IHS Economics Series
Working Paper 56
September 1998

Patents in a Model of Endogenous Growth

Ted O'donoghue
Josef Zweimüller

Institut für Höhere Studien
Institute for Advanced Studies
Vienna

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Reihe Ökonomie / Economics Series No. 56

Patents in a Model of Endogenous Growth

Ted O'Donoghue, Josef Zweimüller

Patents in a Model of Endogenous Growth

Ted O'Donoghue, Josef Zweimüller

Reihe Ökonomie / Economics Series No. 56

September 1998

Ted O'Donoghue
Department of Economics
Cornell University
414 Uris Hall
Ithaca, NY 14853-7601, USA
Phone: ++1-607-255-6287
Fax: ++1-607-255-2818
e-mail: edol@cornell.edu

Josef Zweimüller
University of Zurich and CEPR
Blümlisalpstrasse 10
CH-8006 Zurich, Switzerland
Phone: ++41-1-634-3724
Fax: ++41-1-634-4907
e-mail: zweim@iew.unizh.ch

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Abstract

This paper examines patent protection in an endogenous-growth model. Our aim is twofold. First, we show how the patent policies discussed by the recent patent-design literature can influence R&D in the endogenous-growth framework, where the role of patents has been largely ignored. Second, we explore how the general-equilibrium framework contributes to the results of the patent-design literature. In a general-equilibrium model, both incentives to innovate and monopoly distortions depend on the proportion of industries that conduct R&D. Furthermore, patents affect the allocation of R&D resources across industries, and patents can distort resources away from industries where they are most productive.

Keywords
Innovation, patent policy, intellectual property, patent design

JEL-Classifications
O31. O34, O38

Comments

We thank Reto Föllmi, Gene Grossman and Suzanne Scotchmer for useful comments. O'Donoghue would like to thank the France/Berkeley Fund, the Alfred P. Sloan Foundation, and the Institute for Advanced Studies, Vienna, for financial support.

1. Introduction

The recent endogenous-growth literature has emphasized the role of industrial R&D in economic growth (Romer (1990), Segerstrom *et al* (1990), Aghion and Howitt (1992), Grossman and Helpman (1991), Stokey (1995), Kortum (1997)). In a general-equilibrium framework, growth is driven by technological progress achieved in private firms, and the literature asks what factors stimulate or retard growth. While the literature recognizes the need for patent protection to stimulate industrial R&D, there has been surprisingly little attention paid to the impact of patent policy on growth.[1]

The patent-design literature, on the other hand, addresses exactly the question of how does patent policy affect incentives for industrial R&D.[2] However, the patent-design literature has, for the most part, confined itself to partial-equilibrium analysis. This deficiency seems particularly important: Since a single patent-policy applies to multiple industries, an analysis of how policy affects a single industry seems incomplete.

In this paper, we attempt to merge these two literatures. The aim of the paper is twofold. First, we examine the role of patent policy in the context of endogenous growth. Second, we explore how the general-equilibrium framework contributes to the results of the patent-design literature.

The patent-design literature examines (at least) four tools of patent policy. *Patent life* is the length of time for which a patent is valid. A *patentability requirement* is a minimum innovation size required to receive a patent. A patent's *breadth* puts restrictions on the products other firms can produce (without a license). *Lagging breadth* limits imitation by specifying inferior products that other firms cannot produce. *Leading breadth* limits future innovators by specifying superior products that other firms cannot produce. Notice that lagging breadth represents protection against imitation, whereas leading breadth and a patentability requirement represent protection against future innovators. See O'Donoghue (forthcoming) for a more detailed discussion of how these policy instruments have been used in the literature as well as how these policy instruments relate to the existing patent law.

The endogenous-growth literature has recognized the need for patent protection to stimulate

[1] For exceptions, see Segerstrom (1992), Davidson and Segerstrom (1993), and Helpman (1993), although these papers examine protection against potential imitators and our focus shall be protection against future innovators.

[2] There is a long line of research following Nordhaus (1969) examining patent design for isolated innovations. More recently, the literature has addressed patent design for cumulative innovation. For two-stage models, see Scotchmer (1991,1996a,1996b), Green and Scotchmer (1995), Scotchmer and Green (1990), Chang (1995), Matutes, Regibeau, and Rockett (1996), and Van Dijk (1996). For models of sequential innovation, see O'Donoghue, Scotchmer, and Thisse (1998), O'Donoghue (forthcoming), and Hunt (1995).

growth. For the most part, however, attention has been limited to one simple policy – often called "infinitely-lived patents". More accurately, we interpret the endogenous-growth patent policy as infinitely-lived patents that prevent all imitation (i.e., there is complete lagging breadth), but allow any superior product to displace the innovator (i.e., there is no leading breadth and no patentability requirement).

In Section 2, we outline a model of endogenous growth along the lines of Grossman and Helpman (1991) and Aghion and Howitt (1992). Within this framework, we embed a model of patent policy, which includes the standard endogenous-growth patent policy but also allows for policies with protection against future innovators. In Section 3, we address our first question: What is the role of patents for endogenous growth? We take the endogenous-growth policy as a benchmark, and then show how protection against future innovators can stimulate R&D investment. Specifically, if there is underinvestment under the benchmark policy, then policy can stimulate R&D with either a patentability requirement (as suggested by Hunt (1995) and O'Donoghue (forthcoming)), or with leading breadth (as suggested by O'Donoghue, Scotchmer, and Thisse (1998)).

In Section 4, we address our second question: What is the patent-design literature missing by using partial-equilibrium analysis? A partial-equilibrium analysis of patent policy ignores the fact that policy changes affect multiple industries. A general-equilibrium analysis enables us to incorporate this feature of patent policy. In doing so, we find two important factors missing from partial-equilibrium analysis. First, there is a general-equilibrium effect from policy changes. In a partial-equilibrium framework, stronger patents imply increased profits for successful firms. In a general-equilibrium framework, however, these increased profits imply higher aggregate income and therefore increased demand for all industries, which increases profits further. Hence, there is a multiplier effect (or pecuniary externality) which reinforces the partial-equilibrium effect of patents. This pecuniary externality is stronger when more industries use patents, since stronger patents imply increased profits in innovative industries only. The second factor missing from most partial-equilibrium analyses is a formal model of the static inefficiency (or output distortions) associated with patents. The fact that multiple industries use patents can imply that the output distortions created by patents are small – indeed there may essentially be no output distortions in the extreme cases where almost all industries use patents or very few industries use patents. Hence, partial-equilibrium analyses may be overemphasizing the importance of monopoly distortions created by patents.

In Section 5, we extend our model to explore an issue largely ignored by both the endogenous-growth literature and the patent-design literature: What are the implications of there being asymmetric R&D capabilities across industries? The empirical R&D literature suggests there are significant cross-industry differences in R&D productivity (see the survey by Cohen and Levin (1989)). If there are asymmetric R&D capabilities across industries, then in addition to the aggregate level of R&D, policy must also concern itself with the allocation of R&D resources across industries. We find that the private equilibrium tends to distort R&D resources away from those industries where these resources are more productive. Furthermore, stronger patent protection can exacerbate these distortions. These results are driven by a higher rate of creative destruction in the more productive industries, which induces firms to invest less than desired in those industries.

We conclude in Section 6 by discussing some limitations of our model and also some general lessons to take away from our analysis.

2. A Model of Endogenous Growth

In this section, we lay out a model of endogenous growth that is similar to those in Aghion and Howitt (1992) and Grossman and Helpman (1991).[3] Within this model we embed a model of patent policy that includes the standard endogenous-growth patent policy as a special case but also allows for protection against future innovators. Before introducing the model, however, we briefly describe the essential characteristics.

We consider a simple economy where there are two types of industries. First, there is a *high-technology sector* – a set of industries that conduct R&D to improve product quality. Second, there is a *noninnovative sector* – a set of industries where quality improvements are not possible. For simplicity we assume that labor is the only productive input in all sectors and that there is a fixed set of (labor) resources. Within this setting there are two allocative questions: (i) how to allocate labor between production (for consumption) and R&D; and (ii) how to allocate production labor between the high-technology sector and the noninnovative sector. In addition, there is a third question that will affect the performance of this economy: (iii) How ambitious should the R&D projects be in

[3] In fact, Jones (1995) has rejected these models because they exhibit scale effects that are inconsistent with time-series evidence, and Kortum (1997) has proposed a more general quality-ladder model that is more consistent with empirical evidence. Even so, the basic intuitions we identify in our simple model extend to more sophisticated models such as Kortum's, as we discuss in Section 6.

the high-technology sector (i.e., should firms pursue small or large quality improvements)?

The Underlying Model

The underlying model has three components: the R&D process, intertemporal preferences, and the resource constraint.

We follow Aghion and Howitt (1992), Grossman and Helpman (1991), and Segerstrom *et al* (1990) by supposing that economic growth is driven by endogenous product improvements. There is a continuum of goods indexed by $\omega \in [0,1]$, each produced within its own industry. Individual goods may be available in multiple qualities. For good ω, let $q_\omega(t)$ be the maximum technologically feasible quality at time t. At time t, all firms are capable of producing any quality $q \leq q_\omega(t)$ (i.e., imitation is costless), and no firm is capable of producing any quality $q > q_\omega(t)$.[4]

The evolution of $q_\omega(t)$ for each ω is determined by R&D behavior. We assume that quality improvements occur in only a fraction of industries. Specifically, there is an $\bar{\omega} \in [0,1]$ such that in industries with $\omega \in (\bar{\omega}, 1]$, $q_\omega(t) = 1$ for all t. We refer to these industries as the *noninnovative sector*. The industries with $\omega \in [0, \bar{\omega}]$ constitute the *high-technology sector*.[5] In each high-technology industry, firms conduct R&D to repeatedly increase the maximum feasible quality in that industry. For simplicity, we assume $q_\omega(0) = 1$ for each $\omega \in [0, \bar{\omega}]$, and that the i^{th} innovation in industry ω increases the maximum feasible quality by factor $\gamma_{i\omega} > 1$. Hence, if there have been exactly I innovations in industry ω before date t, then $q_\omega(t) = \Pi_{i=1}^{I} \gamma_{i\omega}$. The *innovation size* $\gamma_{i\omega}$ is endogenous, as described below.

Innovations occur according to a Poisson process. If a firm has arrival rate of innovations ϕ, then the date of success τ has cumulative distribution $F(\tau) = 1 - e^{-\phi\tau}$. Each firm's arrival rate depends on the number of research workers it hires and on the innovation size it pursues. If a firm hires n research workers and pursues innovation size γ, it will have Poisson arrival rate $\phi = \lambda(\gamma)n$, where $\frac{d\lambda}{d\gamma} < 0$ and $\frac{d^2\lambda}{d\gamma^2} \leq 0$. As in the endogenous-growth literature, there are constant returns to scale for R&D labor. In addition, the Poisson arrival rate is decreasing in the innovation size – that is, larger innovations are more difficult to achieve. We assume that the research technology is identical in all industries and at all times. Furthermore, we assume that firms' R&D processes are independent, so the arrival rate of innovations in industry ω, denoted ϕ_ω, is the sum of the individual firms' arrival

[4] We consider a model along the lines of Grossman and Helpman (1991) where product improvements occur in consumption goods. Everything is essentially the same in a model where product improvements occur in intermediate goods, as in Aghion and Howitt (1992).

[5] In their basic model, Grossman and Helpman (1991) consider the case where all industries are innovative, or $\bar{\omega} = 1$.

rates.

Product quality matters because consumers prefer to consume higher-quality goods. We assume there are L consumers with identical intertemporal preferences that can be represented by the intertemporal utility function

$$U = \int_0^\infty e^{-\rho t} \, ln \, u(t) \, dt \tag{1}$$

where t is an index of continuous time, ρ is the rate of time preference, and $ln \, u(t)$ is instantaneous utility at time t. The instantaneous utility function is

$$ln \, u(t) = \int_0^1 ln \, [q_\omega(t) \, x_\omega(t)] \, d\omega \tag{2}$$

where $x_\omega(t)$ is the *quantity* consumed of quality $q_\omega(t)$ of good ω at date t. This formulation assumes that a person consumes only the maximum feasible quality from each industry. Even so, we require that the maximum feasible quality $q_\omega(t)$ have the lowest quality-adjusted price in industry ω (because otherwise consumers would purchase an inferior quality). In other words, if good ω is also available in quality q' at price p', then the price of quality $q_\omega(t)$, denoted $p_\omega(t)$, must satisfy $\frac{p_\omega(t)}{q_\omega(t)} \leq \frac{p'}{q'}$.[6]

The final component of the underlying model is the resource constraint. We assume each person supplies one unit of labor, so that the total labor supply is L. We assume that there are constant returns to scale in output production, and that in each industry labor is the only input with a unit labor requirement a for all qualities.[7] If each consumer consumes $x_\omega(t)$ from industry ω, total consumption from industry ω is $Lx_\omega(t)$, and the total labor requirement is $Lax_\omega(t)$. Economy-wide employment in production is therefore $\int_0^1 Lax_\omega(t) \, d\omega$. Let $N(t)$ be the total number of research workers economy-wide, and $n_\omega(t)$ be the number of research workers in industry ω, which implies $N(t) = \int_0^{\bar{\omega}} n_\omega(t) d\omega$. Then for all t the *resource constraint* is

$$L = La \int_0^1 x_\omega(t) \, d\omega + \int_0^{\bar{\omega}} n_\omega(t) d\omega = La \int_0^1 x_\omega(t) \, d\omega + N(t). \tag{3}$$

6 The instantaneous utility function and the assumption about when consumers purchase the maximum feasible quality represent a reduced form of underlying instantaneous utility function

$$ln \, u(t) = \int_0^1 ln \left[\int_0^{q_\omega(t)} q \, x_{\omega q}(t) \, dq \right] d\omega$$

where $x_{\omega q}(t)$ is the *quantity* consumed of quality q of good ω at date t.

7 We could allow for a different labor requirement in the noninnovative sector than in the high-technology sector. Such an assumption would not change the qualitative nature of our results.

We can summarize the underlying model as follows. The exogenous parameters are the number of consumers L, the intertemporal time preference ρ, the labor requirement for production a, and the fraction of industries that are innovative $\bar{\omega}$. The endogenous variables are the labor allocations $x_\omega(t)$ and $n_\omega(t)$, and the innovation sizes $\gamma_{i\omega}$.

The Social Optimum

A social planner will choose the endogenous variables to maximize intertemporal utility (equations (1) and (2)) subject to the resource constraint (equation (3)). Given the stationary nature of the model, the socially optimal labor allocations will be stationary – that is, for all ω we have $x_\omega(t) = x_\omega$ and $n_\omega(t) = n_\omega$ for all t. Furthermore, the socially optimal innovation size will be constant, or $\gamma_{i\omega} = \gamma$ for all i and ω.

