentage point per year. Chari et al. (1996) assess the quantitative performance of a number of endogenous growth models, and conclude that none are able to match the data even closely.[7] They introduce a financial intermediary that faces reserve requirements, and match the data if both the growth rate of the money supply and the reserve ratio are increased at the same time. However, as noted by Stockman (1996) in his comment on the paper, no evidence supports such a simultaneous change in the policy variables. Moreover, they have also to rely on a rather high labor supply elasticity to obtain the desired growth effect.

It is also important to emphasize that there is no mechanism in most of the models which ensures the type of non-linearity we observe in the data. In particular, the mechanism in several models generate a relationship between inflation and growth which is close to linear [for example, De Gregorio (1993), and Chari et al. (1996)], or the growth effect of inflation becomes stronger at a higher rate of inflation [for example Jones and Manuelli (1995)].

[7] The growth effect of inflation is also week in Dotsey and Ireland (1996), Wu and Zhang (1998). It is higher than in other models in Love and Wen (1999), however, it is still below 0.2 percentage point.

3 Economic Environment

The representative consumer maximizes the present value of momentary utilities defined over consumption c_t, and leisure x_t

$$\bar{U} = \int_0^\infty e^{-\rho t} \frac{(c_t x_t^\alpha)^{1-\theta}}{1-\theta} dt. \tag{1}$$

It is assumed that consumption purchases can be financed either by using money or by using credit services. Let $a_t \in (0, 1]$ be the fraction of consumption goods bought with money at time t. Then the consumer faces the cash-in-advance constraint $M_t = a_t p_t c_t$ where M_t is the money holdings of the consumer and p_t is the price level at time t. The cash-in-advance constraint in real terms can be written as

$$m_t = a_t c_t, \tag{2}$$

where $m_t \equiv M_t/p_t$. Note that the fraction of consumption financed with money corresponds to the inverse velocity of money.

The fraction $1 - a_t$ of consumption goods is financed using credit services. These services are produced with the technology

$$1 - a_t = \zeta \left(\frac{b_t h_t}{c_t} \right)^\gamma \qquad \zeta > 0, \quad \gamma \in (0, 1), \tag{3}$$

where ζ is a productivity parameter, and b_t is the time spent producing credit services. The assumption underlying this technology is that the flow of credit services required to buy a fraction $(1 - a_t)$ of the consumption goods is increasing in the effective labor $b_t h_t$ relative to the level of consumption. Moreover, it is assumed that for a given consumption level c_t, the consumer as "banker" faces increasing marginal costs when increasing the proportion of goods that are bought with credit. This is modeled by having diminishing returns to the effective labor in producing the credit share $(1-a_t)$, thus $\gamma \in (0, 1)$ which is the crucial assumption of our model. The assumption about diminishing returns implies that, in addition, to the wage, the consumer as the producer of credit services also receives a return on this activity. In particular, a fraction γ of the income flow to the credit service production can be viewed as wage income while a fraction $(1-\gamma)$ can be viewed as the return to the producer of credit services.[8]

The consumption good is produced with a constant returns to scale technology

$$c_t = wh_t l_t \tag{4}$$

where w denotes the marginal product of human capital in goods production, h_t is the stock of human capital, and l_t is the amount of time spent working in goods production. The consumer's budget constraint in real terms can be written as

$$\dot{m}_t = wh_t l_t + v_t - c_t - \pi_t m_t, \tag{5}$$

where $m_t \equiv M_t/p_t$ denotes real balances, $v_t \equiv V_t/p_t$ is the lump-sum money transfer from the government in real terms, and $\pi_t \equiv \dot{p}_t/p_t$ is the rate of inflation. The budget constraint states that the income from effective labor and the lump-sum transfer from the government is spent on consumption, on offsetting the effect of inflation on real balances, and on the accumulation of real balances.

The accumulation of human capital depends on the time spent in accumulating human capital, on the level of human capital, and on the rate of depreciation of human capital in a linear fashion

$$\dot{h}_t = \phi[1 - l_t - x_t - b_t]h_t - \delta h_t, \qquad \phi > 0 \quad \delta > 0 \tag{6}$$

[8] Appendix A shows formally the total wage bill in the sector producing credit services is $\gamma R_t(1-a_t)c_t$ while the profit is $(1-\gamma)R_t(1-a_t)c_t$ where R_t is the nominal interest rate which equals the equilibrium relative price of credit services in terms of the consumption goods.

where ϕ is a productivity parameter, δ is the depreciation rate for human capital, and $1 - l_t - x_t - b_t$ is the study time, i.e. the time devoted to human capital accumulation.

Finally, to close the model, we assume that the government prints money at rate $\sigma = \dot{M}_t / M_t$, and it uses the revenues from money creation to finance the lump-sum transfer V_t to the consumer, thus, $V_t = \sigma M_t$, or $v_t = \sigma m_t$ in real terms.

4 Balanced Growth Path

4.1 Competitive Equilibrium

Under the assumption that the consumer operates all technologies directly, we can simplify the consumer's problem. Let

$$\tilde{\tilde{b}}(a_t) \equiv \left(\frac{1 - a_t}{\zeta} \right)^{\frac{1}{\gamma}}. \tag{7}$$

Inspecting equation (3) reveals that this is the effective labor per unit of consumption required to finance a fraction $1 - a_t$ of the consumption goods with credit services. Put differently, this is the cost function in the credit service sector. Now the time spent in producing credit services, b_t, can be written as

$$b_t = \tilde{\tilde{b}}(a_t) \frac{c_t}{h_t}. \tag{8}$$

Using this equation, we can now rewrite the law of motion for human capital (6) as

$$\dot{h}_t = \phi \left[1 - l_t - x_t - \tilde{\tilde{b}}(a_t) \frac{c_t}{h_t} \right] h_t - \delta h_t, \qquad \phi > 0 \tag{6'}$$

The consumer chooses a consumption, credit service purchase, time allocation, real balances and human capital $\{c_t, a_t, x_t, l_t, m_t, h_t\}_{t=0}^{\infty}$, to maximize the life time utility (1) subject to the cash-in-advance constraint (2), the budget constraint (5), the constraint for the human capital accumulation ($6'$), and the credit service technology (7).