Equation (2) can be written as $ln\ u(t) = \int_0^1 ln\ x_\omega d\omega + \int_0^1 ln\ q_\omega(t)\ d\omega$, and we can then write intertemporal utility as

$$U = \frac{\int_0^1 ln\ x_\omega d\omega}{\rho} + \int_0^1 \left[\int_0^\infty e^{-\rho t}\ ln\ q_\omega(t)\ dt\right] d\omega.$$

Since for any noninnovative industry $\omega \in (\bar{\omega}, 1]$, $q_\omega(t) = 1$ for all t, $\int_0^\infty e^{-\rho t}\ ln\ q_\omega(t)\ dt = 0$. For any high-technology industry $\omega \in [0, \bar{\omega}]$, if the stationary Poisson arrival rate of innovations is $\lambda(\gamma)n_\omega$, then we have[8]

$$\int_0^\infty e^{-\rho t}\ ln\ q_\omega(t)\ dt = \frac{ln\ q_\omega(0)}{\rho+\lambda(\gamma)n_\omega} + \left(\frac{\lambda(\gamma)n_\omega}{\rho+\lambda(\gamma)n_\omega}\right)\frac{ln\ [\gamma q_\omega(0)]}{\rho+\lambda(\gamma)n_\omega} + \left(\frac{\lambda(\gamma)n_\omega}{\rho+\lambda(\gamma)n_\omega}\right)^2\frac{ln\ [\gamma^2 q_\omega(0)]}{\rho+\lambda(\gamma)n_\omega} + \cdots$$

$$= \frac{ln\ q_\omega(0)}{\rho} + \frac{ln\ \gamma}{\rho}\frac{\lambda(\gamma)n_\omega}{\rho}.$$

Since for each $\omega \in [0, \bar{\omega}]$ $q_\omega(0) = 1$, we have

$$\rho U = \int_0^1 ln\ x_\omega d\omega + \frac{ln\ \gamma}{\rho}\lambda(\gamma)\int_0^{\bar{\omega}} n_\omega d\omega = \int_0^1 ln\ x_\omega d\omega + \frac{ln\ \gamma}{\rho}\lambda(\gamma)N.$$

If the social planner allocates labor aX to production, each consumer receives total consumption $\int_0^1 x_\omega d\omega = \frac{X}{L}$. Given this constraint, and the identical production technology in all industries, $\int_0^1 \ln x_\omega d\omega$ is maximized by consuming equal quantities from all industries. Hence, for any X, a social planner will choose $x_\omega = \frac{X}{L}$ for all ω. The resource constraint then implies $X = \frac{L-N}{a}$, and

8 This equation uses the following calculations:
(a) If the flow profit π is received *until* uncertain time t that has Poisson arrival rate ϕ, then it has expected value

$$\int_0^\infty \left(\pi\frac{1-e^{-\rho t}}{\rho}\right)\phi e^{-\phi t}dt = \frac{\pi}{\rho+\phi}.$$

(b) If the payoff v is received *at* uncertain time t that has Poisson arrival rate ϕ, then it has expected value

$$\int_0^\infty \left(ve^{-\rho t}\right)\phi e^{-\phi t}dt = \frac{\phi}{\rho+\phi}v.$$

the social planner's problem becomes choosing γ and N to maximize

$$\rho U = \ln\left(\frac{L-N}{La}\right) + \frac{\ln\gamma}{\rho}\lambda(\gamma)N.$$

If we let γ^* denote the socially optimal innovation size, and N^* denote the socially optimal level of aggregate R&D, then γ^* and N^* satisfy

$$\frac{\lambda(\gamma^*)}{-\frac{d\lambda}{d\gamma}} = \gamma^* \ln\gamma^* \tag{4}$$

$$\text{and} \qquad N^* = L - \frac{1}{\ln\gamma^*}\frac{\rho}{\lambda(\gamma^*)}. \tag{5}$$

γ^* and N^* are both independent of $\bar{\omega}$, the fraction of industries that conduct R&D. Furthermore, the social planner cares only about aggregate R&D N^* and not how R&D labor is allocated across industries (i.e., any set of n_ω such that $\int_0^{\bar{\omega}} n_\omega d\omega = N^*$ will do). These results follow from the separability of quality and quantity in the instantaneous utility function, and the constant returns to scale in R&D. Separability implies consumers are indifferent to where quality improvements occur, and constant returns imply R&D is equally good in all industries.

Patent Policy

Our focus is the performance of this economy relative to the social optimum when policymakers are constrained to create incentives for R&D with patents. We now outline patent policy in detail. Following O'Donoghue, Scotchmer, and Thisse (1998) and O'Donoghue (forthcoming), when innovation improves products along a unidimensional quality measure, we can identify four distinct tools of patent design: Patent life, a patentability requirement, lagging breadth, and leading breadth. *Patent life* is the length of time for which a patent is valid. A *patentability requirement* specifies a minimum innovation size required to patent a new product. A patent's breadth specifies products that would infringe upon the patent, or products that cannot be produced without the patentholder's permission (in the form of a license). *Lagging breadth* specifies inferior products that cannot be produced, and *leading breadth* specifies superior products that cannot be produced.

A main goal of this paper is to illustrate how protection against future innovators can stimulate R&D relative to patents without such protection. Except for occasional discussions, we simplify the analysis by fixing patent life and lagging breadth, so that the policy tools of interest are a patentability requirement and leading breadth. Specifically, we focus on patent policies where patent life is infinite and there is "complete lagging breadth". In our model, patents are effectively terminated by future innovations, not by when the patent expires. (O'Donoghue, Scotchmer, and Thisse (1998) define this as "effective patent life".) Hence, infinite patent life is meant to be a proxy representing that there is a very small probability of a patent having value when it expires. "Complete lagging breadth" means that all qualities made feasible by an innovator are protected. In other words, if a firm has a patent on quality q_o and its innovation size was γ, then no other firm can produce any quality $q \in (\frac{1}{\gamma}q_o, q_o]$ during the life of the patent without a license. An implication of infinite patent life and complete lagging breadth is that at all times the most recent innovator will produce the maximum feasible quality $q_\omega(t)$, and the nearest rival will be the previous innovator in that industry.

We denote a patentability requirement by $P \in [1, \infty)$, where a firm can receive a patent only if it has innovation size $\gamma \geq P$. Since imitation is costless, firms will only pursue innovations on which they can receive a patent. Hence, a patentability requirement represents a lower bound on innovation size.

We denote leading breadth by $K \in [1, \infty)$ such that the patent on quality q_o prevents other firms from producing any quality $q \in [q_o, Kq_o)$ during the life of the patent.

There is an important distinction between a patentability requirement and leading breadth, both of which put restrictions on future innovators. A patentability requirement restricts what future innovators can *patent*, and leading breadth restricts what they can *produce* without infringing. Hence, for example, even if an innovator can get a patent on his new product, he may have to pay licensing fees to some previous innovators in order to produce.

We will denote a specific patent policy by $\psi \equiv (P, K)$. Under policy ψ, all patents have infinite patent life, complete lagging breadth, leading breadth K, and the patentability requirement for each generation is P. Different patent policies will induce different outcomes.

The endogenous-growth models of Grossman and Helpman (1991) and Aghion and Howitt (1992) assume a very simple patent policy: Each successful firm receives an infinitely-lived patent that prevents other firms from producing its quality. There is no explicit patent breadth, but these models effectively assume complete lagging breadth since product quality is discrete – discrete product quality implies that the nearest *feasible* competing product is the previous state-of-the-art product. Importantly, however, there is no patentability requirement and no leading breadth (i.e., there is no protection against future innovators). Hence, we interpret the endogenous-growth patent policy as the policy $(P = 1, K = 1) \equiv \psi_o$.[9] We next solve for the private equilibrium under patent policy ψ_o. We will use this outcome as a benchmark against which to compare policies with protection against future innovators in Section 3.

The Private Equilibrium under Policy ψ_o

We now examine how the endogenous variables – $x_\omega(t)$ and $n_\omega(t)$ for each ω and t, and $\gamma_{i\omega}$ for each i and ω – are determined in private markets under patent policy ψ_o. To do so, we must examine output markets (from which all profits are derived), and R&D markets (where firms innovate in order to gain an advantageous output market position).

We begin with the demand functions for consumption goods. Consumers maximize utility (equations (1) and (2)) subject to their budget constraint. We suppose all consumers have identical wealth $A(0)$ (e.g., all consumers own equal shares of all firms). In addition, each consumer supplies one unit of labor at all times that earns wage w. At time t, for each industry ω each consumer purchases quantity $x_\omega(t)$ of quality $q_\omega(t)$ at price $p_\omega(t)$. Letting r denote the rate of interest, each consumer's budget constraint is

$$\int_0^\infty e^{-rt}\left[\int_0^1 p_\omega(t)\, x_\omega(t)\, d\omega\right] dt \;\leq\; A(0) + \int_0^\infty e^{-rt} w\, dt.$$

Wealth evolves according to $\dot{A}(t) = [rA(t) + w] - \left[\int_0^1 p_\omega(t)\, x_\omega(t)\, d\omega\right]$. Restricting attention to balanced growth steady states where $\dot{A}(t) = 0$ for all t, we have $r = \rho$, and for all t *instantaneous income* is $w + \rho A(0) \equiv Y$. Each consumer spends exactly Y at any date t, and chooses consumption bundle $\{x_\omega(t)\}_{\omega\in[0,1]}$ to maximize instantaneous utility. Given Cobb-Douglas utility, consumers allocate income Y to each industry ω, which means that for each industry ω each consumer demands quantity $\frac{Y}{p_\omega(t)}$. Since there are L consumers, the demand function for each consumption industry is $\frac{LY}{p_\omega(t)}$.

Next, we examine the optimal behavior of firms in output markets. We assume each noninnovative industry $\omega \in (\bar{\omega}, 1]$ is competitive. Firms price at marginal cost, so given labor requirement a and wage w, $p_\omega(t) = wa$ for all t. Hence, at all times each consumer purchases $x_\omega(t) = \frac{Y}{wa}$ for each $\omega \in (\bar{\omega}, 1]$.

In each high-technology industry $\omega \in [0, \bar{\omega}]$, patent protection creates imperfect competition. At time t, the most recent innovator produces quality $q_\omega(t)$, and patent protection will determine

9 A few papers consider weaker patent policy. For example, Segerstrom (1992) and Davidson and Segerstrom (1993) explore patent protection where they assume stronger patent protection implies a decreased probability of imitation, and Helpman (1993) explores property rights where he assumes that tighter property rights imply imitation is more costly. With our terminology, these papers explore lagging breadth, and assume no leading breadth.

the largest quality $q' < q_\omega(t)$ that a rival can produce without a license. We shall refer to the most recent innovator as the *market leader.* The market leader will serve the entire market, but the price $p_\omega(t)$ is constrained in that it must be the lowest quality-adjusted price in industry ω. Since rivals are willing to price at marginal cost wa, the market leader's price will be $p_\omega(t) = \mu wa$ where the markup $\mu = \frac{q_\omega(t)}{q'}$. As we shall see, throughout the markup μ will be the same in all high-technology industries and at all times. Given demand function $\frac{LY}{p_\omega(t)}$, at all times the market leader in each industry $\omega \in [0, \bar{\omega}]$ earns profit $\pi = LY\left(\frac{\mu-1}{\mu}\right)$.

We next examine behavior in R&D markets. We assume perfect competition in the market for R&D. As a result, it will turn out that the market leader in industry ω will not conduct R&D in industry ω. As in Grossman and Helpman (1992) and Aghion and Howitt (1991), it is more profitable to gain a one-step advantage than to extend a one-step advantage to a two-step advantage. Consider the payoff to an R&D firm that chooses innovation size γ and hires R&D labor n. The firm's instantaneous wage cost is wn. The firm has success with Poisson arrival rate $\lambda(\gamma)n$. If we let V represent the reward to success, then the firm's expected instantaneous payoff is $-wn + \lambda(\gamma)nV$.

What is the reward to success V? When market leaders do not conduct R&D, at all times every market leader has a one-step advantage. This means that if a firm has an innovation of size γ, the firm is able to charge markup $\mu = \gamma$ (given complete lagging breadth). Hence, an R&D firm will earn flow of profit $\pi = LY\left(\frac{\gamma-1}{\gamma}\right)$ following a success. Since market leaders do not conduct R&D, the reward to success consists of earning flow profit π until the first subsequent success in the industry. If ϕ is the equilibrium Poisson arrival rate in the industry, then this flow of profit has discounted expected value $\frac{\pi}{\rho+\phi}$ (see the calculation in footnote 8). In other words, the reward to success $V = \frac{\pi}{\rho+\phi}$.

We can now rewrite an R&D firm's instantaneous payoff as

$$-wn + \lambda(\gamma)n\frac{\pi}{\rho+\phi} \quad = \quad -wn + \lambda(\gamma)nLY\left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\rho+\phi}$$

where the R&D firm takes L, Y, w, ρ, and ϕ as given and chooses γ and n. Since all R&D firms in all industries will choose γ to maximize $\lambda(\gamma)\left(\frac{\gamma-1}{\gamma}\right)$, at all times all firms conducting R&D will choose innovation size γ_o defined by

$$\frac{\lambda(\gamma_o)}{-\frac{d\lambda}{d\gamma}} \quad = \quad \gamma_o(\gamma_o - 1). \tag{6}$$

Since there are constant returns to scale in the number of research workers n, the individual research venture is of indeterminate size. Even so, free entry requires

$$\lambda(\gamma_o)\frac{\pi}{\rho+\phi} = w. \tag{7}$$

We shall denote the private equilibrium level of R&D labor by $N_o = \bar{\omega} n_o$, where n_o is the number of R&D workers hired in each high-technology industry and N_o is the number of R&D workers hired economy-wide.

At this point, it is convenient to combine the demand functions for consumption goods, optimal firm behavior in output markets, and the resource constraint (equation (3)) into a single equation that expresses market profits π as a function of the markup μ, aggregate R&D N, and exogenous parameters. As described above, for all t, $x_\omega(t) = \frac{Y}{\mu w a}$ for $\omega \in [0,\bar{\omega}]$ and $x_\omega(t) = \frac{Y}{wa}$ for $\omega \in (\bar{\omega},1]$. Plugging these equations into equation (3) and solving for instantaneous income Y, we obtain $Y = \frac{L-N}{L}\frac{\mu}{(1-\bar{\omega})\mu+\bar{\omega}}$. Since $\pi = LY\left(\frac{\mu-1}{\mu}\right)$, we can conclude that

$$\pi = \frac{(L-N)(\mu-1)}{(1-\bar{\omega})\mu+\bar{\omega}}. \tag{8}$$

Note that equation (8) is a condition that must hold in equilibrium, but does not represent market profits as perceived by individual firms. Substituting π from equation (8) into the no-profit condition (equation (7)), we derive the following expression for N_o in terms of exogenous parameters:

$$N_o = \bar{\omega} n_o = \begin{cases} \bar{\omega}\left[\frac{\gamma_o-1}{\gamma_o}L - \frac{(1-\bar{\omega})\gamma_o+\bar{\omega}}{\gamma_o}\frac{\rho}{\lambda(\gamma_o)}\right] & \text{if } L \geq \frac{(1-\bar{\omega})\gamma_o+\bar{\omega}}{\gamma_o-1}\frac{\rho}{\lambda(\gamma_o)} \\ 0 & \text{if } L \leq \frac{(1-\bar{\omega})\gamma_o+\bar{\omega}}{\gamma_o-1}\frac{\rho}{\lambda(\gamma_o)}. \end{cases} \tag{9}$$

We summarize this outcome in the following lemma:

Lemma 1 *Under policy ψ_o:*
(i) The market leader in industry $\omega \in [0,\bar{\omega}]$ does not conduct R&D.
(ii) Firms choose innovation size γ_o defined by equation (6).
(iii) The markup in industry $\omega \in [0,\bar{\omega}]$ is $\mu = \gamma_o$.
(iv) The equilibrium level of R&D N_o is given by equation (9).

Consider how the outcome under policy ψ_o compares to the socially optimal outcome. A comparison of equation (6) to equation (4) reveals that $\gamma_o < \gamma^*$. And a comparison of equations (5) and (9) reveals that for any $\bar{\omega}$ the private market outcome can involve too little or too much R&D. These results are identical to those in Grossman and Helpman (1991) and Aghion and Howitt (1992), and the intuition is as described there.

Constrained Social Welfare

Patents create markups in the high-technology sector, whereas price equals marginal cost in the noninnovative sector. These markups distort consumption towards the noninnovative sector. We close this section by deriving a constrained social welfare function that takes these markups into account. Suppose a patent policy induces economy-wide R&D N, but creates markup μ in each high-technology industry. Then consumption from each noninnovative industry is $x_N \equiv \frac{Y}{wa}$, and consumption from each high-technology industry is $x_H \equiv \frac{Y}{\mu wa}$. Plugging $x_N = \mu x_H$ into the resource constraint (equation (3)) yields $x_H = \frac{1}{(1-\bar{\omega})\mu+\bar{\omega}} \frac{L-N}{La}$. We can then write intertemporal utility as

$$\rho U = (1-\bar{\omega})\, ln\left(\frac{\mu}{(1-\bar{\omega})\mu+\bar{\omega}} \frac{L-N}{La}\right) + \bar{\omega}\, ln\left(\frac{1}{(1-\bar{\omega})\mu+\bar{\omega}} \frac{L-N}{La}\right) + \frac{ln\,\gamma}{\rho}\lambda N \quad (10)$$

$$\text{or} \quad \rho U = ln\left(\frac{\mu^{(1-\bar{\omega})}}{(1-\bar{\omega})\,\mu+\bar{\omega}}\right) + ln\left(\frac{L-N}{L\,a}\right) + \frac{ln\,\gamma}{\rho}\lambda N.$$

In other words, $\rho U = \Omega - D$ where

$$D \equiv -ln\left(\frac{\mu^{(1-\bar{\omega})}}{(1-\bar{\omega})\,\mu+\bar{\omega}}\right) \geq 0 \quad (11)$$

$$\text{and} \quad \Omega \equiv ln\left(\frac{L-N}{L\,a}\right) + \frac{ln\,\gamma}{\rho}\lambda N. \quad (12)$$

D represents the *static inefficiency* associated with markup μ in the high-technology sector. With Cobb-Douglas utility, the static inefficiency is independent of the level of consumption. Ω represents *dynamic social welfare* resulting from the allocation of labor between R&D and consumption. Notice that Ω is maximized at N^*. Writing constrained welfare in this way makes clear the trade-off between dynamic and static efficiency. Patents stimulate R&D by allowing successful firms to earn market profits. Stronger patent protection implies increased R&D, and therefore increased dynamic social welfare when $N < N^*$. However, stronger patent protection also implies larger markups, and therefore increased static inefficiency. In other words, patents can increase growth only at the cost of higher static inefficiencies. Optimal patent policy must weigh this trade-off.

3. Protection Against Future Innovators

In the previous section, we showed that under the endogenous-growth patent policy ψ_o, firms choose suboptimally small innovation size and might hire too little or too much R&D labor. In this section, we ask how different patent policies can correct incentives for R&D.

Before we discuss the use of patent policy to correct R&D incentives, however, a brief discussion of subsidies is in order.[10] In the endogenous-growth literature, the limited attention to how policy can affect the incentive to innovate has focused on R&D subsidies (and taxes). For example, Grossman and Helpman (1991) and Stokey (1995) correctly point out that an R&D subsidy or tax can induce the first-best level of R&D. If policymakers can use R&D subsidies, however, there is no reason for patents. In fact, if there is a noninnovative sector so markups create output distortions, optimal policy would involve essentially *no* patent protection and a very large R&D subsidy.[11]

Furthermore, there is reason to believe that R&D subsidies are not a feasible policy tool. If marginal subsidies (or taxes) are applied to R&D costs, there can be informational problems since firms are likely to have some flexibility in what they claim to be an "R&D cost". Patents, on the other hand, are granted only if a firm actually achieves an innovation. Lump-sum subsidies for successful firms (i.e., prizes – see Wright (1983)) can also be problematical because there may be uncertainty over the value of each innovation. The problem of "picking winners" arises. Again, patents may avoid such informational problems. If firms know the value of their innovations, patents may be useful as a revelation device (see Cornelli and Schankerman (1995) and Scotchmer (1997)). If nobody knows the value of innovations, patents are a way of letting the marketplace decide the value.[12]

The point is that there can be problems with R&D subsidies that are less prevalent with patents. After all, patents are used extensively, and the use of R&D subsidies is limited. In this paper, we take the perspective that R&D subsidies are not an effective policy instrument. Instead,we focus on a situation where patents are the only instrument for policy makers to stimulate R&D.

10 See Aghion and Howitt (1998) for further discussion of policies that subsidize R&D.

11 Segerstrom (1992) discusses such a policy in the context of uncertain innovation size. With certain innovation size, he shows that if policymakers can also tax the noninnovative sector, they can counteract the markups created by patents. However, an output tax on the noninnovative sector seems to be out of the realm of R&D policy.

12 See also Kremer (1996), who proposes a mechanism that combines R&D subsidies and patents with the goal of letting the marketplace reveal the value of an innovation but then having the government purchase the patent and make the innovation freely available.

Recall that a specific patent policy is $\psi \equiv (P, K)$, where P is the patentability requirement and K is leading breadth. We now define some notation to denote the equilibrium outcome as a function of patent policy. Let $\hat{\gamma}(\psi)$ be the equilibrium innovation size under policy ψ, $\hat{N}(\psi)$ be the equilibrium level of aggregate R&D labor under policy ψ, $\hat{n}(\psi)$ be the equilibrium level of R&D labor per industry under policy ψ, and $\hat{\phi}(\psi)$ be the equilibrium industry arrival rate under policy ψ. In the previous section, we examined the endogenous-growth patent policy $\psi_o = (P = 1, K = 1)$. Converting notation, $\hat{\gamma}(\psi_o) = \gamma_o$, $\hat{N}(\psi_o) = N_o$, $\hat{n}(\psi_o) = n_o$, and $\hat{\phi}(\psi_o) = \lambda(\gamma_o)n_o$.

Although we found that there can be underinvestment or overinvestment under policy ψ_o, our focus will be the case of underinvestment. We do not examine the case of overinvestment for two reasons. First, there is evidence that suggests there is too little R&D (see in particular Jones and Williams (1996)), so the overinvestment case is perhaps empirically less relevant. Second, if there is overinvestment under ψ_o, correcting incentives is somewhat trivial. Clearly, ψ_o is not the weakest patent policy. In particular, shorter patent life and/or weaker lagging breadth will decrease the reward to success and therefore reduce the level of R&D. The question arises which technique is better, but this question has been well-studied by Klemperer (1990) and Gilbert and Shapiro (1990) (although in the framework of isolated innovation).

We now suppose there is underinvestment under policy ψ_o, and ask how protection against future innovators, in the form of a patentability requirement or leading breadth, can stimulate R&D relative to policy ψ_o.

A Patentability Requirement

In a partial-equilibrium framework, Hunt (1995) and O'Donoghue (forthcoming) show that a patentability requirement can stimulate R&D. A patentability requirement induces firms to pursue larger innovations which take longer to achieve. Hence, a successful firm earns market profits for a longer period of time. This increases the reward to success and therefore stimulates R&D. We now demonstrate that this result can hold in the endogenous-growth model.

When there is a patentability requirement $P \geq \gamma_o$ and no leading breadth, the outcome of the model is exactly analogous to the outcome under policy ψ_o. The only difference is that firms will choose innovation size P – recall that P represents a lower bound on the innovation size that firms will pursue. The following lemma is exactly analogous to Lemma 1:

Lemma 2 *For any policy* $\psi = (P, K)$ *with* $P \geq \gamma_o$ *and* $K = 1$*:*

(i) The market leader in industry $\omega \in [0, \bar{\omega}]$ *does not conduct R&D.*

(ii) Firms choose innovation size $\hat{\gamma}(\psi) = P$.

(iii) The markup in industry $\omega \in [0, \bar{\omega}]$ *is* $\mu = \hat{\gamma}(\psi)$.

(iv) The equilibrium level of R&D is

$$\hat{N}(\psi) = \bar{\omega}\hat{n}(\psi) = \begin{cases} \bar{\omega}\left[\frac{P-1}{P}L - \frac{(1-\bar{\omega})P+\bar{\omega}}{P}\frac{\rho}{\lambda(P)}\right] & \text{if } L \geq \frac{(1-\bar{\omega})P+\bar{\omega}}{P-1}\frac{\rho}{\lambda(P)} \\ 0 & \text{if } L \leq \frac{(1-\bar{\omega})P+\bar{\omega}}{P-1}\frac{\rho}{\lambda(P)}. \end{cases}$$

Now consider how imposing a patentability requirement influences the behavior of R&D firms. Proposition 1 establishes that a patentability requirement can stimulate R&D in the endogenous-growth framework.

Proposition 1 ***(A patentability requirement can stimulate R&D)***
Consider a policy $\psi = (P, K)$ *with* $P \geq \gamma_o$ *and* $K = 1$*:*

(i) Suppose $N_o > 0$. *Then there exists* $P' > P'' > \gamma_o$ *such that* $\hat{N}(\psi)$ *is increasing in* P *for all* $P \in [\gamma_o, P')$ *and* $\hat{\phi}(\psi)$ *is increasing in* P *for all* $P \in [\gamma_o, P'')$.

(ii) Suppose $N_o = 0$. *Then either (a)* $\hat{N}(\psi) = \hat{\phi}(\psi) = 0$ *for all* $P \geq \gamma_o$*; or (b) there exists* $P' > P'' > \underline{P} \geq \gamma_o$ *such that* $\hat{N}(\psi) = \hat{\phi}(\psi) = 0$ *for all* $P \in [\gamma_o, \underline{P}]$, $\hat{N}(\psi)$ *is increasing in* P *for* $P \in [\underline{P}, P')$, *and* $\hat{\phi}(\psi)$ *is increasing in* P *for* $P \in [\underline{P}, P'')$.

Proof: See Appendix B.

Part (i) of Proposition 1 states that as long as there is investment under policy ψ_o, imposing a patentability requirement can induce firms to hire more R&D labor, and that doing so can lead to an increased rate of innovation. Part (ii) then states that if there is no investment under policy ψ_o, imposing a patentability requirement *may* stimulate investment; however, it is possible that investment might not occur for any patentability requirement.

Proposition 1 by itself does not establish whether a patentability requirement is welfare enhancing. Imposing a patentability requirement $P \geq \gamma_o$ has three effects on social welfare. First, a patentability requirement induces firms to pursue larger innovations. Second, a patentability requirement induces firms to hire more R&D labor (at least in the relevant range). Since under ψ_o firms pursue suboptimally small innovations, in the case where there is underinvestment under ψ_o these two effects will both lead to increased dynamic social welfare. However, the third effect is that a patentability requirement creates increased industry markups, and therefore decreased static efficiency. The optimal patentability requirement must weigh the trade-off between dynamic and static efficiency.

Leading Breadth

An alternative way to stimulate R&D is proposed by O'Donoghue, Scotchmer, and Thisse (1998). Again in a partial-equilibrium framework, they show that leading breadth can stimulate R&D by allowing firms to consolidate market power through licensing agreements. To illustrate this intuition, consider the policy $\psi = (P = \gamma_o, K = \gamma_o^2)$, and assume for the time-being that all firms choose innovation size γ_o. Suppose the most-recent innovator has a patent on quality q_o. Quality q_o infringes the patent of the second-most-recent innovator (who has a patent on quality $\frac{1}{\gamma_o}q_o$), but does not infringe the patent of the third-most-recent innovator (who has a patent on quality $\frac{1}{\gamma_o^2}q_o$). If the most-recent innovator and the second-most-recent innovator enter a licensing agreement to produce quality q_o, their nearest competitor will be the third-most-recent innovator. Hence, the two firms in the licensing contract can charge markup γ_o^2, compared to markup γ_o under policy ψ_o.

As the discussion in the previous paragraph suggests, an important question concerning leading breadth in our model is how many future innovations are required before a given patent is no longer infringed. To simplify our analysis, we will focus on policies where $P \geq \gamma_o$. As we show below, under such policies, firms target innovation size P. We then define $\alpha(\psi)$ as the integer α such that for each patent the α^{th} subsequent innovation will not infringe. Formally, for any policy $\psi = (P, K)$, $\alpha(\psi)$ is the integer m such that $K \in (P^{m-1}, P^m]$.

Of course, the reward to success will depend on how profit is shared in licensing agreements. O'Donoghue, Scotchmer, and Thisse (1998) avoid dealing with the division of profit by assuming firms can bargain over R&D costs. They focus on a model without patent races, however, and with patent races bargaining over R&D costs seems inappropriate. Indeed, the basic premise of the patent race model is that there is nothing to bargain over until some firm has a success, but at that point the R&D costs have already been incurred. Even so, we do not wish to explore here the intricacies of licensing. Hence, we use a vastly simplified model of licensing to make the analysis tractable and to provide some intuition as to how leading breadth affects the incentive to innovate.

Suppose that each time an innovation occurs in an industry, firms write a licensing contract to supply the new higher quality. If an innovation creates quality q_o, any firm with a patent on which quality q_o would infringe must sign the licensing contract. If these firms were to include additional firms with patents not infringed by q_o, they could charge a higher price. However, we assume that the antitrust authorities prevent such participation – that is, the antitrust authorities permit participation only if it is required to get the highest feasible quality produced. In addition, we simplify the analysis by assuming that firms who participate in the licensing contract do not conduct R&D.