The first order conditions for the consumer's problem along with the constraints (2), (5), ($6'$), (7) and the transversality condition, are

$$R_t = \frac{w}{\gamma\zeta}\left(\frac{1-a_t}{\zeta}\right)^{\frac{1}{\gamma}-1} \tag{9a}$$

$$\frac{1}{\alpha}\frac{x_t}{c_t} = \frac{1 + a_t R_t + w\tilde{\tilde{b}}(a_t)}{wh_t} \tag{9b}$$

$$\frac{\dot{c}_t}{c_t} = \frac{\phi(1-x_t) - \delta - \rho}{\theta}. \tag{9c}$$

where R_t is the nominal interest rate defined by

$$R_t \equiv \phi(1-x_t) - \delta + \pi_t, \tag{10}$$

where $\phi(1-x_t)-\delta$ is the real interest rate, i.e. the net return on human capital.[9] Equation (9a) equates the opportunity cost of holding money R_t to the marginal cost of credit services (similar to Baumol (1952)). Equation (9b) sets the marginal rate of substitution between consumption and leisure time equal to the marginal cost of consumption to the marginal product of working time. Note that the cost of one unit of consumption consists of the one unit of resources required for the consumption itself, the cost of holding a_t units of money, $a_t R_t$, and the cost of $1 - a_t$ units of credit services, $w\tilde{\tilde{b}}(a_t)$, used to purchase consumption. Equation (9c) is the standard intertemporal Euler-equation for the optimal consumption growth.

We focus on the competitive equilibrium along the balanced growth path which is a price $\{R_t\}_{t=0}^{\infty}$, an allocation $\{c_t, a_t, x_t, l_t, m_t, h_t\}_{t=0}^{\infty}$, and a set of initial conditions $\{m_0, h_0\}$ such that given the price $\{R_t\}_{t=0}^{\infty}$ the allocation $\{c_t, a_t, x_t, l_t, m_t, h_t\}_{t=0}^{\infty}$ solves the consumer's problem, i.e it satisfies equations (2), (5), ($6'$), (7) and (9a)-(9c), the goods

[9] The bond market that determines R_t as a deterministic Fisher equation of interest is suppressed for brevity.

market clears, i.e (4) satisfied, c_t, m_t and h_t grows at a common constant rate g, and $a_t, x_t, l_t, R_t, \pi_t$ are constant over time.[10] Note that along the balanced growth path inflation is simply the difference between the growth rate of the money supply and the growth rate of consumption,

$$\pi_t = \sigma - g. \tag{11}$$

4.2 Credit Services and Money Demand

The model thus stated represents a way to incorporate a full bank sector into a growth model in which the demand for money is linked to the technology of credit service production.[11] The optimizing consumer equates the equilibrium price of credit services, i.e. the nominal interest rate, with its marginal cost. Viewed from the marginal product instead of the marginal cost, the consumer equates the marginal product of labor in credit service production to the real wage in terms of credit services, i.e. to w/R_t. The equilibrium supply of credit services is pinned down by its relative price, R_t, as is standard for decreasing returns to scale technologies. Therefore the equilibrium money demand is determined by the amount of consumption goods which is not financed by credit services. Since higher interest rate implies larger equilibrium supply of credit services,[12] the equilibrium money demand is decreasing in the nominal interest rate.

Under the assumption $\gamma \in (0, 1)$, the equilibrium demand for real balances as the fraction of goods not financed with credit services is implied by equations (9a) as

$$a_t = a(R_t) = 1 - \zeta^{\frac{1}{1-\gamma}} \left(\frac{\gamma R_t}{w} \right)^{\frac{\gamma}{1-\gamma}}, \tag{12}$$

[10] See the Appendix B for a proof of the existence of the equilibrium along the balanced growth path.

[11] Appendix A shows that the decentralized allocation where the credit service sector and the consumer optimize independently leads to the same equilibrium condition.

[12] The model implies that the hours worked in banking rise with inflation which is consistent with the evidence found by Aiyagari et al. (1998).

for $R_t \in [0, \bar{R})$ where $\bar{R}$ is the nominal interest rate at which $a_t = 0$,[13] i.e. $\bar{R}$ is defined by

$$\bar{R} = \frac{w}{\gamma}\left(\frac{1}{\zeta}\right)^{\frac{1}{1-\gamma}}. \tag{13}$$

Moreover, it also follows from equations (7) and (9a) that the effective labor devoted to produce credit services is given by

$$\tilde{b}(R_t) \equiv \tilde{\tilde{b}}(a(R_t)) = \zeta^{\frac{1}{1-\gamma}}\left(\frac{\gamma R_t}{w}\right)^{\frac{1}{1-\gamma}}. \tag{14}$$

Since the marginal cost in the credit service sector equals the nominal interest rate R_t, the above relationship also highlights that the supply of credit services underlies the money demand. This implies that the output of the financial sector is closely linked to money demand that in turn determines the nature of the inflation-growth effect.

5 Balanced Growth Path: The Log-utility Case

This simple model is already far too complicated to obtain results analytically. However, before presenting the numerical results, it is useful to get more insights about how the financial technology and the money demand are related to the growth effect of inflation. Therefore, we look at the log-utility case, $\theta = 1$, with zero depreciation $\delta = 0$ in more detail now.

[13] Hence our model implies that consumption is entirely financed with credit services at some high but finite nominal interest rate. This could by viewed as the approximation of real economies which use very little cash at high inflation. However, we restrict our attention to the case when $R_t < \bar{R}$. Our calibration indicates that $\bar{R}$ is about 650%, implying that focusing on $R_t < \bar{R}$ is not too much of a restriction.

5.1 Consumption, Leisure and the Growth Effect of Inflation

The main focus of our paper is the growth effect of inflation and its non-linear nature. Since the growth rate on the balanced growth path for $\theta = 1$ and $\delta = 0$ is

$$g = \phi(1 - x_t) - \rho \tag{15}$$

by equation (9c), we have only to determine how leisure is affected by the growth rate of the money supply.[14]

The definition of the nominal interest rate (10), the balanced growth path relationship for inflation (11), and the equation for the consumption growth rate imply that

$$R_t = \rho + \sigma \tag{16}$$

for $\theta = 1$ and $\delta = 0$. Since the nominal interest rate uniquely determines the inverse velocity a_t through (12), we can express all variables as a function of R_t.