Under policy ψ, exactly $\alpha(\psi)$ patentholders sign the licensing contract. These firms will split market profits according to some *bargaining solution*. We suppose there is a set of stationary bargaining solutions $(\mathrm{s}^1, \mathrm{s}^2, ...)$ where for each $\alpha \in \{1, 2, ...\}$ we have $\mathrm{s}^\alpha \equiv (s_1^\alpha, s_2^\alpha, ..., s_\alpha^\alpha) \in [0,1]^\alpha$ and $\sum_{i=1}^{\alpha} s_i^\alpha = 1$. Then if $\alpha(\psi) = \alpha$, the most-recent innovator gets share s_1^α, the second-most-recent innovator gets share s_2^α, and so on. In addition, we assume for simplicity that the bargaining solution is the same in all industries.

Although we have not yet defined an equilibrium when there is leading breadth, we are in a position to discuss some features that must hold in any outcome with this simplified model of licensing:

Lemma 3 *Suppose the patent policy is $\psi = (P, K)$ with $P \geq \gamma_o$, $K \geq 1$, and $\alpha(\psi) = \alpha$. Conditional on innovation rate ϕ, any equilibrium will be characterized by the following features:*
(i) Firms in industry $\omega \in [0, \bar{\omega}]$ target innovation size $\hat{\gamma}(\psi) = P$,
(ii) The markup in industry $\omega \in [0, \bar{\omega}]$ is $\mu = P^\alpha$, and
(iii) The reward to success in industry $\omega \in [0, \bar{\omega}]$ is $V = \pi \cdot B(\phi, \alpha)$, where π is given by equation 8 and $B(\phi, \alpha) \equiv \sum_{i=1}^{\alpha} s_i^\alpha \frac{\phi^{i-1}}{(\rho+\phi)^i}$.

Proof: See Appendix B.

When there is no leading breadth (i.e., $K = 1$), any patentability requirement $P \geq \gamma_o$ will be binding. Part (i) of Lemma 3 establishes that even for the case where $K > 1$, any patentability requirement $P \geq \gamma_o$ will be binding. The intuition is that leading breadth creates an incentive for firms to choose even smaller innovation sizes. Part (ii) of Lemma 3 simply reflects the fact that increased leading breadth creates increased market profits (as shown by O'Donoghue, Scotchmer, and Thisse (1998)). Specifically, the markup μ is increasing in the amount of leading breadth (because α is weakly increasing in K), and market profits are increasing in the markup μ.

Part (iii) of Lemma 3 illustrates that in addition to market profits, the reward to success depends on the bargaining solution. We have defined $B(\phi, \alpha) \equiv \sum_{i=1}^{\alpha} s_i^\alpha \frac{\phi^{i-1}}{(\rho+\phi)^i}$ to be the discounted share of market profits that each innovator receives. We often refer to $B(\phi, \alpha)$ as the *bargaining discount factor*. The magnitude of $B(\phi, \alpha)$ depends on the bargaining solution s^α.

O'Donoghue (1996) argues that licensing outcomes for cumulative innovation will tend to yield a backloaded payoff stream: An innovator should receive small payoffs early in the life of her patent when her product infringes previous patents and hence she is in a weak bargaining position; and an innovator should receive large payoffs late in the life of her patent when previous patents have effectively expired, subsequent innovations now infringe her patent, and hence she is in a strong bargaining position. Formally, that payoffs are backloaded means that the bargaining solution s^α satisfies $s_1^\alpha \leq s_2^\alpha \leq ... \leq s_\alpha^\alpha$. The more backloaded are payoffs, the smaller is $B(\phi, \alpha)$. For any α, $B(\phi, \alpha)$ is maximized under bargaining solution $s^\alpha = (1, 0, ..., 0)$ when $B(\phi, \alpha) = \frac{1}{\rho+\phi}$, and $B(\phi, \alpha)$ is minimized under bargaining solution $s^\alpha = (0, 0, ..., 1)$ when $B(\phi, \alpha) = \frac{\phi^{\alpha-1}}{(\rho+\phi)^\alpha}$. Of course, the backloading of payoffs matters only because firms discount the future (i.e., $\rho > 0$). Note that for any s^α, $\lim_{\rho\to 0} B(\phi, \alpha) = \frac{1}{\phi}$.

In addition to the conditions in Lemma 3, an equilibrium with leading breadth must satisfy the no-profit condition $\lambda V = w$, which we can rewrite as $\lambda \pi B(\phi, \alpha) = w$. When there is leading breadth and $\alpha \geq 2$, however, there can be multiple industry arrival rates ϕ that satisfy this no-profit condition. See Appendix A for a more complete discussion. We define the equilibrium to be the largest industry arrival rate that satisfies the no-profit condition. In other words,[13]

$$\hat{\phi}(\psi) \equiv \begin{cases} \max\{\phi \mid \lambda V = w\} & \text{If the set } \{\phi \mid \lambda V = w\} \text{ is nonempty} \\ 0 & \text{If the set } \{\phi \mid \lambda V = w\} \text{ is empty.} \end{cases}$$

Now consider how introducing leading breadth influences the behavior of R&D firms. Increased leading breadth has two effects on the reward to success $V = \pi B(\phi, \alpha)$. First, increased leading breadth leads to a larger markup μ and therefore increased market profits π. This effect will tend to stimulate R&D spending. But increased leading breadth also changes the bargaining discount factor $B(\phi, \alpha)$. O'Donoghue (1996) argues that the stronger is leading breadth the more backloaded are payoff streams: The stronger is leading breadth, the more previous patents any given innovation will infringe, and therefore the weaker is each innovator's bargaining position early in the life of her patent. This logic suggests that $B(\phi, \alpha) \geq B(\phi, \alpha + 1)$ for all α and ϕ. Indeed, this inequality must hold for $\alpha = 1$. Much of our discussion will assume this inequality holds in general, in which case the backloading effect of increased licensing counteracts the profit effect.[14]

Proposition 2 establishes that leading breadth can stimulate R&D as long as increased leading breadth does not cause the bargaining solution to become excessively backloaded.

---

13 We show in Appendix A that this definition is well-defined.

14 Of course for $\alpha \geq 2$ it could be that $B(\phi, \alpha+1) > B(\phi, \alpha)$ (for example if $s^2 = (0, 1)$ and $s^3 = (1, 0, 0)$). In this case, however, our result that leading breadth can stimulate R&D is *strengthened.*

Proposition 2 *(Leading breadth can stimulate R&D)*
Consider two policies $\psi = (P, K)$ *and* $\psi' = (P, K')$ *with* $P \geq \gamma_o$ *and* $K' > K$*, and suppose* $\hat{N}(\psi) > 0$.

(i) Suppose $\alpha(\psi) = 1$ *and* $\alpha(\psi') = 2$*. If the bargaining solution* $\mathrm{s}^2 = (s, 1-s)$*, then there exists* $\bar{s} \leq \frac{P}{P+1}$ *such that* $\hat{N}(\psi') \geq \hat{N}(\psi)$ *as long as* $s \geq \bar{s}$.

(ii) Suppose $\alpha(\psi) = \alpha$ *and* $\alpha(\psi') = \alpha + 1$*. For any bargaining solutions* s^{α} *and* $\mathrm{s}^{\alpha+1}$ *there exists* $\overline{\rho} > 0$ *such that* $\hat{N}(\psi') \geq \hat{N}(\psi)$ *as long as* $\rho < \overline{\rho}$.

Proof: See Appendix B.

Part (i) of Proposition 2 establishes that, relative to policy ψ_o, increasing leading breadth so that $\alpha(\psi) = 2$ will stimulate R&D as long as the first-period share of market profits is big enough. Part (ii) of Proposition 2 establishes that increased leading breadth can always stimulate R&D investment as long as people are patient enough. Proposition 2 illustrates that one of two things must be true for the profit effect of increased leading breadth to dominate the backloading effect. First, it could be that increased leading breadth does not significantly increase backloading (as illustrated by part (i)). Second, it could be that people are patient enough so that backloading is not too costly (as illustrated by part (ii)).

Just as Proposition 1 does not establish whether a patentability requirement is welfare enhancing, Proposition 2 does not establish whether leading breadth is welfare enhancing. Imposing leading breadth has two effects on social welfare. First, leading breadth can induce firms to hire more R&D labor. In the case where there is underinvestment under ψ_o this effect will lead to increased dynamic social welfare. Second, leading breadth allows firms to consolidate market power and thereby creates increased industry markups. This effect will lead to decreased static efficiency. Like the optimal patentability requirement, optimal leading breadth must weigh the trade-off between dynamic and static efficiency.

How does a patentability requirement compare to leading breadth? Since leading breadth allows firms to consolidate market power and create large market profits, it can be much more effective than a patentability requirement at stimulating R&D. However, these large market profits may make leading breadth significantly worse than a patentability requirement in terms of static efficiency, and in addition the effectiveness of leading breadth can be undermined by licensing inefficiencies. A careful comparison of the two types of policy would require much further specification of the model, and is beyond the scope of this paper. Furthermore, the appropriate question is not whether a patentability requirement or leading breadth is better, but rather is how to tailor the two instruments together to improve incentives for R&D. Indeed, Proposition 2 is written in a way that illustrates how leading breadth can be effective in addition to a patentability requirement.[15]

4. Partial Equilibrium vs. General Equilibrium

Our goal in Section 3 was to describe what the endogenous-growth literature can learn from the patent-design literature. Our goal in this section is to describe what the patent-design literature can learn from the endogenous-growth literature. By confining itself to partial-equilibrium analyses, the patent-design literature examines how patents affect a single industry assuming nothing changes elsewhere in the economy. However, patent policy is an economy-wide phenomenon. Patents are used in many industries, and therefore any policy change will affect many industries. The general-equilibrium approach of the endogenous-growth literature therefore permits a more complete analysis of patent design.[16]

One feature that arises in a general-equilibrium model that is absent from partial-equilibrium models is that there is a general-equilibrium effect which makes patent protection more effective at stimulating R&D. In a partial-equilibrium framework, stronger patents imply increased profits. In a general-equilibrium framework, however, these increased profits imply higher aggregate income and therefore increased demand, which increases profits further. Hence, there is a multiplier effect (or pecuniary externality) which reinforces the partial-equilibrium effect of patents.

We can interpret this pecuniary externality in terms of the industry demand curve. Partial-equilibrium analyses generally assume the industry demand curve is independent of patent protection, in which case stronger patent protection permits firms to charge a higher price *along a given demand curve*. In a general-equilibrium framework, the multiplier effect described above implies that the industry demand curve increases. Hence, stronger patent protection permits firms to charge a higher price *along a higher demand curve*. As a result, patent protection can be more effective than it appears from partial-equilibrium analyses.

The presence of this pecuniary externality implies that the effectiveness of patents depends crucially on the size of the high-technology sector. Stronger patents imply increased profits in high-

---

15 For more discussion of these issues, see O'Donoghue (forthcoming).

16 In fact, our discussion of what the general-equilibrium framework contributes to the patent-design literature applies equally well to cumulative innovation and isolated innovation. Our discussion revolves around the cumulative-innovation interpretation only because we focus on a model of endogenous growth.

technology industries, but not in noninnovative industries. On the other hand, the higher income and higher demand is spread across all industries. A partial-equilibrium analysis is, in a sense, equivalent to assuming $\bar{\omega}$ close to 0. A larger high-technology sector implies that patents have a stronger impact on income, and therefore patents become more effective. Proposition 3 captures this intuition by showing that for any patent policy, the equilibrium industry arrival rate is increasing in the size of the high-technology sector.

Proposition 3 *(Patent policy is more effective as more industries innovate)*
For any patent policy $\psi = (P, K)$ *with* $P \geq \gamma_o$, $\hat{\phi}(\psi)$ *is increasing in* $\bar{\omega}$.

Proof: See Appendix B.

The economy-wide effects of patents also has implications for static efficiency. The core theme of the patent-design literature is the trade-off between incentives for R&D and monopoly distortions: Patents create markups that lead to increased incentives to innovate, but these markups reduce static efficiency. However, there is often no formal model of static efficiency. The usual measure is the deadweight loss associated with the assumed demand curve, but we have seen that the assumption of a fixed demand curve may not be valid. To better understand the monopoly distortions created by patent protection, we must examine the economy-wide effects of patents.

The general-equilibrium framework allows us to formally model the static inefficiency. As is the explicit or implicit assumption in much of the patent-design literature, static inefficiency arises in the form of output distortions.[17] The markups created by patents distort consumption away from high-technology industries and towards noninnovative industries. In our model, the static inefficiency is given by equation (11).

The perhaps surprising result is that the social cost of output distortions may be smaller than partial-equilibrium analyses assume. In particular, if $\bar{\omega}$ is close to 0, which means few industries conduct R&D, or if $\bar{\omega}$ is close to 1, which means most industries conduct R&D, then output distortions are not very costly. For $\bar{\omega}$ close to 0, patents create markups in only a few industries, so relative prices are efficient except for the few innovative industries. In this case, output distortions involve too little consumption in the few innovative industries, but very little change in the non-innovative industries. Since the innovative industries produce a small portion of the consumption bundle, these output distortions are not too costly. For $\bar{\omega}$ close to 1, patents create an identical

[17] Some patent-design papers concentrate on other static inefficiencies such as delayed diffusion or high cost firms persisting in the market.

markup in almost all industries, so relative prices are efficient except for the few noninnovative industries. Again, the resultant output distortions are not too costly. Proposition 4 summarizes this discussion.

Proposition 4 ***(Output distortions are small for few or many innovative industries)***
For any policy $\psi = (P, K)$ with $P \geq \gamma_o$, there exists $\omega' \in (0, 1)$ such that the static inefficiency D is increasing in $\bar{\omega}$ for all $\bar{\omega} < \omega'$ and decreasing in $\bar{\omega}$ for all $\bar{\omega} > \omega'$.

Proof: See Appendix B.

An obvious question arises in our model: How does optimal patent policy depend on the proportion of industries that use patents? Unfortunately, solving for the optimal patent policy in our model is quite difficult. Two problems arise. First, the backloading of payoffs when there is leading breadth complicates equations, as well as raises the issue of how does backloading depend on the amount of leading breadth. Second, the implications of a patentability requirement for social welfare depend heavily on the functional form of λ.

To provide some intuition, however, we consider a special case that abstracts away from these issues. We first assume that for each α the bargaining solution is $s^\alpha \equiv (1, 0, ..., 0)$. We can interpret this assumption as a proxy for the rate of time preference ρ being very small relative to the industry arrival rate ϕ, in which case backloading becomes irrelevant. Then we examine the (constrained) socially optimal leading breadth for a fixed patentability requirement. Proposition 5 establishes that optimal leading breadth is (weakly) increasing in the size of the high-technology sector.

Proposition 5 ***(Optimal patent protection is stronger as more industries innovate)***
Let ψ^* denote the socially optimal patent policy conditional on having patentability requirement $\bar{P}$. If the bargaining solution $s^\alpha \equiv (1, 0, ..., 0)$ for all α, then $\alpha(\psi^*)$ is nondecreasing in $\bar{\omega}$ for any $\bar{P} \geq \gamma_o$.

Proof: See Appendix B.

5. R&D Distortions

An important issue largely ignored by both the endogenous-growth literature and the patent-design literature is the allocation of R&D across industries. The endogenous-growth assumption of identical R&D capabilities in all innovative industries implies that the allocation of R&D labor is irrelevant. As a result, the only policy concern in the previous two sections was the allocation of labor between R&D and production. This assumption, however is highly questionable. Empirical evidence suggests there are significant cross-industry differences in R&D productivity and in R&D behavior (see the survey by Cohen and Levin (1989)). Klenow (1996) builds an endogenous-growth model in which he allows industries to vary along several dimensions in order to explore the source of these cross-industry differences. Comparing the implications of his model to the empirical evidence, he concludes that industry differences in market size and technological opportunities best explain industry differences in R&D behavior.

In this section, we posit that there are asymmetric R&D capabilities across industries (i.e., asymmetric technological opportunities), and ask what are the implications for patent policy. For simplicity, we shall assume that all industries are innovative (i.e., $\bar{\omega} = 1$), and that a uniform patent policy applies for all industries.

We suppose asymmetric R&D capabilities arise from sector-specific capital. (It will become clear that our basic results would hold for other sources of asymmetric R&D capabilities.) Suppose capital is used only in R&D. If an R&D firm employs labor n and capital h, it will have arrival rate of innovations $\phi \equiv \lambda(\gamma)\, h^{1-\beta} n^{\beta}$, $\beta \in (0,1)$. The function λ is exactly as in the basic model, and in the discussion that follows we often suppress its argument. There are two types of capital, *sector-1 capital* and *sector-2 capital*, where all industries with $\omega \in [0, \frac{1}{2}]$ can use only sector-1 capital and industries with $\omega \in (\frac{1}{2}, 1]$ can use only sector-2 capital. All results easily generalize to the case where the two sectors are of unequal sizes, but the notation would be more complicated. The total supply of sector-1 capital is H_1, the total supply of sector-2 capital is H_2, and all consumers own equal shares of each type of capital (which is part of their wealth $A(t)$). The remainder of the model is unchanged.

With two sectors conducting R&D, we need some new notation. Let n_1 denote R&D labor employed by each industry in sector 1, and let n_2 denote R&D labor employed by each industry in sector 2. Let N_1 and N_2 denote total labor employed in sector-1 R&D and sector-2 R&D, respectively, so $N_1 = \frac{1}{2}n_1$ and $N_2 = \frac{1}{2}n_2$. Finally, let N denote the aggregate level of labor employed in R&D, so $N \equiv N_1 + N_2$.

The Social Optimum

A social planner must allocate labor between production, sector-1 R&D, and sector-2 R&D – that is, a social planner must choose x_ω for each $\omega \in [0, 1]$, N_1, and N_2. Since there are constant returns to scale in R&D, for any N_1 a social planner will allocate sector-1 R&D resources in the ratio

$\frac{H_1}{N_1}$ (i.e., all sector-1 firms that conduct R&D will receive capital h and labor n such that $\frac{h}{n} = \frac{H_1}{N_1}$). As a result, the total arrival rate of innovations in sector 1 is $\lambda H_1^{1-\beta} N_1^{\beta}$. Analogously, for any N_2 a social planner will allocate sector-2 R&D resources in the ratio $\frac{H_2}{N_2}$, and the total arrival rate of innovations in sector 2 is $\lambda H_2^{1-\beta} N_2^{\beta}$. Using a logic similar to that in Section 2, we can derive the following expression for intertemporal utility:

$$\rho U = \ln\left(\frac{L-(N_1+N_2)}{La}\right) + \frac{\ln\gamma}{\rho}\lambda(\gamma)\left[H_1^{1-\beta}N_1^{\beta} + H_2^{1-\beta}N_2^{\beta}\right]. \tag{13}$$

A social planner will choose the endogenous variables N_1, N_2, and γ to maximize equation (13). In doing so, a social planner has two concerns: (1) how much labor to allocate to R&D (i.e., the choice of N), and (2) how to divide R&D labor between sector-1 industries and sector-2 industries (i.e., the choice of $\frac{N_1}{N_2}$). Our focus in this section is the latter decision. It is straightforward to show that the optimal distribution of R&D labor is $\left[\frac{N_1}{N_2}\right]^* = \frac{H_1}{H_2}$. Moreover, conditional on any aggregate level of R&D, intertemporal utility is maximized when $\frac{N_1}{N_2} = \left[\frac{N_1}{N_2}\right]^*$.

The Private Equilibrium

The only feature of the economy different from the basic model is the R&D production function. The demand functions for consumption goods, optimal firm behavior in output markets, and the resource constraint (for labor) are all unchanged. We again incorporate these features into equation (8), and since $\bar{\omega} = 1$ we have

$$\bar{\pi} = (L-N)(\mu-1) \tag{14}$$

where μ is the industry markup determined by patent policy. Since there is a uniform patent policy economy-wide, the markup μ will be the same in all industries, and therefore market profits will be the same in all industries.

The introduction of sector-specific capital creates important changes in a firms' R&D decisions. As in the basic model, there is perfect competition in R&D and the R&D production function has constant returns to scale. As a result, the individual research venture is of indeterminate size. However, since all industries within a given sector are symmetric, the amount of capital and labor employed in R&D at the industry level will be the same across all industries within a given sector. In other words, there exists h_1, n_1, h_2, and n_2 such that every industry in sector 1 will employ capital h_1 and labor n_1 and every industry in sector 2 will employ capital h_2 and labor n_2. Every industry in sector 1 will have industry arrival rate $\phi_1 = \lambda h_1^{1-\beta} n_1^{\beta}$, and every industry in sector 2 will have industry arrival rate $\phi_2 = \lambda h_2^{1-\beta} n_2^{\beta}$.

In the basic model we expressed the reward to success for a given industry as a function of the patent policy ψ, the available market profits π, and the industry arrival rate ϕ. The introduction of sector-specific capital changes this formulation only in the sense that sector-1 industries and sector-2 industries have different arrival rates and therefore different rewards to success. The reward to success for each industry in sector j is

$$V_j = \bar{\pi} \sum_{i=1}^{\alpha} s_i^{\alpha} \frac{\phi_j^{i-1}}{(\rho + \phi_j)^i} = \bar{\pi} \cdot B(\phi_j, \alpha). \tag{15}$$

Now consider an individual firm's decision. Let w_j be the rental rate for sector-j capital. If a firm in sector j employs labor n and capital h, the net payoff to R&D is

$$-\,n\,w \;-\; h\,w_j \;+\; \lambda\, h^{1-\beta} n^{\beta} V_j.$$

Profit maximization implies that all firms in sector j employ capital h and labor n in the same ratio, which must be $\frac{h}{n} = \frac{h_j}{n_j}$.[18] Perfect competition in capital markets implies that the rental rate for sector-j capital is equal to its marginal product, or $w_j = (1-\beta)\lambda h_j^{-\beta} n_j^{\beta} V_j$. Using $\frac{h}{n} = \frac{h_j}{n_j}$, $w_j = (1-\beta)\lambda h_j^{-\beta} n_j^{\beta} V_j$, and $\phi_j = \lambda h_j^{1-\beta} n_j^{\beta}$, we can write the no-profit condition for a firm in industry j as

$$\frac{\beta \phi_j}{n_j} V_j = w.$$

Substituting for n_j we can rewrite the no-profit conditions as

$$V_1 = \hat{C}_1 \phi_1^{\frac{1-\beta}{\beta}} \tag{16}$$

$$V_2 = \hat{C}_2 \phi_2^{\frac{1-\beta}{\beta}} \tag{17}$$

where $\hat{C}_j \equiv w \left[\beta \lambda^{\frac{1}{\beta}} h_j^{\frac{1-\beta}{\beta}} \right]^{-1}$.

The specific patent policy ψ determines μ and α, and then equations (16) and (17) determine the equilibrium industry arrival rates for each sector ϕ_1 and ϕ_2. Then

$$\hat{N}_j(\psi) = \frac{1}{2}\hat{n}_j(\psi) = \frac{1}{2}\left(\hat{\phi}_1(\psi)\right)^{\frac{1}{\beta}} \;/\; \left(\lambda^{\frac{1}{\beta}} h_j^{\frac{1-\beta}{\beta}}\right).$$

As in the basic model, for any patent policy ψ with $\alpha = 1$, a unique industry arrival rate satisfies the no-profit condition in each industry; whereas for any patent policy ψ with $\alpha \geq 2$, there can be multiple industry arrival rates that satisfy the no-profit condition. See Appendix A for a more complete discussion. As before, we define the equilibrium industry arrival rate to be the largest

[18] Note that the choice of innovation size is identical to that in the basic model. Hence, for any $\alpha(\psi) \geq 1$ any patentability requirement $P \geq \gamma_o$ will be binding.

arrival rate that satisfies the no-profit condition – that is,

$$\hat{\phi}_j(\psi) \equiv \begin{cases} \max\{\phi \mid V_j = \hat{C}_j \phi^{\frac{1-\beta}{\beta}}\} & \text{If the set } \{\phi \mid V_j = \hat{C}_j \phi^{\frac{1-\beta}{\beta}}\} \text{ is nonempty} \\ 0 & \text{If the set } \{\phi \mid V_j = \hat{C}_j \phi^{\frac{1-\beta}{\beta}}\} \text{ is empty.} \end{cases}$$

Appendix A establishes that for any patent policy ψ, $\hat{\phi}_1(\psi)$ and $\hat{\phi}_2(\psi)$ are unique, and that $h_1 > h_2$ implies $\hat{\phi}_1(\psi) > \hat{\phi}_2(\psi)$.

The Role of Patents

We focus on the following two questions: What R&D distortions are associated with the use of patents? How do R&D distortions depend on the strength of patent protection?

Given R&D production function $\phi = \lambda h^{1-\beta} n^{\beta}$, R&D labor is more productive in the sector with more available sector-specific capital *per industry*. For the remainder of this section, we assume without loss of generality that $h_1 > h_2$, so R&D labor is more productive in sector-1 industries. We first consider policy $\psi_o = (P = 1, K = 1)$. It is straightforward to show that under policy ψ_o firms pursue suboptimally small innovations (firms in fact choose innovation size γ_o exactly as in the basic model). It is also straightforward to show that under ψ_o there can be too little or too much aggregate labor allocated to R&D. As emphasized above, however, our focus in this section is the allocation of R&D labor between sector 1 and sector 2. The following proposition establishes that under policy ψ_o the private equilibrium allocates too little R&D labor to the more productive sector.

Proposition 6 ***(Patents distort R&D labor away from productive industries)***
Suppose $h_1 > h_2$, so R&D labor is more productive in sector-1 industries. Under policy ψ_o, $\frac{\hat{N}_1(\psi)}{\hat{N}_2(\psi)} < \left[\frac{N_1}{N_2}\right]^*$.

Proof: See Appendix B.

Proposition 6 is driven by differential rates of creative destruction in the two sectors. Since R&D labor is more productive in sector-1 industries, these industries will have a higher rate of innovation, and therefore a higher rate of creative destruction. Since the reward to success is decreasing in the rate of creative destruction, the private equilibrium will distort R&D labor away from the industries where it is more productive and towards industries where it is less productive.

Now suppose that under policy ψ_o, the private equilibrium allocates too little labor to R&D. Protection from future innovators, in the form of a patentability requirement and/or leading breadth may be able to stimulate R&D investment and the rate of innovation. If industries differ in their R&D capabilities, however, we must ask how such a policy will affect R&D distortions. The answer to this question is complicated in that it can depend on the bargaining solution. To illustrate the forces at work, we first ask what happens under the extreme bargaining solution where $s^\alpha = (1, 0, ..., 0)$ for each α. Since the bargaining solution for $\alpha = 1$ is trivially $s^1 = (1)$, this case includes all policies with a patentability requirement and no leading breadth. In addition, for policies with leading breadth we can interpret this assumption as a proxy for the rate of time preference ρ being very small relative to the industry arrival rate ϕ, in which case backloading becomes irrelevant. The following proposition shows that in this case stronger patent protection exacerbates R&D distortions.

Proposition 7 *(Stronger patent protection can exacerbate R&D distortions)*
Suppose $h_1 > h_2$, so R&D labor is more productive in sector-1 industries. Suppose also that the bargaining solution is $s^\alpha = (1, 0, ..., 0)$ for each α.
If either $\hat{\phi}_1(\psi) > \hat{\phi}_1(\psi')$ or $\hat{\phi}_2(\psi) > \hat{\phi}_2(\psi')$ (i.e., if policy ψ increases the industry arrival rate in either sector relative to policy ψ'), then $\frac{\hat{N}_1(\psi)}{\hat{N}_2(\psi)} < \frac{\hat{N}_1(\psi')}{\hat{N}_2(\psi')} < \left[\frac{N_1}{N_2}\right]^*$.

Proof: See Appendix B.

Proposition 7 is also driven by the differential rates of creative destruction in the two sectors. By imposing stronger patent protection in order to induce firms to hire more R&D labor, policymakers increase the rate of creative destruction in all industries. But R&D labor is more productive in sector-1 industries, and as a result the increase in the rate of creative destruction is disproportionately large in sector-1 industries relative to sector-2 industries. Since the reward to success depends negatively on the rate of creative destruction, the private equilibrium will allocate the additional R&D labor with a bias towards sector-2 industries. Hence, stronger patent protection exacerbates the R&D distortion.

For policies with a patentability requirement (and no leading breadth), Proposition 7 implies that stimulating R&D investment will exacerbate R&D distortions. For policies with leading breadth, in contrast, Proposition 7 only tells part of the story, because the proposition relies on the extreme bargaining solution where the most recent innovator gets the entire bargaining surplus. The following example illustrates the importance of this assumption by demonstrating that the result can be overturned if we consider the alternative extreme:

Example: Assume $L = 2.5$ and $\rho = .10$. Suppose that under policy ψ_o, firms choose innovation size $\gamma_o = 1.1$, and suppose further that $\lambda(\gamma_o) = 1$. Assume R&D production function $\phi = \lambda(\gamma)h^{\frac{1}{2}}n^{\frac{1}{2}}$, and suppose $h_1 = 2H_1 = 4$ and $h_2 = 2H_2 = 1$. Then we have $\phi_1 = 2n_1^{\frac{1}{2}}$ and $\phi_2 = n_2^{\frac{1}{2}}$, from which we can derive $N_1 = \frac{1}{2}n_1 = \frac{1}{8}\phi_1^2$, $N_2 = \frac{1}{2}n_2 = \frac{1}{2}\phi_2^2$, and $N = \frac{1}{8}\phi_1^2 + \frac{1}{2}\phi_2^2$.