Observe that the goods market equilibrium (4), and the law of motion for human capital (6′) imply

$$g = \phi(1 - x_t) - \phi[1 + w\tilde{b}(R_t)]l_t.$$

Using equation (15), we obtain

$$l_t = \frac{1}{1 + w\tilde{b}(R_t)} \frac{\rho}{\phi}, \tag{17a}$$

for the time spent in production of consumption good along the balanced growth path. Finally, substituting the goods market clearing condition (4) into (9b) for c_t/h_t, and

[14] Note however that inflation-induced changes in leisure equal the negative of changes in time spent in human capital accumulation, which equals $1 - x_t - [\rho/(\theta\phi)]$, a monetary analogue to Lucas's (1988) endogenous growth rate that depends on time spent in human capital accumulation.

combining the result with the above relationship for l_t leads to the equation for leisure along the balanced growth path

$$x_t = \frac{\alpha\rho}{\phi}\frac{1 + w\tilde{b}(R_t) + a(R_t)R_t}{1 + w\tilde{b}(R_t)}. \tag{17b}$$

The condition says that leisure is proportional to the individual cost of one unit of consumption good relative to its social resource cost. The individual cost of consuming one unit of consumption good consists of the real cost of the consumption plus the exchange costs $w\tilde{b}(R_t) + a(R_t)R_t$ which is the cost of $(1 - a_t)$ units of credit services, and the opportunity cost of holding a_t units of money. The social resource cost is made up by the cost of producing the consumption good itself and cost of producing $1 - a_t$ units of credit services required for consumption purchases.[15]

The growth rate of the economy along the balanced growth path depends on leisure only as indicated by (15). Therefore the growth effect of monetary policy can be obtained through evaluation of the effect of money growth on leisure. Moreover, since an increase in the growth rate of the money supply leads to a one-to-one increase in the nominal interest rate [compare (16)], the effect of money growth on leisure can simply be evaluated by taking the derivative of (17b) with respect to R_t,

$$\frac{\partial x_t}{\partial R_t} = \frac{\alpha\rho}{\phi}\frac{a(R_t) - \gamma[1 + w\tilde{b}(R_t)]}{(1-\gamma)[1 + w\tilde{b}(R_t)]^2}. \tag{18}$$

As we can see, an increase in the nominal interest rate increases or decreases leisure depending on the sign of the numerator. To assess its sign, consider $\hat{R}$ which is defined by

$$a(\hat{R}) = \gamma[1 + w\tilde{b}(\hat{R})]. \tag{19}$$

[15] With a_t set equal to 1 as in Lucas (1980), as R_t increases the exchange cost (the cost of holding money) rises at a constant rate. In our model a_t falls as R_t rises, so the exchange cost (the cost of holding money and the credit service cost) rises at a decreasing rate.

Clearly, $\hat{R}$ exists, and it is unique. This is because $a(R_t)$ is monotonically decreasing while $\tilde{b}(R_t)$ is monotonically increasing in R_t, and $a(R_t) \in (0, 1]$ while $\tilde{b}(R_t) \in [0, \infty)$. It follows now from equation (18) that leisure increases in R_t for $R_t \in [0, \hat{R})$, it decreases in R_t for $R_t \in [\hat{R}, \bar{R})$. Consequently, inflation has a negative effect on growth for $R_t \in [0, \hat{R})$ and it has a positive one for $R_t \in [\hat{R}, \bar{R})$.

The intuition behind the non-monotonicity of leisure in the nominal interest rate can be understood as follows. An increase in the nominal interest rate has two effects. First, it increases the cost of consumption thereby inducing a substitution from goods production to leisure. This is the substitution effect. Second, the social resource cost $w\tilde{b}(R_t)$ rises as R_t rises, and reduces consumption of both goods and leisure. This is the income effect. The two effects go to the opposite direction, and the substitution effect dominates as long as $R_t \in [0, \hat{R})$, while the income effect dominates as long as $R_t \in [\hat{R}, \bar{R})$.

Finally, we can also show that the effect of the nominal interest rate on leisure is decreasing as the nominal interest rate rises. Taking the derivative of (18) with respect to R_t, we obtain that

$$\frac{\partial^2 x_t}{\partial R_t^2} = \frac{\gamma[1 + w\tilde{b}(R_t)]^2 - \gamma[2 - a(R_t)][1 + w\tilde{b}(R_t)] - 2w\tilde{b}(R_t)a(R_t)}{R_t(1-\gamma)^2[1 + w\tilde{b}(R_t)]^3}. \tag{20}$$

It is easy to show that the numerator is negative for $R_t \in [0, \hat{R})$ where $a(R_t) > \gamma[1 + w\tilde{b}(R_t)]$. Applying the result for the growth rate, we conclude that the effect of inflation on growth weakens as inflation rises.

5.2 Interest Elasticity and the Growth Effect of Inflation

To get further insight about the model, we analyze how the interest elasticity of money demand per unit of human capital, $\eta_m(R_t)$, is related to the growth effect of inflation. This will also give a precise interpretation of the threshold nominal interest rate $\hat{R}$ at which the growth effect inflects from negative to positive.

The interest elasticity of money demand per unit of human capital is the sum of the interest elasticity of consumption per human capital, and the interest elasticity of the inverse velocity, $\eta_m(R_t) = \eta_c(R_t) + \eta_a(R_t)$. Using the goods market clearing condition (4) and (17a) leads to

$$\eta_c(R_t) = -\eta_b(R_t) \frac{w\tilde{b}(R_t)}{1 + w\tilde{b}(R_t)}, \tag{21a}$$

where $\eta_b(R_t)$ is the interest elasticity of the effective banking time per unit of consumption. It is easy to see from equations (12) and (14) that

$$\eta_a(R_t) = -\frac{\gamma}{1-\gamma} \frac{1-a(R_t)}{a(R_t)}, \qquad \eta_b(R_t) = \frac{1}{1-\gamma}. \tag{21b}$$

Putting the pieces together, we obtain that the inverse elasticity of money demand is given by

$$\eta_m(R_t) = -\frac{1}{1-\gamma} \frac{w\tilde{b}(R_t)}{1 + w\tilde{b}(R_t)} - \frac{\gamma}{1-\gamma} \frac{1-a(R_t)}{a(R_t)}. \tag{22}$$

We can express now the effect of the nominal interest rate on leisure, and hence economic growth in terms of the interest elasticity of money demand. Plugging equations (21a) and (21b) into (18) leads to

$$\frac{\partial x_t}{\partial R_t} = \frac{\alpha \rho}{\phi} \frac{a(R_t)}{1 + w\tilde{b}(R_t)} \left[1 + \eta_m(R_t)\right]. \tag{18$'$}$$

Equation (18$'$) highlights that the direction of the effect of the nominal interest rate on leisure and hence on growth depends on the size of the interest elasticity of money demand.