Under policy ψ_o, the no-profit conditions are

$$(L-N)(\gamma-1)\frac{1}{\rho+\phi_1} = \frac{\phi_1}{2} \qquad \text{and} \qquad (L-N)(\gamma-1)\frac{1}{\rho+\phi_2} = 2\phi_2.$$

These no profit-conditions are satisfied by $\phi_1 = .645$ and $\phi_2 = .300$, which yields $\frac{\phi_1}{\phi_2} = 2.15$.

Now consider policy $\psi' \equiv (P = \gamma_o, K = \gamma_o^2)$, and suppose the bargaining solution is $\mathrm{s}^2 = (0,1)$. Then the no-profit conditions are

$$(L-N)(\gamma^2-1)\frac{\phi_1}{(\rho+\phi_1)^2} = \frac{\phi_1}{2} \qquad \text{and} \qquad (L-N)(\gamma^2-1)\frac{\phi_2}{(\rho+\phi_2)^2} = 2\phi_2.$$

These no profit-conditions are satisfied by $\phi_1 = .886$ and $\phi_2 = .393$, which yields $\frac{\phi_1}{\phi_2} = 2.25$.

In this example, leading breadth stimulates the rate of innovation in both sectors, and moreover decreases the R&D distortion (a social planner would set $\frac{\phi_1}{\phi_2} = 4$). Why does Proposition 7 fail to hold when we relax the assumption of $\mathrm{s}^\alpha = (1, 0, ..., 0)$? To answer this question, recall that increased leading breadth has two effects on the reward to success. On one hand, it increases available market profits. But on the other hand, increased leading breadth can cause payoffs to become backloaded. The backloading of payoffs is more costly the smaller is the industry arrival rate. Under the assumption $\mathrm{s}^\alpha = (1, 0, ..., 0)$, only the first effect of increased leading breadth is present, whereas once we relax this assumption, the backloading effect arises. It is the backloading effect that tends to counteract the result in Proposition 7. Since the industry arrival rate is larger in sector-1 industries than in sector-2 industries, the backloading of payoffs will be more costly in sector-2 industries. As a result, increased leading breadth has a bigger effect on sector-1 industries, which is exactly what is needed to reduce R&D distortions.

In sum, then, our analysis in this section suggests that when industries have asymmetric R&D capabilities, patent protection will tend to distort R&D investment away from those industries where it would be most productive. In addition, if policymakers attempt to stimulate aggregate R&D investment with protection from future innovators, they will alter this R&D distortion. A patentability requirement tends to increase R&D distortions. In contrast, leading breadth may increase or decrease R&D distortions, depending on the backloading of payoffs.

6. Discussion and Conclusion

In this paper, we have examined patent policy in a model of endogenous growth. For simplicity, we have couched our analysis within a relatively simple quality-ladder model along the lines of Grossman and Helpman (1991) and Aghion and Howitt (1992). Within this model, we are forced to make a number of simplifying assumptions to keep the analysis tractable. For instance, we use a very simple model of licensing, we assume there is no scope to have different patent policies apply to different industries, and we consider a world where market leaders do not have an incentive to conduct R&D. Moreover, the simple quality-ladder models of Aghion and Howitt and Grossman and Helpman may not be the most realistic of endogenous-growth models. Jones (1995) rejects R&D-based models of endogenous-growth because they exhibit scale effects that are inconsistent with time-series evidence, and Kortum (1997) synthesizes the quality-ladder models and search-theoretic models to build an endogenous-growth model that is more consistent with empirical evidence.

Even so, the basic intuitions we identify are not special to our model. We conclude by discussing some general lessons that the reader should take away from our analysis.

Perhaps the most basic lesson to take away from our analysis is that whenever R&D firms face a threat from future innovators, there may be a role for patents to provide protection against future innovators. Since most R&D-based models of endogenous growth assume that successful R&D firms have their market profits eroded when other firms come along with new inventions, there can be a role for protection against future innovators in those models. Even in Kortum's model, for example, imposing a patentability requirement so that fewer ideas can earn patents will slow the rate of market turnover and thereby increase the value of patents.

A second lesson is that, in addition to stimulating R&D investment, patent policy can also be useful for influencing the direction of firms' inventive activity. In our model, this took the form of imposing a patentability requirement to counteract firms' tendencies to pursue suboptimally small innovations. In a similar fashion, a patentability requirement and/or leading breadth could influence the characteristics of new products, or the types of cost reductions that firms pursue.

A third lesson is that any examination of government policy with regard to R&D must carefully assess the static efficiency implications of any policy proposal, and to do so one must consider the economy-wide implications of the policy. Our analysis suggests that any policy that will affect all industries equally or that will affect only a few industries may have small static efficiency implications, whereas any policy that has asymmetric effects across industries can have large static efficiency implications.

A fourth lesson is that the theoretical R&D literature may be missing an important issue when it assumes symmetric R&D capabilities across industries. Our model shows that patent policy can cause R&D distortions, and that the magnitude of those distortions depends on the specific patent policy. However, this lesson does not apply only to analyses of patent policy. Any analysis of government policy for R&D should ask what are the implications for R&D distortions.

Perhaps our most important contribution is the merging of the patent-design literature and the endogenous-growth literature. The patent-design literature focuses on specific policy instruments, with attention paid to institutional detail. The endogenous-growth literature builds careful models of R&D at the economy-wide level, with attention paid to empirical calibration. By merging these literatures, we hope this paper will help improve the study of government policy with regard to R&D.

Appendix A

Equilibrium Conditions in the Basic Model (Sections 2-4)

Combining Lemmas 2 and 3, under any patent policy $\psi = (P, K)$ with $P \geq \gamma_o$ and $\alpha(\psi) = \alpha$, the markup in high-technology industries will be $\mu = P^\alpha$ and the reward to success will be $V = \pi \cdot B(\phi, \alpha)$ where $B(\phi, \alpha) \equiv \sum_{i=1}^{\alpha} s_i^\alpha \frac{\phi^{i-1}}{(\rho+\phi)^i}$ and $\pi = (L - N)\frac{\mu - 1}{(1-\bar{\omega})\mu + \bar{\omega}}$. Any ϕ that satisfies the no-profit condition $\lambda V = w$ is a candidate for an "equilibrium". Given $N = \bar{\omega} n = \frac{\bar{\omega}}{\lambda}\phi$, define $\pi(\phi) = (L - \frac{\bar{\omega}}{\lambda}\phi)\frac{\mu-1}{(1-\bar{\omega})\mu+\bar{\omega}}$. We can then write the no-profit condition as

$$B(\phi, \alpha) = \frac{w}{\lambda \pi(\phi)}.$$

Closer inspection of this equation reveals that ϕ is the only endogenous variable, as should be the case. Note that $\pi(\phi)$ is decreasing in ϕ, and that $\lim_{\phi \to \frac{\lambda}{\bar{\omega}}L} \pi(\phi) = 0$ (where $\phi = \frac{\lambda}{\bar{\omega}}L$ implies $N = L$). This implies $\left(\frac{w}{\lambda\pi(\phi)}\right)$ is increasing in ϕ and $\lim_{\phi \to \frac{\lambda}{\bar{\omega}}L} \left(\frac{w}{\lambda\pi(\phi)}\right) = \infty$.

What is $B(\phi, \alpha)$? If $\alpha = 1$ then $B(\phi, \alpha) = \frac{1}{\rho+\phi}$, which is everywhere decreasing in ϕ. As a result, for any policy without leading breadth (i.e., $\alpha = 1$) there is a unique ϕ that satisfies the no-profit condition, as illustrated in Figure 1.

If $\alpha > 1$ then $B(\phi, \alpha)$ can sometimes be everywhere decreasing in ϕ, in which case a unique ϕ satisfies the no-profit condition. For example, $s^\alpha = (\frac{1}{\alpha}, \frac{1}{\alpha}, ..., \frac{1}{\alpha})$ implies $B(\phi, \alpha) = \sum_{i=1}^{\alpha} \frac{1}{\alpha} \frac{\phi^{i-1}}{(\rho+\phi)^i} = \frac{1}{\alpha\rho}\left[1 - \left(\frac{\phi}{\rho+\phi}\right)^\alpha\right]$, which is everywhere decreasing in ϕ. But $\alpha > 1$ can also yield $B(\phi, \alpha)$ not everywhere decreasing in ϕ. For example, if the bargaining solution $s^2 = (0, 1)$ then $B(\phi, \alpha) = \frac{\phi}{(\rho+\phi)^2}$, which is initially increasing in ϕ and then decreasing in ϕ. In this case, multiple ϕ can satisfy the no-profit condition, as illustrated in Figure 2. When multiple ϕ satisfy the no-profit condition, we define the largest such ϕ to be the equilibrium.[19]

For all α, $B(\phi, \alpha)$ is continuous in ϕ, and also $\lim_{\phi \to \frac{\lambda}{\bar{\omega}}L} B(\phi, \alpha)$ is finite (to see the latter note that $B(\phi, \alpha) \leq \frac{1}{\rho+\phi}$ for any α). Two results follow. First, our equilibrium definition is well-defined – that is, there must be some $\hat{\phi} < \frac{\lambda}{\bar{\omega}}L$ such that $B(\hat{\phi}, \alpha) = \frac{w}{\lambda\pi(\hat{\phi})}$ and $B(\phi, \alpha) < \frac{w}{\lambda\pi(\phi)}$ for all $\phi > \hat{\phi}$. Second, if $B(\phi', \alpha) > \frac{1}{\lambda\pi(\phi')}$, then $\hat{\phi}(\psi) > \phi'$. In words, if for some ϕ' the reward to success is larger than the wage, then the equilibrium industry arrival rate is larger than ϕ'. This second result will be useful in proving Propositions 2 and 3.

---

[19] In Figure 2, $\hat{\phi}_1$ is unstable in the sense that ϕ slightly larger implies $\lambda V > w$, which would lead all firms to increase R&D spending further until $\hat{\phi}_2$ is reached. The largest ϕ that satisfies the no-profit condition is always stable.

Equilibrium Conditions in the Asymmetric-R&D Model (Section 5)

Under any patent policy $\psi = (P, K)$ with $P \geq \gamma_o$ and $\alpha(\psi) = \alpha$, the markup in all industries will be $\mu = P^\alpha$ and the reward to success will be $V = \bar{\pi} \cdot B(\phi, \alpha)$ where $B(\phi, \alpha) \equiv \sum_{i=1}^{\alpha} s_i^\alpha \frac{\phi^{i-1}}{(\rho+\phi)^i}$ and $\bar{\pi} = (L - N)(\mu - 1)$. Any combination (N, ϕ_1, ϕ_2) that satisfies the no-profit conditions $\frac{\beta\phi_1}{n_1} V_1 = w$ and $\frac{\beta\phi_2}{n_2} V_2 = w$ and

$$N = \frac{1}{2}\lambda^{\frac{-1}{\beta}} h_1^{\frac{1-\beta}{-\beta}} \phi_1^{\frac{1-\beta}{\beta}} + \frac{1}{2}\lambda^{\frac{-1}{\beta}} h_2^{\frac{1-\beta}{-\beta}} \phi_2^{\frac{1-\beta}{\beta}} \tag{A-1}$$

is a candidate for an "equilibrium".

Define $\hat{V}(\phi, \alpha, N) \equiv (L - N)(P^\alpha - 1)B(\phi, \alpha)$, and note that this function is the same for both sectors, so any differences in the reward to success across sectors are driven by different industry arrival rates. As in the text, define $\hat{C}_j \equiv w \left[\beta\lambda^{\frac{1}{\beta}} h_j^{\frac{1-\beta}{\beta}}\right]^{-1}$, and note that $h_1 > h_2$ implies $\hat{C}_1 < \hat{C}_2$. We can then rewrite the no-profit conditions as

$$\hat{V}(\phi_1, \alpha, N) = \hat{C}_1 \phi_1^{\frac{1-\beta}{\beta}} \qquad \text{and} \qquad \hat{V}(\phi_2, \alpha, N) = \hat{C}_2 \phi_2^{\frac{1-\beta}{\beta}}.$$

These no-profit conditions are graphed in Figure 3. As in the basic model, $B(\phi, \alpha)$ can be non-monotonic in ϕ and therefore $\hat{V}(\phi, \alpha, N)$ can be non-monotonic in ϕ. As a result, for any fixed N multiple ϕ can satisfy the no-profit condition for sector j. We again define the largest such ϕ to be the equilibrium.

Since $\frac{\partial \hat{V}}{\partial N} < 0$ and $\hat{C}_1 \phi^{\frac{1-\beta}{\beta}}$ and $\hat{C}_2 \phi^{\frac{1-\beta}{\beta}}$ are independent of N, the no-profit conditions imply that (all else equal) ϕ_1 and ϕ_2 are decreasing in N. We can therefore conclude that for any patent policy ψ a unique combination $(\hat{N}(\psi), \phi_1(\psi), \phi_2(\psi))$ satisfies the no-profit conditions and equation (A-1). Moreover, since $\hat{C}_1 < \hat{C}_2$ implies $\hat{V}(\hat{\phi}_2(\psi), \alpha, \hat{N}(\psi)) > \hat{C}_1 \left(\hat{\phi}_2(\psi)\right)^{\frac{1-\beta}{\beta}}$, we can conclude that $h_1 > h_2$ implies $\hat{\phi}_1(\psi) > \hat{\phi}_2(\psi)$.