Equation (22) implies that at the Friedman-optimum $\eta_m(0) = 0$. It follows from the definition of $\hat{R}$ in equation (19) that $\eta_m(R_t) \in [0, -1)$ for $R_t \in [0, \hat{R})$, and $\eta_m(\hat{R}) \leq -1$

for $R_t \in [\hat{R}, \bar{R})$. Moreover, recall that the revenue maximizing nominal interest rate is the one at which $\eta_m(R_t) = -1$ [Friedman (1971)]. Therefore $\hat{R}$ is not only the nominal interest rate at which leisure and the growth rate inflect from a negative to a positive effect, but also the seigniorage maximizing nominal interest rate.

The nature of the interest elasticity of money demand determines the non-linear nature of the effect of inflation on leisure and growth rates. To highlight the mechanism, it is useful to decompose the elasticity in the following way

$$\eta_m(R_t) = \eta_c(R_t) + [1 - a(R_t)]\varepsilon(R_t) \tag{23}$$

where ε is the elasticity of substitution between money and credit services, i.e.

$$\varepsilon(R_t) \equiv \frac{\partial}{\partial R_t}\left(\frac{a(R_t)}{1 - a(R_t)}\right)\frac{R_t[1 - a(R_t)]}{a(R_t)} = -\frac{\gamma}{1-\gamma}\frac{1}{a(R_t)}$$

Both terms in equation (23) are negative and increase in magnitude as the nominal interest rate increases. Viewing money and credit as an input for consumption[16], the above equation is the same as in the input price theory for two factors. The own price elasticity of a factor input [money] is equal to the share of the other factor input [credit services] multiplied by the elasticity of substitution between the factor inputs, $\varepsilon(R_t)$, plus the scale effect, $\eta_c(R_t)$, the effect on consumption in our case. This compares to Alfred Marshall's factor-elasticity law that as the share of the input, i.e. money as input for consumption, declines, the own-price elasticity becomes greater in absolute value.

The role of the interest elasticity of money demand in determining the inflation-growth effect can be inferred from equation (23). At a low nominal interest rate, the elasticity of substitution between money and credit services is low in absolute value, i.e. it is difficult to substitute from money to credit. Therefore the consumers substitute mainly toward

[16] This interpretation is similar to one in Becker (1965) where the consumption of goods requires both the goods and exchange for the goods.

leisure to avoid the costs of inflation leading to the strong growth effect. In contrast, as inflation rises, $\varepsilon(R_t)$ rises in absolute value implying that the use of credit services becomes increasingly important in escaping the inflation tax. The substitution from goods production toward credit service production becomes stronger which weakens the substitution towards leisure thereby reducing the effect of inflation on growth.

5.3 The Welfare Cost of Inflation and the Cost of Credit Services

The calculation of the welfare cost of inflation also highlights the way in which credit services determine the inflation-growth effect, relative to standard models. A large portion of the welfare cost of inflation is the use of resources in production of credit services. And the rest of the welfare cost is approximately due to a lower growth rate. Thus the larger magnitude of the inflation-growth effect at low inflation rates is because of the time being used in banking that both uses up resources and leads to less human capital accumulation.

Now we show that in the absence of growth effect, the welfare cost of inflation equals the resource cost of banking. And also, already knowing that the growth effect gets smaller as the interest rate rises, we show that the resource cost part becomes higher at a higher nominal interest rate. In the next section, calibrations of the welfare cost indicate that the resource effect, while large, can be dominated by the growth effect.

Setting $\alpha = 0$ eliminates both the labor-leisure choice and the growth effect of inflation from our model. The growth rate reduces to $g = (\phi - \rho)/\theta$. The welfare cost of inflation can be found by including an endowment of the consumption good per unit of human capital, denoted by e^*, in the budget constraint (5) which becomes on the balanced growth path

$$c^* \equiv \frac{c_t}{h_t} = wl^* + e^*$$

where * denotes the balanced growth path values per unit of human capital for the corresponding variable. For $\theta = 1$ and $\alpha = \delta = 0$, the law of motion for human capital (6′) can be written as

$$g = \phi - \phi e^* \tilde{b}(R^*) - \phi[1 + w\tilde{b}(R^*)]l^*.$$

This equation together with (15) implies that

$$l^* = \frac{\rho - \phi e^* \tilde{b}(R^*)}{1 + w\tilde{b}(R^*)} \frac{1}{\phi}. \tag{24}$$

The goods market clearing condition can be written as

$$c^*(e^*, R^*) = \frac{w\rho/\phi + e^*}{1 + w\tilde{b}(R^*)} \tag{25}$$

Since $c^*(e^*, R^*) = c_t/h_t$ and $\ln(h_t) = gt$ for $h_0 = 1$, the momentary utility along the balanced growth path becomes $u(e^*, R^*) = \ln(c(e^*, R^*)) + gt$ implying that the lifetime utility becomes

$$\bar{U}(e^*, R^*) = \frac{\ln(c^*(e^*, R^*))}{\rho} + \frac{g}{\rho^2}. \tag{26}$$

The real goods endowment e^* necessary to compensate for suboptimal inflation is then determined from the equation

$$\bar{U}(e^*, R^*) = \bar{U}(0, 0). \tag{27}$$

As standard, we express e^* as a percent of income (which equals consumption in our setup) at the Friedman optimum which can be obtained from solving equation (27)

$$\bar{e} \equiv \frac{e^*}{c(0,0)} = w\tilde{b}(R^*),$$

thus, the welfare cost of inflation as a percent of income at the Friedman optimum is the value of resources used in credit service production. Inspecting (14) also reveals that the closer is the banking technology to constant returns, $\gamma = 1$, the faster do welfare costs rise. As this resource cost rises, it is because of heavy credit use. This use provides an escape valve from the inflation tax that decreases the incentive to inefficiently use leisure.

This welfare cost exactly equals the area under the money demand function normalized by consumption, i.e. the inverse velocity $a(R^*)$. To see this consider the following

$$\int_{a(R^*)}^{a(0)} R da(R) = -\int_0^{R^*} R a'(R) dR = \left(\frac{\gamma\zeta}{w^\gamma}\right)^{\frac{1}{1-\gamma}} (R^*)^{\frac{1}{1-\gamma}} = w\tilde{b}(R^*) \qquad (28)$$

Exactly as in Lucas (1993), without leisure, the resources used up in avoiding the inflation tax are exactly the welfare cost of inflation. However, in contrast to the general transaction cost of the McCallum and Goodfriend (1989) framework, here the interpretation is more specific. This specificity allows the conclusion that the welfare cost, the integral under the marginal cost of banking function, and the integral under the money demand function are all the same in this case. And the elasticity of these functions determine how quickly the consumer substitutes to credit instead of having to avoid the tax through more leisure.

6 Calibration

In this section we calibrate the model by assigning values to the model parameters and by calculating the steady state values of variables for different money growth rates. We use the U.S economy as the benchmark. Thus we assume that the benchmark output growth rate is $g = 0.02$ and the inflation rate is $\pi = 0.05$. The implied benchmark money growth rate is the sum of the output growth rate and the inflation rate, i.e. $\sigma = 0.07$. The proportions of time allocated to leisure and to work are set at $x_t = 0.7$ and $l_t = 0.17$.[17].

[17] Jones, Manuelli and Rossi (1993) have used similar values. See also King and Rebelo (1999).

Further, the subjective discount rate, the depreciation rate, and the degree of risk aversion are set in the following way: $\rho = 0.054$, $\delta = 0.025$, and $\theta = 1.3$.

Equation (10) determines the productivity parameter of technology for the human capital accumulation, $\phi = 0.35$, and the definition of the nominal interest rate in (10) implies that $R_t = 0.13$. Using equation (6) yields the time devoted to accumulate human capital, i.e. study time, $1 - l_t - x_t - b_t = (g + \delta)/\phi \equiv u_t = 0.1286$.[18] The time spent in credit service production is the residual part of time allocation, $b_t = 1-l_t-x_t-(g+\delta)/\phi = 0.0014$.[19]

To specify the production function for credit services, we set $\gamma = 0.265$ and $\zeta = 0.75$ as estimated by Gillman and Otto (1998) for Australia. Using the goods market equilibrium condition (4), the equilibrium banking time from (8) and (14) can be written as

$$b_t = \tilde{b}(R_t)\frac{c_t}{h_t} = w l_t \zeta^{\frac{1}{1-\gamma}} \left(\frac{\gamma R_t}{w}\right)^{\frac{1}{1-\gamma}}. \tag{29}$$

Together with the already determined parameters, this gives us the efficiency wage rate, $w = 0.5824$. Now equation (12) can be used the compute the fraction of goods bought by using cash, $a_t = 0.7561$. Finally, the value of the parameter for the weight of leisure in the utility function is obtained from (9b) as $\alpha = 3.7207$.[20]

The numerical results from the calibration are given in Table 1.[21] The non-linearity of the inflation effect is in line with the evidence. The results are sensitive to γ. As γ goes up, the inflection point at which the output growth rate stops falling, occurs at a lower inflation rate. The effect of a changing money growth rate, from the optimal rate

[18] Jones et al. (1993) have used similar value, $u_t = 0.12$.

[19] This corresponds to the values for the fraction of labor force in finance equal to 0.0028 for the 4% inflation rate used in Dotsey and Ireland (1996).

[20] This falls into the range used in the business cycle literature [compare King and Rebelo (1999)], and it is lower than the values used by Jones et al. (1993) and Chari et al. (1996).

[21] The calibrations have taken into account the following restrictions on the model parameters: $R_t \in [0, \bar{R})$ for $a_t \in (0, 1]$ where $\bar{R}$ is defined in (13), and $\bar{R} = 6.5076$ for our parameter values. The feasible values for the money growth rate must satisfy $\sigma \in [\underline{\sigma}, \bar{\sigma})$ where $\underline{\sigma}$ and $\bar{\sigma}$ are defined in equation (B.6a) and (B.6b) in Appendix B, and $\underline{\sigma}$ is the money growth rate of the Friedman-optimum.

Table 1: Inflation and growth

Inflation rate in %	Percentage point change in the growth rate due to 10% increase in inflation
0	−0.54
10	−0.45
20	−0.39
30	−0.33
40	−0.29
50	−0.25
100	−0.01

to the rate of 40%, on the steady state values of important variables is documented in Table 2.[22]

For the calibration of the welfare cost of inflation, we consider the measure from Cooley and Hansen (1991) that increases consumption to offset suboptimal inflation.[23] This measure can be computed using the formula

$$\left(\frac{e}{c}\right)^{*} = \frac{c_0}{c^*}\left(\frac{x_0}{x^*}\right)^{\alpha}\left[\frac{\rho + (\theta - 1)g^*}{\rho + (\theta - 1)g_0}\right] - 1 \tag{30}$$

where * denotes the equilibrium with a suboptimal inflation rate, as just qualified above, and the 0 subscript denotes the Friedman optimum, $R = 0$. The last line of Table 2 shows the value of this welfare cost measure is 2.5% for $R \simeq 17\%$, for example. This estimate is significantly higher than in Gomme (1993) and in similar models with zero or exogenous growth rate [see Cooley and Hansen (1989, 1991)]. It is of similar size found by Love and Wen (1999), and Wu and Zhang (1998). For example, Wu and Zhang (1998) shows in an endogenous growth model with a 10% growth rate of the money supply, that the welfare cost is 2.65% with cash-in-advance for consumption, and 5.98% with cash-in-advance for both consumption and indivisible labor.