Appendix B

Proof of Proposition 1: (i) Define $\tilde{N} \equiv \bar{\omega}\left[\frac{P-1}{P}L - \frac{(1-\bar{\omega})P+\bar{\omega}}{P}\frac{\rho}{\lambda(P)}\right]$, and then Lemma 2 implies $\hat{N}(\psi) = \max\{\tilde{N}, 0\}$ and $\hat{\phi}(\psi) = \lambda(P)\hat{N}(\psi) = \max\{\lambda(P)\tilde{N}, 0\}$. In addition, whenever $\tilde{N} \geq 0$, $\hat{N}(\psi)$ is increasing in P if and only if $\frac{d\tilde{N}}{dP} > 0$, and $\hat{\phi}(\psi)$ is increasing in P if and only if $\frac{d[\lambda(P)\tilde{N}]}{dP} > 0$. Differentiating:

$$\frac{d\tilde{N}}{dP} = \bar{\omega}\left[\frac{1}{P^2}L + \frac{\bar{\omega}}{P^2}\frac{\rho}{\lambda} - \frac{(1-\bar{\omega})P+\bar{\omega}}{P}\frac{\rho}{\lambda}\left(\frac{-\frac{d\lambda}{dP}}{\lambda}\right)\right]$$

$$\text{and} \quad \frac{d[\lambda(P)\tilde{N}]}{dP} = \bar{\omega}\left[\frac{d\lambda}{dP}\left(\frac{P-1}{P}\right)L + \frac{\lambda}{P^2}L + \frac{\bar{\omega}}{P^2}\rho\right].$$

$N_o > 0$ implies $L \geq \frac{(1-\bar{\omega})\gamma_o+\bar{\omega}}{\gamma_o - 1}\frac{\rho}{\lambda}$, and at $P = \gamma_o$ we have $-\frac{d\lambda}{dP} = \frac{\lambda}{\gamma_o(\gamma_o-1)}$. It is then straightforward to show $\left.\frac{d\tilde{N}}{dP}\right|_{\gamma_o} > 0$ and $\left.\frac{d[\lambda(P)\tilde{N}]}{dP}\right|_{\gamma_o} > 0$, from which it follows that there exists $P' > \gamma_o$ and $P'' > \gamma_o$ such that $\hat{N}(\psi)$ is increasing in P for all $P \in [\gamma_o, P')$ and $\hat{\phi}(\psi)$ is increasing in P for all $P \in [\gamma_o, P'')$. That $P' > P''$ follows from $\frac{d[\lambda(P)\tilde{N}]}{dP} = \lambda\frac{d\tilde{N}}{dP} + \frac{d\lambda}{dP} < 0$ at $P = P'$.

ii) This result follows from $\left.\frac{d\tilde{N}}{dP}\right|_{\gamma_o} > 0$ and $\left.\frac{d[\lambda(P)\tilde{N}]}{dP}\right|_{\gamma_o} > 0$ combined with the fact that $\tilde{N}$ is continuous and differentiable, because then either $\tilde{N} \leq 0$ for all $P \geq \gamma_o$ or there exists $\underline{P} \geq \gamma_o$ such that $P = \underline{P}$ implies $\tilde{N} = 0$ and $\frac{d\tilde{N}}{dP} > 0$. □

Proof of Lemma 3: Suppose for now that all R&D firms choose innovation size γ, and we will prove in the end that $\hat{\gamma}(\psi) = P$. Since there are α firms in each licensing agreement, $\mu = \gamma^\alpha$ in all high-technology industries . Given the markup μ, market profits in all high-technology industries are given by π (see equation (8). Given s$^\alpha$ and π, a successful firm receives flow payoff $s_1^\alpha\pi$ until the first subsequent innovation, then flow payoff $s_2^\alpha\pi$ until the second subsequent innovation, and so on. Since the reward to success V is the discounted value of this payoff stream, if the industry arrival rate is ϕ then $V = \pi\sum_{i=1}^{\alpha} s_i^\alpha \frac{\phi^{i-1}}{(\rho+\phi)^i} = \pi \cdot B(\phi, \alpha)$ (see the calculations in footnote 8).

It remains to prove that firms will choose innovation size $\hat{\gamma}(\psi) = P$. Consider an individual R&D firm's choice of γ when all other firms choose innovation size $\gamma' \geq 1$. If this firm has a success of size γ, then the industry markup while the firm is part of the bargaining group will be $\mu = \Gamma\gamma$, where $\Gamma = (\gamma')^{\alpha-1} \geq 1$. The R&D firm therefore has instantaneous payoff

$$-wn + \lambda(\gamma)nLY\left(\frac{\Gamma\gamma - 1}{\Gamma\gamma}\right)B(\phi, \alpha)$$

where the firm takes L, Y, w, Γ, and $B(\phi, \alpha)$ as given and chooses γ and n subject to $\gamma \geq P$. Since $P \geq \gamma_o$, $\lambda(\gamma)\left(\frac{\gamma-1}{\gamma}\right)$ is decreasing at $\gamma = P$. It is then straightforward to show that $\Gamma \geq 1$ implies

$\lambda(\gamma)\left(\frac{\Gamma\gamma-1}{\Gamma\gamma}\right)$ is also decreasing at $\gamma = P$, and hence innovation size $\gamma = P$ is optimal (given the constraint $\gamma \geq P$). The result follows. □

Proof of Proposition 2: Define $\pi(\phi, \alpha) = (L - \frac{\bar{\omega}}{\lambda}\phi)\frac{P^\alpha - 1}{(1-\bar{\omega})P^\alpha + \bar{\omega}}$ and let $\bar{\phi} = \hat{\phi}(\psi)$, which implies $\lambda\pi(\bar{\phi}, \alpha(\psi)) \cdot B(\bar{\phi}, \alpha(\psi)) = w$. Using the logic from Appendix A, if $\lambda\pi(\bar{\phi}, \alpha(\psi')) \cdot B(\bar{\phi}, \alpha(\psi')) \geq \lambda\pi(\bar{\phi}, \alpha(\psi)) \cdot B(\bar{\phi}, \alpha(\psi)) = w$, then $\hat{\phi}(\psi') > \hat{\phi}(\psi)$.

(i) If $\alpha(\psi) = 1$ and $\alpha(\psi') = 2$, then $\hat{\phi}(\psi') > \hat{\phi}(\psi)$ if

$$(L - \frac{\bar{\omega}}{\lambda}\bar{\phi})\frac{(P^2 - 1)}{(1 - \bar{\omega})P^2 + \bar{\omega}}\frac{s\rho + \bar{\phi}}{(\rho + \bar{\phi})^2} \geq (L - \frac{\bar{\omega}}{\lambda}\bar{\phi})\frac{(P - 1)}{(1 - \bar{\omega})P + \bar{\omega}}\frac{1}{(\rho + \bar{\phi})}.$$

We can rewrite this inequality as $s \geq \Gamma - \frac{\bar{\phi}}{\rho}[1 - \Gamma] \equiv \bar{s}$, where $\Gamma \equiv \frac{1}{P+1}\frac{(1-\bar{\omega})P^2+\bar{\omega}}{(1-\bar{\omega})P+\bar{\omega}} < 1$. $\bar{s}$ is maximized at $\bar{\omega} = 0$, where $\bar{s} = \frac{P}{P+1} - \frac{\bar{\phi}}{\rho}\left[\frac{1}{P+1}\right] \leq \frac{P}{P+1}$.

(ii) Suppose $\alpha(\psi) = \alpha$ and $\alpha(\psi') = \alpha + 1$. Since $\frac{\phi^{\alpha-1}}{(\rho+\phi)^\alpha} \leq B(\phi, \alpha) \leq \frac{1}{\rho+\phi}$ for all α and ϕ, we must have $\hat{\phi}(\psi') > \hat{\phi}(\psi)$ if

$$(L - \frac{\bar{\omega}}{\lambda}\bar{\phi})\frac{(P^{\alpha+1} - 1)}{(1 - \bar{\omega})P^{\alpha+1} + \bar{\omega}}\frac{\bar{\phi}^\alpha}{(\rho + \bar{\phi})^{\alpha+1}} \geq (L - \frac{\bar{\omega}}{\lambda}\bar{\phi})\frac{(P^\alpha - 1)}{(1 - \bar{\omega})P^\alpha + \bar{\omega}}\frac{1}{(\rho + \bar{\phi})}.$$

We can rewrite this inequality as

$$\rho \leq \bar{\phi}\left[\left(\frac{(P^{\alpha+1} - 1)}{(P^\alpha - 1)}\frac{(1 - \bar{\omega})P^\alpha + \bar{\omega}}{(1 - \bar{\omega})P^{\alpha+1} + \bar{\omega}}\right)^{\frac{1}{\alpha}} - 1\right] \equiv \bar{\rho}.$$

It is straightforward to show $\bar{\rho} > 0$, and the result follows. □

Proof of Proposition 3: Appendix A shows that the equilibrium industry arrival rate $\hat{\phi}$ must satisfy the no-profit condition $B(\hat{\phi}, \alpha) = \frac{w}{\lambda\pi(\hat{\phi})}$, and moreover that if $B(\phi', \alpha) > \frac{w}{\lambda\pi(\phi')}$ then $\hat{\phi} > \phi'$. Since $B(\phi, \alpha)$ and λ are independent of $\bar{\omega}$, the result follows if $\pi(\phi)$ is increasing in $\bar{\omega}$. From Appendix A, $\pi(\phi) = (L - \frac{\bar{\omega}}{\lambda}\phi)\frac{\mu-1}{(1-\bar{\omega})\mu+\bar{\omega}}$, and differentiating yields $\frac{\partial\pi(\phi)}{\partial\bar{\omega}} = \frac{\mu-1}{[(1-\bar{\omega})\sigma+\bar{\omega}]^2}\left[(\mu - 1)L - \mu\frac{\phi}{\lambda}\right]$, so $\frac{\partial\pi(\phi)}{\partial\bar{\omega}} > 0$ if and only if $\phi < \frac{\mu-1}{\mu}\lambda L$. Since $B(\phi, \alpha) \leq \frac{1}{\rho+\phi}$ for all ϕ and α, we must have $\hat{\phi} < \bar{\phi}$ where $\bar{\phi}$ is defined by $\frac{1}{\rho+\bar{\phi}} = \frac{1}{\lambda\pi(\bar{\phi})}$. Solving this latter equation for $\bar{\phi}$ yields $\bar{\phi} = \frac{\mu-1}{\mu}\lambda L - \frac{(1-\bar{\omega})\mu+\bar{\omega}}{\mu}\rho < \frac{\mu-1}{\mu}\lambda L$, and the result follows. □

Proof of Proposition 4: Under any policy $\psi = (P, K)$ with $P \geq \gamma_o$ and $\alpha(\psi) = \alpha$, the markup $\mu = P^\alpha$ regardless of $\bar{\omega}$. Hence, the static inefficiency is increasing in $\bar{\omega}$ if and only if $\frac{\partial D}{\partial\bar{\omega}} > 0$, and the static inefficiency is decreasing in $\bar{\omega}$ if and only if $\frac{\partial D}{\partial\bar{\omega}} < 0$. Differentiating equation (11) yields

$$\frac{\partial D}{\partial\bar{\omega}} = \frac{\mu\ln\mu - (\mu - 1)(1 + \bar{\omega}\ln\mu)}{(1 - \bar{\omega})\mu + \bar{\omega}}.$$

$\frac{\partial D}{\partial \bar{\omega}} > 0$ if and only if $\bar{\omega} < \frac{\mu}{\mu-1} - \frac{1}{\ln \mu}$, and $\frac{\partial D}{\partial \bar{\omega}} < 0$ if and only if $\bar{\omega} > \frac{\mu}{\mu-1} - \frac{1}{\ln \mu}$. For any $\mu > 1$, we have $0 < \frac{\mu}{\mu-1} - \frac{1}{\ln \mu} < 1$, and the result follows. □

Proof of Proposition 5: Under any policy ψ such that $\bar{P} = \gamma \geq \gamma_o$ and $\alpha(\psi) = \alpha$, the markup will be $\mu = \gamma^\alpha$, regardless of $\bar{\omega}$. Let $W(\alpha, \bar{\omega})$ denote social welfare as a function of α and $\bar{\omega}$ taking as given that $\bar{P} = \gamma$. For any α, bargaining solution $s^\alpha \equiv (1, 0, ..., 0)$ implies $B(\phi, \alpha) = \frac{1}{\rho+\phi}$. The no-profit condition is then $\frac{1}{\rho+\phi} = \frac{1}{\lambda\pi(\phi)}$, which yields $N = \bar{\omega}\frac{\phi}{\lambda} = \bar{\omega}\left[\frac{\mu-1}{\mu}L - \frac{(1-\bar{\omega})\mu+\bar{\omega}}{\mu}\frac{\rho}{\lambda}\right]$. >From this, $L - N = \frac{(1-\bar{\omega})\mu+\bar{\omega}}{\mu}\left(L + \bar{\omega}\frac{\rho}{\lambda}\right)$, and substituting into equation (10) yields social welfare function

$$W(\alpha, \bar{\omega}) = \ln\left(\frac{L + \bar{\omega}\frac{\rho}{\lambda}}{La}\right) + \bar{\omega}\ln\left(\frac{1}{\mu}\right) + \frac{\ln \gamma}{\rho}\lambda\bar{\omega}\left[\frac{\mu-1}{\mu}L - \frac{(1-\bar{\omega})\mu + \bar{\omega}}{\mu}\frac{\rho}{\lambda}\right].$$

Note that in this equation, L, a, ρ, γ, and λ are all independent of α and $\bar{\omega}$. Hence, for $\alpha > \alpha'$ we have

$$W(\alpha, \bar{\omega}) - W(\alpha', \bar{\omega}) \quad = \quad \bar{\omega}\left[\ln\left(\frac{1}{\gamma^{\alpha-\alpha'}}\right) \quad + \quad \frac{\lambda \ln \gamma}{\rho}\left(\frac{\gamma^{\alpha-\alpha'} - 1}{\gamma^\alpha}\right)\left(L + \bar{\omega}\frac{\rho}{\lambda}\right)\right].$$

Since $\gamma^{\alpha-\alpha'} > 1$, we can conclude that for any $\alpha > \alpha'$, if $W(\alpha, \bar{\omega}') - W(\alpha', \bar{\omega}') \geq 0$ then $W(\alpha, \bar{\omega}) - W(\alpha', \bar{\omega}) > 0$ for all $\bar{\omega} > \bar{\omega}'$. The result follows. □

Proof of Proposition 6: Combining equations (16) and (17), in the private equilibrium we must have

$$\frac{V_1}{V_2} = \left(\frac{h_2}{h_1}\right)^{\frac{1-\beta}{\beta}}\left(\frac{\phi_1}{\phi_2}\right)^{\frac{1-\beta}{\beta}}.$$

Substituting $\phi_j = \lambda h_j^{1-\beta} n_j^\beta$, $n_j = 2N_j$, and $h_j = 2H_j$, and rearranging, this becomes

$$\frac{N_1}{N_2} = \frac{H_1}{H_2}\left(\frac{V_1}{V_2}\right)^{\frac{1}{1-\beta}}.$$

Equation (15) implies that under policy ψ_o, $\frac{V_1}{V_2} = \frac{\rho+\phi_2}{\rho+\phi_1}$. In Appendix A, we show that $h_1 > h_2$ implies $\phi_1 > \phi_2$, from which we can conclude $\frac{V_1}{V_2} < 1$. Since $\left[\frac{N_1}{N_2}\right]^* = \frac{H_1}{H_2}$, $V_1 < V_2$ implies $\frac{\hat{N}_1(\psi)}{\hat{N}_2(\psi)} < \left[\frac{N_1}{N_2}\right]^*$. □

Proof of Proposition 7: From the proof of Proposition 6, $\frac{N_1}{N_2} = \frac{H_1}{H_2}\left(\frac{V_1}{V_2}\right)^{\frac{1}{1-\beta}}$. Hence, $\frac{N_1}{N_2}$ is strictly increasing in $\frac{V_1}{V_2}$. If $s^\alpha = (1, 0, ..., 0)$ for each α, then for any patent policy we have

$$\frac{V_1}{V_2} = \frac{\rho + \phi_2}{\rho + \phi_1} = \frac{\rho + \frac{1}{z}\phi_1}{\rho + \phi_1}$$

where $z \equiv \frac{\phi_1}{\phi_2} > 1$. It is straightforward to show that $\frac{V_1}{V_2}$ is decreasing in both ϕ_1 and z.