[22] The optimal rate of money growth is equal to $\underline{\sigma}$ as defined in equation (B.6a) in in Appendix B.

[23] Here, the consumption endowment is increased to get the same utility when the consumer faces suboptimal inflation as when there is no endowment and the consumer is at the optimum. But with suboptimal inflation the leisure is suboptimally kept the same with or without the added endowment.

Table 2: Values of variables along the balanced growth path for different money growth rates.

	$-6.27\%^c$	0%	10%	20%	30%	40%	60%
Real output[a]	0.1043	0.1014	0.0977	0.0943	0.0913	0.0885	0.0836
Working time	0.1791	0.1741	0.1677	0.162	0.1568	0.152	0.1435
Leisure time	0.6666	0.6858	0.7076	0.7254	0.7404	0.7531	0.7732
Banking time	0	0.0006	0.002	0.0036	0.0053	0.0072	0.011
Study time	0.1543	0.1395	0.1227	0.109	0.0975	0.0877	0.0722
Cash-good[b]	1	0.8064	0.7332	0.6846	0.6464	0.6144	0.5615
Credit-good[b]	0	0.1936	0.2668	0.3154	0.3536	0.3856	0.4385
Real money demand[a]	0.1043	0.0818	0.0716	0.0646	0.059	0.0544	0.0469
Growth rate	0.029	0.0238	0.018	0.0132	0.0091	0.0057	0.0003
Inflation rate	−0.0917	−0.0165	0.0893	0.1941	0.2982	0.4016	0.607
Nominal interest rate	0	0.0684	0.1667	0.2652	0.364	0.463	0.6614
Cost of banking	0	0.0035	0.0118	0.0221	0.0341	0.0473	0.0769
Welfare cost[b]	0	0.0064	0.0251	0.05	0.0787	0.1099	0.1762

[a] As a fraction of human capital.
[b] As a fraction of output.
[c] The welfare maximizing growth rate of money.

It is also useful to compare this welfare cost with the cost of banking in Table 2. As we can see, the total welfare cost of inflation rises faster than the cost of banking. This means that as inflation rises the welfare cost of inflation increases mainly due to the lower growth rate of the economy. This qualifies the recent result of Aiyagari et al. (1998) who found in a neoclassical growth model that the welfare cost of inflation is bounded by about 5% of consumption. Our result obtained from an endogenous growth model indicates that the welfare cost of inflation can be substantially higher due the lower growth rate of the economy.

7 Conclusions and Qualifications

We proposed a monetary growth model to explain the observed magnitude of the growth effect of inflation. In the model, money and credit services incur exchange costs that affect the total cost of consumption. An increase in inflation causes an increase in banking time and leisure use, a lower net return on human capital, and a lower balanced-growth rate.

The calibration results show that the growth effect of inflation is much stronger in the presence of a credit service sector than in a conventional monetary growth model.

Moreover, our model can also explain why the effect of inflation on growth weakens as inflation rises as the data seems to suggest. The magnitude of the change in the growth rate depends inversely on the magnitude of the interest elasticity of money demand. The interest elasticity of money demand increases in magnitude when the ratio of credit services to money usage increases. The increase in interest elasticity coincides with an increase in the elasticity of substitution between money and credit. With such ease of substitution amongst means of exchange, there is a lessor burden on using the leisure channel in order to escape the inflation tax. Substitution towards leisure becomes weaker as the absolute value of the elasticity of substitution between money and credit is bigger. Meanwhile as more credit is used at a rising marginal cost, the social resource loss gets bigger, inducing less leisure (and goods). This adds up to a negative inflation-growth effect that gets weaker as the inflation rate increases. However, the effect is substantial at low inflation rates because an inelastic money demand causes sluggish but significant substitution towards time for producing credit services, plus more leisure time, resulting in significantly less human capital investment.

We also analyzed the determinants of the welfare cost of inflation. The model shows that the welfare costs rise with the nominal interest rate at a rate that depends on the degree of diminishing returns to labor in banking. In contrast, the answer in the framework of McCallum and Goodfriend (1989) is found by setting the general transaction technology after assuming a certain interest elasticity in order to calibrate the transaction parameters.[24] The advantage of the credit services approach is that an upward sloping supply curve provides for calibration based on the structural parameters of credit service production, leaving the behavioral parameters such as the interest elasticity of money

[24] This approach has been recently followed by Love and Wen (1999) who parametrize the McCallum-Goodfriend type technology to fit the money demand elasticities. In contrast, the money demand elasticity is determined by the credit service technology in our model.

demand to vary, as is crucial for calibrating the non-linearity of the inflation-growth effect.

It is useful to compare our results with that of Stokey and Rebelo (1995). They argue that elasticities of substitution between factors of substitution do not effect growth, while factor shares do matter, for the case of no leisure and when leisure is indexed by human capital as it enters the utility function. In these cases in our model, there similarly is no effect of inflation on growth, although this appears counter to empirical evidence. However with raw leisure time entering the utility function, factor elasticities of substitution do effect growth, if money and credit are viewed as factors in producing exchange, the cost of which is part of the shadow price of consumption goods. Meanwhile the factor shares in credit services production play a lesser role. The no-elasticity effect of Stokey and Rebelo (1995) is true in their models in which the absolute value of the elasticity of substitution between factors is assumed to be constant. In our model the elasticity of substitution between money and credit is endogenous. This elasticity depends partly on the assumed Cobb-Douglas share of labor in credit service production (γ), but also on the level of the inflation tax itself. The growth decrease is stronger at low levels of the inflation tax because the elasticity of substitution between money and credit is then low, and so the substitution from goods (and labor) to leisure is stronger. Put differently, dropping the inflation rate from 5% to 0 has a stronger positive effect on growth than dropping it from 15% to 10%. This is because the absolute value of the elasticity of substitution between money and credit is lower at low levels of the inflation rate, and the substitution between labor and leisure is higher. The elasticity of substitution between money and credit (inversely) determines the magnitude of the inflation-growth effect and changes in this elasticity create the non-linearity in the inflation-growth effect. The labor's factor share in credit services (γ) has a generally ambiguous effect on the inflation-growth effect and its non-linearities.

Finally, Stokey and Rebelo (1995) find lower effects of taxes on growth when only human capital is used to produce human capital as in our model, suggesting a dimension in which we may underestimate the inflation-growth effect. By including physical capital, the growth rate effect will depend not only on leisure usage directly, but also on a variable marginal product of human capital in the production of human capital.

Appendix

A Equivalence of Explicit and Implicit Banking Sectors

Proposition 1 *Stating the bank problem separately as a firm maximization problem is equivalent to keeping the bank sector implicit in the consumer problem, except that the credit services price and profit are revealed only with the explicit form.*

Proof. Let Π_t denote the bank profit in nominal terms at time t, and, let p_{ft} be the price of credit services. The bank maximizes total revenue minus total cost with respect to the choice of a_t, which is equivalent to the choice of labor time $b(a_t)$ in credit service production. The bank problem is to maximize profit

$$\max_{a_t} \Pi_t = \max_{a_t} \left[p_{ft}(1-a_t) - p_t w b(a_t) \right] c_t$$

as graphed in Figure 3, and where $b(a_t)$ is defined in (7). This gives the first-order condition of

$$\frac{p_{ft}}{p_t} = \frac{w}{\gamma\zeta} \left(\frac{1-a_t}{\zeta} \right)^{\frac{1}{\gamma}-1} \tag{A.1}$$

which has a standard price-theoretic interpretation: the relative marginal cost of the credit service (p_{ft}/p_t) equals the ratio of the marginal factor cost w divided by the marginal factor product $b'(a_t)$.

Bank profits are returned to the consumer as banker, and the consumer now pays the explicit fee p_{ft} for the credit service. The budget constraint of the consumer, instead of equation (5), is

$$\dot{m}_t = wh_t(l_t + b_t) + v_t + \frac{\Pi_t}{p_t} - \pi_t m_t - c_t - \frac{p_{ft}}{p_t}(1 - a_t)c_t, \tag{A.2}$$

The only difference to the consumer's first-order conditions occurs with respect to a_t, which now yields the relative price of the credit service

$$\frac{p_{ft}}{p_t} = R_t. \tag{A.3}$$

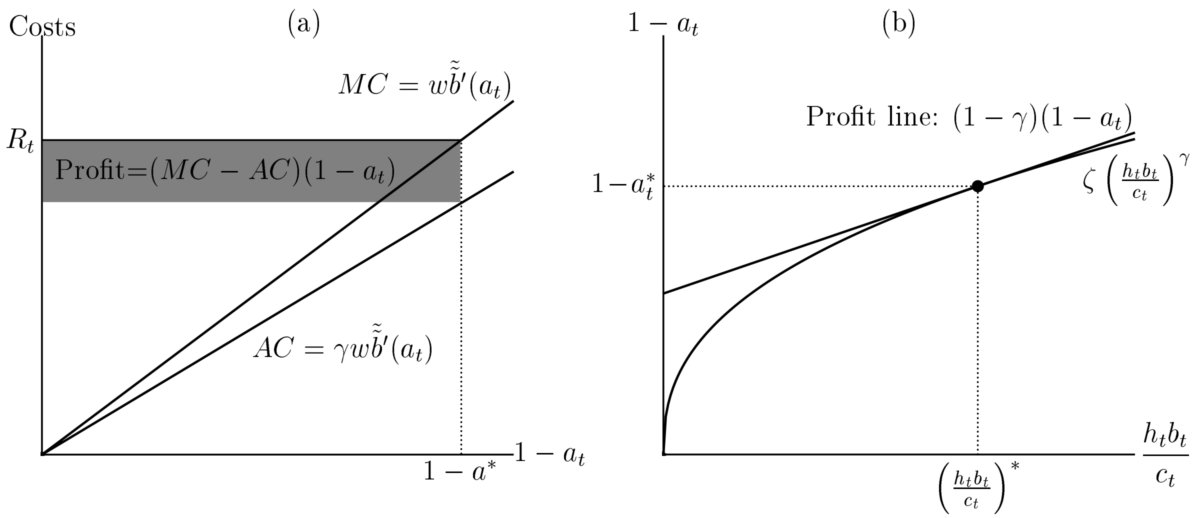
First, combining equations (A.1) and (A.3) yields equation (9a). The budget constraint with the explicit bank sector reduces to the implicit form as in equation (5) because

$$\begin{aligned} &wh_t(l_t + b_t) + \frac{\Pi_t}{p_t} - \frac{p_{ft}}{p_t}(1 - a_t)c_t \\ &\qquad = wh_t\left(l_t + b(a_t)\frac{c_t}{h_t}\right) + \left[\frac{p_{ft}}{p_t}(1 - a_t) - wb(a_t)\right]c_t - \frac{p_{ft}}{p_t}(1 - a_t)c_t \end{aligned} \tag{A.4}$$

which equals wh_tl_t. Therefore all equilibrium conditions are the same and the implicit problem is equivalent to the explicit problem. □

The marginal and average cost functions in the banking sector are upward sloping for any $\gamma \in (0, 1)$, and marginal cost lies everywhere above average cost except at the origin as it is displayed on Figure 3. Equilibrium occurs where the nominal interest rate, which is the equilibrium price of credit services, equals the marginal cost of production. The output-normalized welfare cost of inflation, in this sector, is the area under the marginal

Figure 3: Production of credit services



cost curve, which is the cost of production, $wb(a_t)$. Viewed from the marginal product instead of the marginal cost, Figure 3 graphs the production function for credit services, and the profit line. Equilibrium occurs where the marginal product of labor equals the real wage in the credit services sector, which is the ratio of the real wage to the nominal interest rate, w/R_t. The output-normalized welfare cost of inflation from this sector is the real labor costs, given in the graph as the difference between the output and the real profit, factored by R_t, or $wb(a_t) = \gamma(1 - a_t)R_t$. The production for credit services uses labor only and yields positive profits for the bank owner. The profit in equilibrium is equal to $1 - \gamma$ fraction of the revenues from selling the credit services. And the real wages paid to labor is also γ fraction of the revenues of credit services.