Now suppose that either $\hat{\phi}_1(\psi) > \hat{\phi}_1(\psi')$ or $\hat{\phi}_2(\psi) > \hat{\phi}_2(\psi')$. The claim is that this implies $\frac{\hat{N}_1(\psi)}{\hat{N}_2(\psi)} < \frac{\hat{N}_1(\psi')}{\hat{N}_2(\psi')}$, which is equivalent to $z(\psi) < z(\psi')$. Suppose otherwise. First, note that $\hat{\phi}_2(\psi) > \hat{\phi}_2(\psi')$ and $z(\psi) \geq z(\psi')$ implies $\hat{\phi}_1(\psi) > \hat{\phi}_1(\psi')$. Hence, if either $\hat{\phi}_1(\psi) > \hat{\phi}_1(\psi')$ or $\hat{\phi}_2(\psi) > \hat{\phi}_2(\psi')$, and $z(\psi) \geq z(\psi')$, then $\frac{V_1}{V_2}$ must be smaller under policy ψ. But since $\frac{N_1}{N_2}$ is strictly increasing in $\frac{V_1}{V_2}$, this implies $z(\psi) < z(\psi')$, a contradiction. The result follows. □

References

Aghion, Philippe and Howitt, Peter (1992), "A Model of Growth through Creative Destruction," *Econometrica*, **60**, 323-351.

Aghion, Philippe and Howitt, Peter (1998), *Endogenous Growth Theory*, MIT Press, Cambridge.

Chang, Howard (1995), "Patent Scope, Antitrust Policy and Cumulative Innovation," *RAND Journal of Economics*, **26**, 34-57.

Cohen, W.M. and Levin, R.C.(1989), "Empirical Studies of Innovation and Market Structure," in R. Schmalensee and R.D. Willig, eds., *Handbook of Industrial Organization,* Elsevier, Amsterdam, 1059-1107.

Cornelli, Francesca and Schankerman, Mark (1995), "Optimal Patent Renewals," Mimeo, London Business School and London School of Economics.

Davidson, Carl and Segerstrom, Paul S. (1993), "Patent Enforcement and Economic Growth," FIEF Working Paper No.113, Trade Union Institute for Economic Research, Stockholm.

Gilbert, Richard, and Shapiro, Carl (1990), "Optimal Patent Length and Breadth," *RAND Journal of Economics*, **21**, 106-112.

Green, Jerry and Scotchmer, Suzanne (1995), "On the Division of Profit in Sequential Innovation," *RAND Journal of Economics*, **26**, 20-33.

Grossman, G.M. and Helpman, E. (1991), "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, **58**, 43-61.

Helpman, Elhanan (1993), "Innovation, Imitation, and Intellectual Property Rights," *Econometrica*, **61**, 1247-1280.

Hunt, Robert (1995), "Nonobviousness and the Incentive to Innovate: An Economic Analysis of Intellectual Property Reform," Mimeo, Congressional Budget Office and Department of Economics, University of Pennsylvania.

Jones, Charles I. (1995), "Time Series Tests of Endogenous Growth Models," *Quarterly Journal of Economics*, **110**, 495-525.

Jones, Charles I. and Williams, John C. (1996), "Too Much of a Good Thing? The Economics of Investment in R&D," Mimeo, Stanford University.

Klemperer, Paul (1990), "How Broad Should the Scope of Patent Protection Be?" *RAND Journal of Economics*, **21**, 113-130.

Klenow, Peter J. (1996), "Industry Innovation: Where and Why?" *Carnegie-Rochester Conference Series on Public Policy,* **44**, 125-150.

Kortum, Samuel (1997), "Research, Patenting, and Technological Change," *Econometrica*, **65**, 1389-1419.

Kremer, Michael (1997), "Patent Buyouts: A Mechanism for Encouraging Innovation," Mimeo, Harvard University.

Matutes, Carmen, Regibeau, Pierre, and Rockett, Katharine (1996), "Optimal Patent Design and the Diffusion of Innovations," *Rand Journal of Economics*, **27**, 60-83.

Nordhaus, William (1969), *Invention, Growth, and Welfare*, MIT Press, Cambridge, MA.

O'Donoghue, Ted (forthcoming), "A Patentability Requirement for Sequential Innovation," *RAND Journal of Economics*.

O'Donoghue, Ted (1996), "Licensing Cumulative Innovations: The Problem of Backloaded Payoffs," Mimeo, Cornell University.

O'Donoghue, Ted, Scotchmer, Suzanne, and Thisse, Jacques-Francois (1998), "Patent Breadth, Patent Life, and the Pace of Technological Progress," *Journal of Economics and Management Strategy*, **7**(1), 1-32.

Romer, Paul (1990), "Endogenous Technological Change," *Journal of Political Economy*, **98**, S71-102.

Scotchmer, Suzanne and Green, Jerry (1990), "Novelty and Disclosure in Patent Law," *Rand Journal of Economics*, **21**, 131-146.

Scotchmer, Suzanne (1991), "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law," *Journal of Economic Perspectives*, **5**, 29-41.

Scotchmer, Suzanne (1996a), "Protecting Early Innovators: Should Second-Generation Products be Patentable?" *Rand Journal of Economics*, **27**, 117-26.

Scotchmer, Suzanne (1996b), "Patents as an Incentive System," in B. Allen, ed., *Proceedings of the 1992 World Congress of the International Economic Association*, McMillan Press and St. Martin's Press, Cambridge, MA.

Scotchmer, Suzanne (1997), "On the Optimality of the Patent System," GSPP Working Paper #236, University of California at Berkeley.

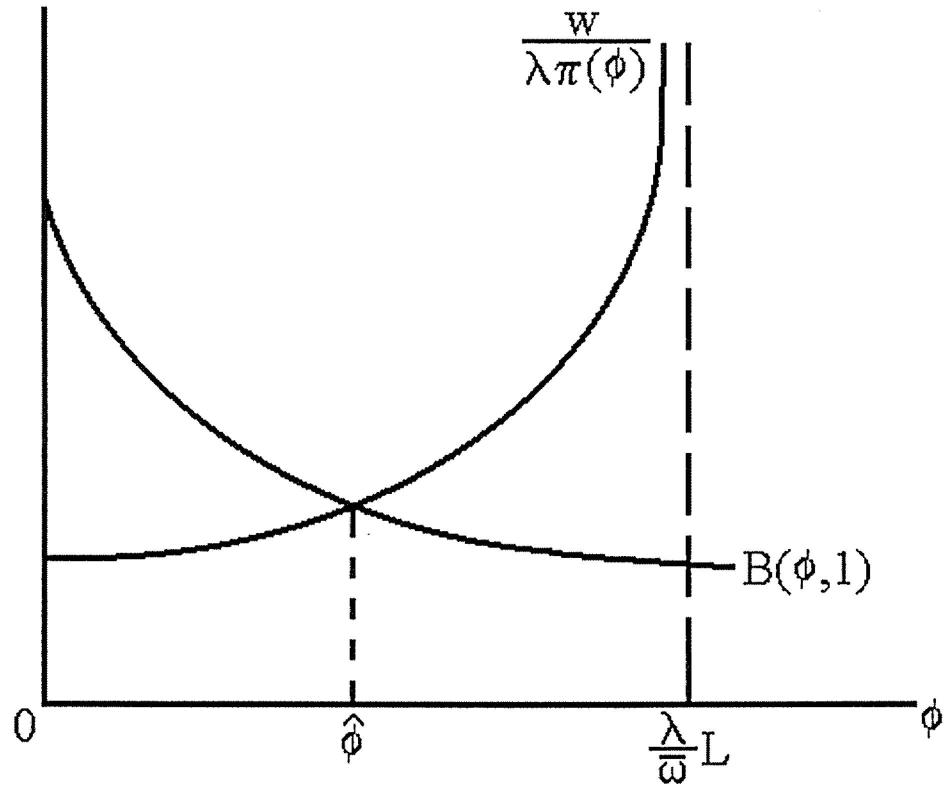
Segerstrom, Paul (1992), "On the Optimal Trade-Off Between Static and Dynamic Efficiency," Mimeo, Michigan State University.

Segerstrom, Paul, Anant, T.C.A., and Dinopoulos, Elias (1990), "A Schumpeterian Model of Product Life Cycle," *American Economic Review*, **80**, 1077-1091.

Stokey, Nancy (1995), "R&D and Economic Growth," *Review of Economic Studies*, **62**, 469-490.

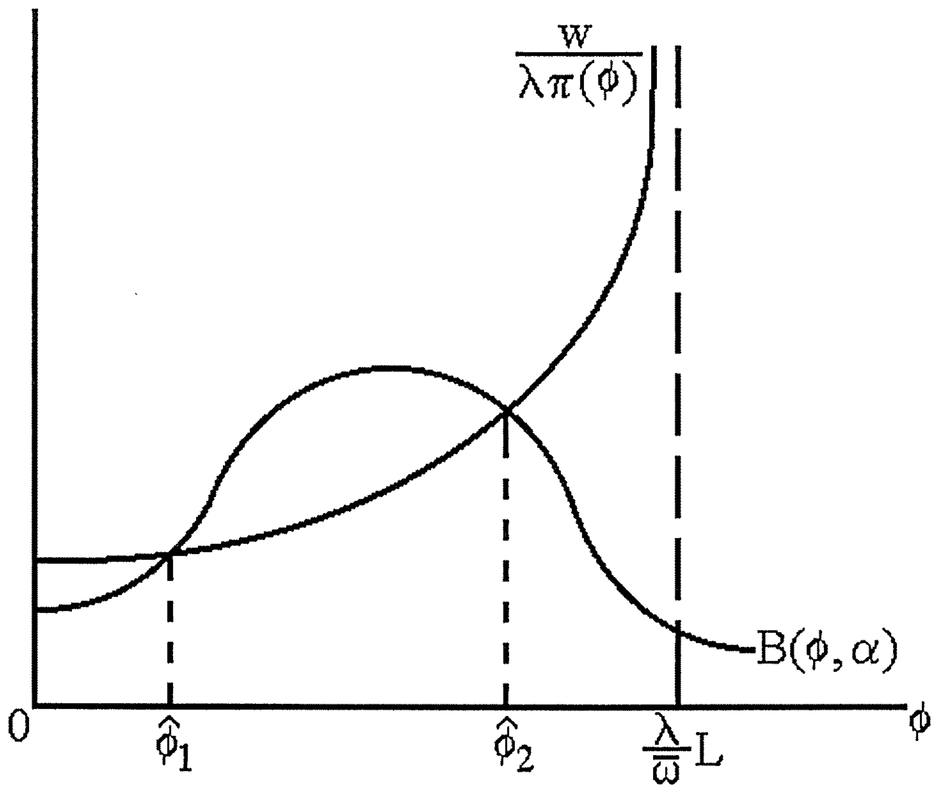
Van Dijk, Theon (1996), "Patent Height and Competition in Product Improvements," *Journal of Industrial Economics*, **44**, 151-167.

Wright, Brian (1983), "The Economics of Invention Incentives: Patents, Prizes, and Research Contracts," *American Economic Review*, **73**, 691-707.



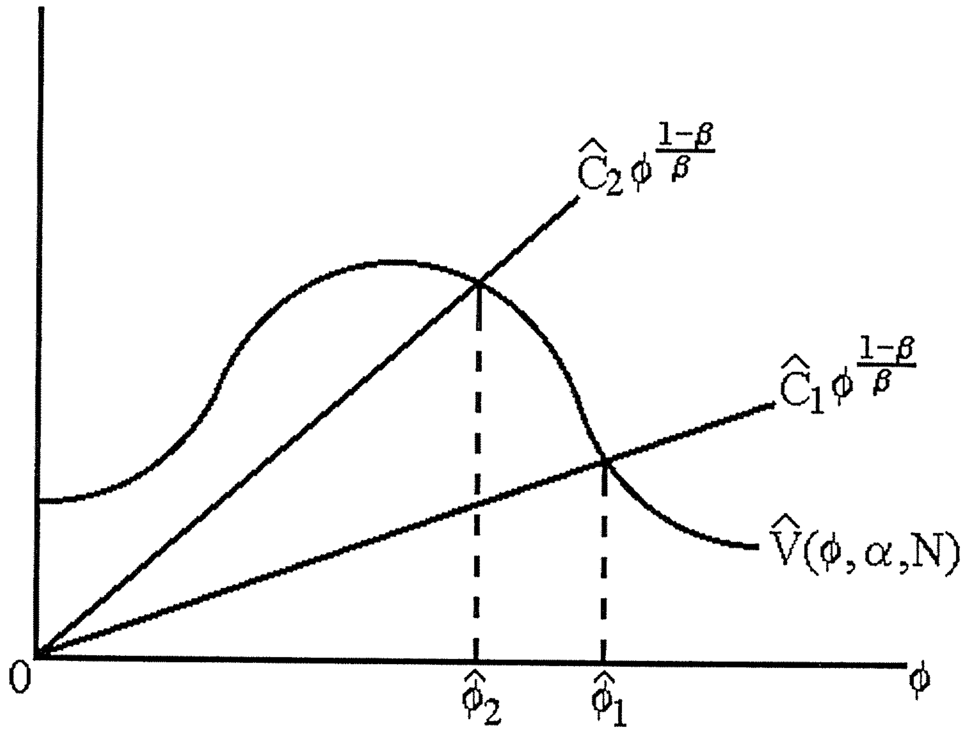
$\frac{w}{\lambda\pi(\phi)}$
$B(\phi,1)$
0
$\hat{\phi}$
$\frac{\lambda}{\bar{\omega}}L$
ϕ

Figure 1



$\frac{w}{\lambda\pi(\phi)}$
$B(\phi,\alpha)$
0
$\hat{\phi}_1$
$\hat{\phi}_2$
$\frac{\lambda}{\overline{\omega}}L$
ϕ

Figure 2



$\hat{C}_2 \phi^{\frac{1-\beta}{\beta}}$
$\hat{C}_1 \phi^{\frac{1-\beta}{\beta}}$
$\hat{V}(\phi, \alpha, N)$
0
$\hat{\phi}_2$
$\hat{\phi}_1$
ϕ

Figure 3

Institut für Höhere Studien
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna
Austria

Phone: +43-1-599 91-145
Fax: +43-1-599 91-163