B The Existence of the Balanced-growth Equilibrium

The equilibrium along a balanced growth path satisfies the following equations:

$$\frac{m_t}{c_t} = a(R_t) \tag{B.5a}$$

$$\frac{c_t}{h_t} = wl_t \tag{B.5b}$$

$$g = \phi\left(1 - l_t - x_t - \tilde{b}(R_t)\frac{c_t}{h_t}\right) - \delta \tag{B.5c}$$

$$g = \frac{\phi(1 - x_t) - \delta - \rho}{\theta} \tag{B.5d}$$

$$R_t = \phi(1 - x_t) - \delta + (\sigma - g) \tag{B.5e}$$

$$\frac{h_t}{c_t} = \frac{\alpha[1 + a(R_t)R_t + w\tilde{b}(R_t)]}{wx_t}, \tag{B.5f}$$

where g is the common growth rate along the balanced growth path, (B.5a) is just the cash-in-advance constraint (2), equation (B.5b) is the budget constraint of the household (5) which has been rewritten using $\pi_t = \sigma - g$, and $v_t = \sigma m_t$. Similarly, stating (B.5f), we also used $\pi_t = \sigma - g$.

Proposition 2 *Suppose $\theta > 1$. Let $\underline{\sigma}$ and $\bar{\sigma}$ be defined by*

$$\underline{\sigma} = -\frac{(\theta - 1)(\phi - \delta) + \rho}{(\theta - 1)\alpha + \theta}, \tag{B.6a}$$

$$\bar{\sigma} = \underline{\sigma} + \bar{R}. \tag{B.6b}$$

where $\bar{R}$ is defined in equation (13). There is a $\breve{\sigma} \in [\underline{\sigma}, \bar{\sigma}]$ such that there is a unique balanced growth path for all $\sigma \in [\underline{\sigma}, \breve{\sigma})$.

Proof. First, the above system of equations implies that the following four equations determine the balanced growth path for the variables g, R_t, x_t, and l_t

$$\theta g = \phi(1 - x_t) - \delta - \rho \tag{B.7a}$$

$$R_t = (\theta - 1)g + \rho + \sigma \tag{B.7b}$$

$$\frac{1}{\alpha}\frac{x_t}{l_t} = 1 + a(R_t)R_t + w\tilde{b}(R_t) \tag{B.7c}$$

$$g = \phi(1 - x_t) - \delta - \phi[1 + w\tilde{b}(R_t)]l_t, \tag{B.7d}$$

where equation (B.7a) follows from (B.5d), equation (B.7b) follows from (B.5d) and (B.5e), equation (B.7c) follows from (B.5b) and (B.5f), and finally, equation (B.7d) follows from (B.5b) and (B.5c).

Next, observe that combining equations (B.7a) and (B.7d) results in

$$(\theta - 1)g + \rho = \phi[1 + w\tilde{b}(R_t)]l_t,$$

which can be plugged together with (B.7b) into equation (B.7c) leading to

$$x_t = \frac{\alpha(R_t - \sigma)}{\phi}\frac{1 + a(R_t)R_t + w\tilde{b}(R_t)}{1 + w\tilde{b}(R_t)}. \tag{B.8a}$$

Moreover, equations (B.7a) and (B.7b) yield

$$\frac{\theta}{\theta - 1}[R_t - \rho - \sigma] = \phi(1 - x_t) - \delta - \rho \tag{B.8b}$$

Finally, plugging equation (B.8a) into equation (B.8b) for x_t leads to

$$(R_t - \sigma)\left[\frac{\theta}{\theta - 1} + \frac{\alpha[1 + a(R_t)R_t + w\tilde{b}(R_t)]}{1 + w\tilde{b}(R_t)}\right] = \phi - \delta + \frac{\rho}{\theta - 1} \tag{B.9}$$

This equation determines the nominal interest rate along the balanced growth path for a given σ if it has a solution. It is easy to see that equations (B.7a)-(B.7d) can be solved uniquely for the variables of interest for a given R_t. Therefore, we only have to show that there is a unique solution of equation (B.9) in R_t for all $\sigma \in [\underset{\sim}{\sigma}, \breve{\sigma})$ for some $\breve{\sigma} \leq \bar{\sigma}$.

It is easy to check that if $\sigma = \underset{\sim}{\sigma}$, then $R_t = 0$ is a solution of the balanced growth path condition (B.9). Similarly if $\sigma = \bar{\sigma}$, then $\bar{R}$ is a solution of (B.9). Next, observe that the left hand side of (B.9) is decreasing in σ. Therefore for any $\sigma > \underset{\sim}{\sigma}$, there is a unique solution of equation (B.9) in terms of R_t if the left hand side is increasing in R_t. Let

$$h(R_t) = (R_t - \sigma)\left[\frac{\theta}{\theta - 1} + \frac{\alpha[1 + a(R_t)R_t + w\tilde{b}(R_t)]}{1 + w\tilde{b}(R_t)}\right].$$

To prove our claim, we have to show that there is a $\breve{\sigma}$ such that $h(R_t)$ is increasing in R_t for all $\sigma \in [\underset{\sim}{\sigma}, \breve{\sigma})$. Taking the derivative of $h(R_t)$ with respect to R_t we obtain

$$\frac{\partial h}{\partial R_t} = \frac{\theta}{\theta - 1} + \frac{\alpha[1 + a(R_t)R_t + w\tilde{b}(R_t)]}{1 + w\tilde{b}(R_t)} + \alpha(R_t - \sigma)\frac{a(R_t) - \gamma[1 + w\tilde{b}(R_t)]}{(1 - \gamma)[1 + w\tilde{b}(R_t)]^2} \tag{B.10}$$

We know from the definition of $\hat{R}$ in equation (19) that $a(R_t) \geq \gamma[1 + w\tilde{b}(R_t)]$ for all $R_t \leq \hat{R}$. Therefore $h'(R_t) > 0$ for all $R_t \leq \hat{R}$. The continuity of $h(R_t)$ implies that there is a $\breve{\sigma}$ such that equation (B.9) has a unique solution in R_t for all $\sigma \in [\underset{\sim}{\sigma}, \breve{\sigma})$. □

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Institut für Höhere Studien
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna
Austria

Phone: +43-1-599 91-149
Fax: +43-1-599 91-163
e-mail: woergoet@ihs.ac.at